PAPER

Vibroacoustic coupling of cylindrical enclosure with excited end plates

Atsushi Kojima^{1,*}, Hiroyuki Moriyama² and Yasuo Oshinoya²

¹*Course of Science and Technology, Graduate School of Tokai University,
4-1-1, Kitakaname, Hiratsuka, 259-1292 Japan*

²*Department of Prime Mover Engineering, Tokai University,
4-1-1, Kitakaname, Hiratsuka, 259-1292 Japan*

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Abstract: In the present paper, we describe vibroacoustic coupling between structural vibrations and the internal sound fields of thin structures. A cylindrical structure with thin plates at both ends is considered, and the coupling between the plate vibrations and the internal sound field is obtained when external harmonic forces are applied to both plates. This coupling is theoretically and experimentally investigated by considering the behavior of the plates and the acoustic characteristics of the internal sound field with variations in the harmonic forces. The acoustic characteristics are evaluated on the basis of the sound pressure level, which is maximized while changing the phase difference between the plate vibrations, when the phase difference and relative amplitude between the harmonic forces are varied. The behavior of the plate vibration is investigated by varying the phase difference with the cylinder length. Comparison of the characteristics of the systems reveals that vibroacoustic coupling is effective in increasing the acoustic energy and that the phase difference depends greatly on the acoustic mode, which contributes to the formation of the sound field. Moreover, the experiment of electricity generation verifies that vibroacoustic coupling is effective in the electricity generation system adopted in this investigation, if the coupling phenomena are promoted.

Keywords: Vibroacoustic coupling, Cylindrical enclosure, Structural vibration, Internal sound field, Phase difference

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1. INTRODUCTION

The efficient use of all sorts of energy that is dissipated into the environment is of great concern. The application of acoustic energy to various source of power, with the goal of efficiently using all forms of energy, has attracted a great deal of attention. Thermoacoustic engines, which use the inherently efficient Stirling cycle and are designed on the basis of a simple acoustic apparatus with no moving parts, are one such example. Thermoacoustic engines can achieve an efficiency of 30%, which is comparable to the efficiency of other heat engines [1]. Recently, electricity generation using resonance phenomena in the thermoacoustic engine has been investigated with the goal of extracting the work done in the engine. The acoustic energy spent on the generation of electricity is extracted from a resonance tube that branches out of the engine, and the appropriate position of the resonance tube for effectively generating

electricity is described [2]. Moreover, electricity generated from the sound of a voice via piezoelectric elements is developed in order for the voice to be a supplementary electric source for a mobile phone [3], and this system is being investigated for application in other devices [4]. Since acoustic energy is extremely small in comparison with vibration energy and the above electricity is generated by both sound and vibration systems, vibroacoustic coupling between both vibration and acoustic systems is one means of amplifying acoustic energy, although vibroacoustic coupling has not been investigated extensively in this field.

In an attempt to control noise in an airplane, Cheng and Nicolas investigated coupling between the sound field in an aircraft cabin and the vibrations of the rear pressure bulkhead [5,6]. They adopted a cylindrical structure as the analytical model, in which the rear pressure bulkhead at one end of the cylinder was assumed to be a circular plate. This analytical model was examined under a variety of conditions. The plate was supported at its edges by springs,

*e-mail: Obtad003@mail.tokai-u.jp

the stiffnesses of which could be adjusted to simulate various support conditions. These investigations clarified the influence of the support conditions on the sound pressure of an internal sound field coupled with the vibration of the end plate. They also found a frequency range in which an intense sound pressure level is generated. The authors used the above-mentioned analytical model [5,6] of a cylindrical structure with plates at both ends to investigate coupling based on the sound pressure level distribution in the cavity. The acoustic characteristics under vibroacoustic coupling were investigated for cases in which excitation forces with different relative amplitudes and phases were applied to both end plates [7]. In addition, the excitation frequency at which the coupling system becomes aperiodic owing to the application of excitation forces of different frequencies to the respective end plates has been investigated [8]. Finally, the effect of the excitation position with respect to nodal lines on the appearance of vibration modes on the plates has also been investigated [9].

In almost all such studies, vibroacoustic coupling was estimated by assuming that the plate and cavity dimensions and the phase difference between the vibroacoustics of the two plates were fixed. However, the eigenfrequencies of the plate and cavity vary with the dimensions of the plate and cavity and the phase difference directly affects the sound field when the medium is much less dense than the plate. Although a good grasp of coupling phenomena had attracted a great deal of attention, the parametric study has rarely been carried out for the above influence up to now, so that their influence on the coupling phenomena almost remains unknown.

In order to develop a new electricity generation system, we herein adopt an analytical model similar to the above-mentioned cylindrical structure with plates at both ends, because the vibration area of the model, on which piezoelectric elements can be installed, is twice as large as that in the case of a single plate. When both plates are excited by point forces and their dimensions are varied, the investigation reveals, theoretically and experimentally, that the excitation of both plates is useful for the promotion of vibroacoustic coupling and for increasing the amount of acoustic energy. In particular, the phase difference between the vibrations of the two plates is considered to be a significant characteristic of vibroacoustic coupling. Furthermore, the effect of vibroacoustic coupling is estimated from the experiment of electricity generation performed with piezoelectric elements.

2. ANALYTICAL METHOD

2.1. Equation of Plate Motion

The analytical model considered herein consists of a cavity with two circular end plates, as shown in Fig. 1.

Fig. 1 Configuration of the analytical model.

The plates are supported by translational and rotational springs distributed at constant intervals and the support conditions are determined by their respective spring stiffnesses, T_1 and T_2 and R_1 and R_2 , where the suffixes 1 and 2 indicate plates 1 and 2, respectively. The radius a and thickness h of plates having Young's modulus E and Poisson ratio ν are fixed. Assumed to be cylindrical, the sound field has the same radius as the plates, and its length is varied since the resonance frequency depends on the length. The boundary conditions are considered to be structurally and acoustically rigid at the lateral wall between the structure and the sound field. The coordinates are the radius r , the angle θ between the planes of the plates and the cross-sectional plane of the cavity, and the distance z along the cylinder axis. The harmonic point forces F_1 and F_2 are applied to the respective plates at $r_1 = r_2$ and $\theta_1 = \theta_2 = 0$ deg.

The flexural displacements w_1 and w_2 of plates 1 and 2 are expressed by substituting Eq. (2) for the plate mode shape into Eq. (1) and are expanded over two sets of suitable trial functions [5]:

$$\left. \begin{aligned} w_1 &= \sum_{s=0}^1 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} B_{1nm}^s X_{nm}^s e^{j\phi_1}, \\ w_2 &= \sum_{s=0}^1 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} B_{2nm}^s X_{nm}^s e^{j\phi_2}, \end{aligned} \right\} \quad (1)$$

$$X_{nm}^s = \sin(n\theta + s\pi/2)(r/a)^m, \quad (2)$$

where n , m , and s are, respectively, the circumferential order, the radial order, and the symmetry index with respect to the plate vibration, B_{1nm}^s and B_{2nm}^s are the coefficients to be determined, and ϕ_1 and ϕ_2 are the phases of the respective plate vibrations. In the present analysis, ϕ_1 is set to 0 deg, and ϕ_2 varies in the range of 0 to 180 deg. In this analysis, because coupling phenomena are assumed to be harmonic and to behave as $e^{j\omega t}$, in which ω is the angular frequency of the harmonic point forces on the plates and t is the elapsed time, we can remove $e^{j\omega t}$. The equations of plate motion are obtained by finding the extremum of Hamilton's function in terms of Eq. (1), as follows [5,6]:

$$\left[\sum_{m'=0}^{\infty} \{K_{1nm m'}^s (1 + j\eta_p) - \omega^2 M_{1nm m'}^s\} + \sum_{m'=0}^{\infty} aF_{sn} \left\{ T_1 + \left(\frac{m}{a}\right) \left(\frac{m'}{a}\right) R_1 \right\} \right] \mathbf{B}_{1nm}^s e^{j\phi_1} = \mathbf{F}_{1nm}^s - \mathbf{P}_{1nm}^s, \quad (3)$$

$$\left[\sum_{m'=0}^{\infty} \{K_{2nm m'}^s (1 + j\eta_p) - \omega^2 M_{2nm m'}^s\} + \sum_{m'=0}^{\infty} aF_{sn} \left\{ T_2 + \left(\frac{m}{a}\right) \left(\frac{m'}{a}\right) R_2 \right\} \right] \mathbf{B}_{2nm}^s e^{j\phi_2} = -\mathbf{F}_{2nm}^s + \mathbf{P}_{2nm}^s, \quad (4)$$

where $K_{1nm m'}^s$ and $K_{2nm m'}^s$ and $M_{1nm m'}^s$, and $M_{2nm m'}^s$ are components of the symmetrical stiffness and mass matrices, respectively, because the index m' is the radial order ($m = m'$), η_p is the structural damping factor of the plate, and F_{sn}^s is a coefficient that is determined by the indices n and s [5].

The first terms on the right-hand sides of Eqs. (3) and (4) are point forces, and the second terms are the acoustic excitation, which also functions as the coupling term between each plate vibration and the sound field. These point force and acoustic excitation terms are, respectively,

$$\left. \begin{aligned} F_{1nm}^s &= \int_{A_1} F_1 \delta(r - r_1) \delta(\theta - \theta_1) X_{nm}^s dA_1, \\ F_{2nm}^s &= \int_{A_2} F_2 \delta(r - r_2) \delta(\theta - \theta_2) X_{nm}^s dA_2, \end{aligned} \right\} \quad (5)$$

$$P_{1nm}^s = \int_{A_1} P_c X_{nm}^s dA_1, \quad P_{2nm}^s = \int_{A_2} P_c X_{nm}^s dA_2, \quad (6)$$

where δ is the delta function that represents point forces on the plates, A_1 and A_2 are the areas of the plates, and P_c is the sound pressure at an arbitrary point on the boundary surface with the plates.

2.2. Coupling Equations between Plate Vibrations and Internal Sound Field

The sound field must be assumed to be cylindrical in this investigation, so that the acoustic modal shape Y_{npq}^s and the angular resonance frequency ω_{npq} in the cavity (where the indexes n , p , and q indicate the circumferential, radial, and longitudinal orders, respectively) are defined as

$$Y_{npq}^s = \sin(n\theta + s\pi/2) J_n(\lambda_{np} r) \cos\{(q\pi/L)z\}, \quad (7)$$

$$\omega_{npq} = c\{\lambda_{np}^2 + (q\pi/L)^2\}^{1/2}, \quad (8)$$

where J_n is the n th-order Bessel function and λ_{np} is the p th solution of an eigenvalue function for a circular sound field having modes (n, p) divided by the radius. Here, Green's function G is employed to predict the sound pressure in a cavity, where the plate vibrations are regarded as the

source. The measurement point is $p = (r, \theta, z)$ in the cavity when the source points are $p_1 = (r_1, \theta_1, z_1)$ and $p_2 = (r_2, \theta_2, z_2)$ on plates 1 and 2, respectively. Moreover, if $\mathbf{u}$ is the unit normal of the boundary surface (positive toward the outside), then $\partial P_c / \partial \mathbf{u}$ is 0 on the lateral wall of the cylinder because the wall remains rigid. On the other hand, the spatial factor $P_c(\mathbf{p})$ of the sound pressure within and on the surface bounding the medium can be obtained only from the surface integral terms, as follows:

$$P_c(\mathbf{p}) = - \int_{A_1} G(\mathbf{p}, \mathbf{p}_1) \frac{\partial P_c(\mathbf{p}_1)}{\partial \mathbf{u}_1} dA_1 - \int_{A_2} G(\mathbf{p}, \mathbf{p}_2) \frac{\partial P_c(\mathbf{p}_2)}{\partial \mathbf{u}_2} dA_2, \quad (9)$$

where subscripts 1 and 2 indicate differentiation and integration with respect to (r_1, θ_1, z_1) and (r_2, θ_2, z_2) coordinates. A detailed procedure for obtaining these equations is given in Ref. [10]. The boundary conditions between the plate vibrations and the sound field on the respective plate surfaces are found by assuming the continuity of velocities on the plates:

$$\frac{\partial P_c}{\partial \mathbf{u}_1} = \rho_c \omega^2 w_1, \quad \frac{\partial P_c}{\partial \mathbf{u}_2} = -\rho_c \omega^2 w_2, \quad (10)$$

where ρ_c is the fluid density in the cavity. Applying Eq. (10) to Eq. (9), since the analytical mode has two boundary surfaces, P_c becomes

$$P_c = - \int_{A_1} G \rho_c \omega^2 w_1 dA_1 + \int_{A_2} G \rho_c \omega^2 w_2 dA_2. \quad (11)$$

In addition, P_c can also be expressed as follows [5,6]:

$$P_c = \rho_c c^2 \sum_{s=0}^1 \sum_{n=0}^{\infty} \sum_{p=1}^{\infty} \sum_{q=0}^{\infty} \frac{P_{npq}^s Y_{npq}^s}{M_{npq}^s}, \quad (12)$$

where P_{npq}^s is the pressure coefficient that is to be determined and the modal generalized mass M_{npq}^s is the mean value of $Y_{npq}^s{}^2$ averaged over the cavity volume V_c .

The equation that relates Eqs. (11) and (12) is obtained for an arbitrary acoustic mode (n, p, q) [5,10]:

$$(\omega_{npq}^2 - \omega^2) P_{npq}^s = - \frac{\omega^2}{V_c} \left(\int_{A_1} Y_{npq}^s w_1 dA_1 - \int_{A_2} Y_{npq}^s w_2 dA_2 \right). \quad (13)$$

Substituting Eq. (1) for w_1 and w_2 and considering a modal damping factor η_c , Eq. (13) can be rewritten as follows:

$$(\omega_{npq}^2 + j\eta_c \omega_{npq} \omega - \omega^2) P_{npq}^s = \frac{A\omega^2}{V_c} \left(- \sum_{m=0}^{\infty} I_1 B_{1nm}^s e^{j\phi_1} + \sum_{m=0}^{\infty} I_2 B_{2nm}^s e^{j\phi_2} \right), \quad (14)$$

$$I_1 = \frac{1}{A} \int_{A_1} X_{nm}^s Y_{npq}^s dA_1, \quad I_2 = \frac{1}{A} \int_{A_2} X_{nm}^s Y_{npq}^s dA_2, \quad (15)$$

where A is the total surface area of the plates and I_1 and I_2 are the spatial coupling coefficients. Moreover, substituting Eq. (12) for P_c and applying I_1 and I_2 to the integrals in Eq. (6), the acoustic excitation terms P_{1nm}^s and P_{2nm}^s can be expressed, with respect to an arbitrary vibration mode (n, m) , as

$$\left. \begin{aligned} P_{1nm}^s &= \rho_c c^2 A \sum_{p=1}^{\infty} \sum_{q=0}^{\infty} \frac{I_1 P_{npq}^s}{M_{npq}^s}, \\ P_{2nm}^s &= \rho_c c^2 A \sum_{p=1}^{\infty} \sum_{q=0}^{\infty} \frac{I_2 P_{npq}^s}{M_{npq}^s}. \end{aligned} \right\} \quad (16)$$

Finally, substituting P_{npq}^s of Eq. (14) into Eq. (16) and substituting P_{1nm}^s and P_{2nm}^s into Eqs. (3) and (4), we obtain the coupling equations, the right-hand sides of which are as follows:

$$F_{1nm}^s - P_{1nm}^s = F_{1nm}^s + \frac{\rho_c c^2 \omega^2 A^2}{V_c} \sum_{m'=0}^{\infty} \sum_{p=1}^{\infty} \sum_{q=0}^{\infty} \times \frac{I_1(I_1 B_{1nm'}^s e^{j\phi_1} - I_2 B_{2nm'}^s e^{j\phi_2})}{M_{npq}^s (\omega_{npq}^2 + j\eta_c \omega_{npq} \omega - \omega^2)}, \quad (17)$$

$$-F_{2nm}^s + P_{2nm}^s = -F_{2nm}^s - \frac{\rho_c c^2 \omega^2 A^2}{V_c} \sum_{m'=0}^{\infty} \sum_{p=1}^{\infty} \sum_{q=0}^{\infty} \times \frac{I_2(I_1 B_{1nm'}^s e^{j\phi_1} - I_2 B_{2nm'}^s e^{j\phi_2})}{M_{npq}^s (\omega_{npq}^2 + j\eta_c \omega_{npq} \omega - \omega^2)}. \quad (18)$$

The second terms on the right-hand sides of Eqs. (17) and (18) represent the acoustic excitations for plates 1 and 2, respectively. The acoustic excitation terms contain both I_1 and I_2 because the acoustic mode of the sound field is coupled to the vibration modes of the respective plates. Before the actual calculation, the natural frequency of the plate is always considered from the viewpoint of convergence for the plate vibration mode (n, m) . In this case, the natural frequency is solved as the eigenvalue of Eq. (3) or (4), the right-hand side of which is set to be 0 under the assumption of free vibration. The actual calculation is performed by taking 15 terms for n , while m is set to be greater than 12 in order to ensure convergence to the natural frequency and to the modal shape of the plate vibration. Using the same truncation for p as for m , the order accounts for acoustic modes greater than $q = 15$, so that the resonance frequency containing q can be more than the excitation frequency. The plate and cavity loss factors are assumed to be constant: $\eta_p = \eta_c = 0.01$ [5,6]. Since $B_{1nm'}^s$ and $B_{2nm'}^s$ can be obtained from the simultaneous equations in Eqs. (3) and (4), which include Eqs. (17) and (18), respectively, as excitation terms, the behavior of the plate vibrations and the sound field under vibroacoustic coupling can be determined. In this investigation, the phases ϕ_1 and ϕ_2 of the respective plate vibrations, which are considered to be influencing factors on the coupling phenomena as well as the cylinder

length and the point force, are related by the phase difference ϕ :

$$\phi = \phi_2 - \phi_1. \quad (19)$$

The sound pressure P_c is obtained by substituting P_{npq}^s , which is obtained from Eq. (14) using $B_{1nm'}^s$ and $B_{2nm'}^s$ as determined above, into Eq. (12). The square of the sound pressure P_v^2 averaged over the entire sound field is defined, and the sound pressure level L_{pv} expressed logarithmically relative to $P_0 = 2 \times 10^{-5}$ Pa is obtained as follows:

$$P_v^2 = \frac{1}{2V_c} \int_{V_c} P_c P_c^* dV_c, \quad L_{pv} = 10 \log \left(\frac{P_v}{P_0} \right)^2, \quad (20)$$

where P_c^* is the conjugate component.

3. THEORETICAL RESULTS

3.1. Effect of Excitation of Both Plates on Acoustic Characteristics

In this investigation, the plates are assumed to be made of aluminum having a Young's modulus E of 71 GPa, a Poisson's ratio ν of 0.33, a radius a of 150 mm, and a thickness h of 3 mm. The length of the cylindrical sound field having the same radius as that of the plates varies in the range 100 to 2,000 mm. The support conditions of the plates, which have flexural rigidity D [$= Eh^3 / \{12(1 - \nu^2)\}$], are expressed by the nondimensional stiffness parameters T_n ($= T_1 a^3 / D = T_2 a^3 / D$) and R_n ($= R_1 a / D = R_2 a / D$). In the present study, these parameters are identical for both plates. Both R_n and T_n are equal to 10^8 . Therefore, the support condition is regarded as clamped support. The natural frequency of the plate corresponding to the (n, m) mode is expressed as f_{nm} and is regarded as the excitation frequency. In order to simplify this vibroacoustic problem, only the $(0, 0)$ mode is taken into consideration. The point forces F_1 and F_2 are applied to the respective plates at $r_1/a = r_2/a = 0.4$ and their magnitude is represented by the excitation ratio $F_R = F_2/F_1$ obtained by varying F_2 when F_1 is set to 1 N. The phase difference ϕ_2 ranges from 0 to 180 deg while ϕ_1 remains constant at 0 deg, so that the phase difference ϕ between the two plate vibrations varies in the range of 0 to 180 deg. The resonance frequency of the cylindrical sound field is denoted as f_{npq} (i.e., the natural frequency corresponding to the (n, p, q) mode).

Figure 2 shows the changes in the sound pressure level L_{pv} averaged over the entire sound field when the plates are excited by the natural frequency, $f_{00} = 340$ Hz, and the cylinder length L is varied between 100 and 2,000 mm. The maximized L_{pv} is obtained when ϕ is varied between 0 and 180 deg and is contrasted between the two methods of excitation, in which the cases of exciting one plate and both plates are examined, in order to confirm the amplification

Fig. 2 Variation in maximized sound pressure level with cylinder length when one end plate and both end plates are excited.

effect of the acoustic energy by the excitation of both plates. Note that L_{pv} varies substantially and exhibits peaks in the vicinity of $L = 510, 1,010$, and $1,520$ mm in both cases, and, over the entire range of L , L_{pv} for the case in which both plates are excited is greater than L_{pv} when one plate is excited. The acoustic mode $(0,0,q)$ causes L_{pv} to have peaks at $L = 510, 1,010$, and $1,520$ mm, since the modal shape on a cross-sectional plane of the acoustic mode is similar to that of the $(0,0)$ mode of the plate vibrations. These peaks occur at integer intervals of q , starting from $q = 1$, with increasing L .

Figures 3(a) and 3(b) show the variations in L_{pv} as ϕ ranges from 0 to 180 deg. These variations differ between changing F_R and L , respectively, because the value of L_{pv} depends on F_R and the peaks are derived from acoustic modes of different longitudinal order. Figure 3(a) shows the variations in L_{pv} for various values of F_R at the first peak at $L = 510$ mm in Fig. 2. L_{pv} increases with ϕ and reaches a maximum, which is plotted by circles and occurs in the vicinity of 90 deg when $F_R = 0$. Here, the values of ϕ at which L_{pv} is maximum are denoted by ϕ_{max} . Although L_{pv} decreases with increasing ϕ beyond ϕ_{max} and exhibits a similar trend irrespective of the value of F_R , ϕ_{max} shifts to a lower value and L_{pv} increases slightly with F_R . Figure 3(b) shows the variations in L_{pv} with respect to ϕ for the first, second, and third peaks that appear at $L = 510, 1,010$, and $1,520$ mm, respectively, when $F_R = 1$. As shown in the figure, ϕ_{max} shifts to a value larger than 90 deg at the second peak and to a value smaller than 90 deg at the first and third peaks. As described above, since the order of L_{pv} peaks is identical to the longitudinal order q of the $(0,0,q)$ mode, even and odd values of q cause ϕ_{max} to increase and decrease, respectively. In the general resonance tube with both ends closed, since the sound pressures at the ends have opposite signs at odd orders and the same signs at even

(a) Change in excitation force ratio at $L = 510$ mm.

(b) Change in cylinder length at $F_R = 1.0$.

Fig. 3 Sound pressure level as a function of cylinder length.

orders, such a shift in ϕ_{max} contributes to the intensification of the sound field, i.e., this shift generates L_{pv} peaks at $L = 510, 1,010$, and $1,520$ mm.

The effect of F_R on the acoustic characteristics, having been estimated from incomplete results, has not yet been clarified and hence is examined for continuous changes in F_R in Figs. 4(a) and 4(b). Figure 4(a) shows ϕ_{max} as a function of F_R . Here, ϕ_{max} of $L = 510$ and $1,010$ mm (i.e., $q = 1$ and 2) is chosen because ϕ_{max} of $q = 1$ and 3 almost overlap. In addition, ϕ_{max} of $q = 1$ decreases with increasing F_R , and is 90 deg at $F_R = 0$, whereas this decrease is suppressed with increasing F_R and makes ϕ_{max} of $q = 1$ asymptotic in the vicinity of 70 deg. Moreover, ϕ_{max} of $q = 2$ has the opposite tendency to that of $q = 1$ and is approximately symmetric for $q = 1$ at approximately 90 deg. Figure 4(b) shows L_{pv} as a function of F_R , and L_{pv} is chosen for $L = 510, 1,010$, and $1,520$ mm (i.e., $q = 1, 2$, and 3). Here, L_{pv} increases with F_R over the entire range of F_R . The increase becomes proportional at around $F_R = 0.4$, below which the increase is suppressed, whereas changes in ϕ_{max} are facilitated. In the case of the excitation of one end ($F_R = 0$), the value of L_{pv} increases by 3 dB

(a) Phase difference based on maximized sound pressure.

(b) Maximized sound pressure level.

Fig. 4 Phase difference and sound pressure level as functions of cylinder length.

upon applying twice the vibration energy to plate 1 because of the acoustics. However, L_{pv} of $F_R = 1$ becomes approximately 3.7 dB larger than that of $F_R = 0$. This increase is caused by the enhanced efficiencies of energy transfer between the acoustic systems as a result of the excitation of both plates.

3.2. Phase Difference between Plate Vibrations under Vibroacoustic Coupling

Figure 5(a) shows the changes in $\phi_{\max}$ for $F_R = 0, 0.2, 0.4$, and 1 in order to clarify the effect of the excitation conditions on $\phi_{\max}$. For $F_R = 0$, $\phi_{\max}$ is approximately 83 deg at $L = 100$ mm and decreases gradually with increasing L up to approximately 380 mm, where $\phi_{\max}$ suddenly increases to over 90 deg and subsequently decreases with increasing L . This behavior of $\phi_{\max}$ is repeated in a similar manner as L increases to 2,000 mm. The abrupt changes in $\phi_{\max}$ are centered approximately on 90 deg. This repeated behavior is caused by the shifting of the acoustic mode, which contributes greatly to the formation of the sound field. The (0,0,1) mode affects the sound field between $L = 390$ and 740 mm, and the periodic

(a) Based on maximized sound pressure.

(b) Based on maximized flexural displacement.

Fig. 5 Variation in phase difference with cylinder length.

changes in $\phi_{\max}$, which occur between $L = 750$ and 1,290 mm and between $L = 1,300$ and 1,770 mm, are caused primarily by the (0,0,2) and (0,0,3) modes, respectively. By increasing F_R , $\phi_{\max}$ is made to deviate from the normal behavior, such as that for $F_R = 0$. As a result, $\phi_{\max}$ tends to shift toward the in-phase side ($\phi_{\max} = 0$ deg) in the range of L where the (0,0,1) and (0,0,3) modes affect the sound field at $F_R = 0$ and toward the out-of-phase side ($\phi_{\max} = 180$ deg) in the range of L where the (0,0,2) mode affects the sound field at $F_R = 0$.

Since F_R greatly affects the values of L_{pv} , as shown in Fig. 4(b), the magnitude of the point forces should be related to the values of the flexural displacements w_1 and w_2 ; hence the effects of w_1 and w_2 on L_{pv} should also be investigated. In Fig. 5(b), the phase differences are considered at $F_R = 1$ when w_1 and w_2 are maximized and are denoted as ϕ_{w1} and ϕ_{w2} ; $\phi_{\max}$ is also indicated in the figure. Moreover, the range of the dominant acoustic mode is classified according to $\phi_{\max}$ at $F_R = 0$ by color. Between $L = 100$ and 330 mm, ϕ_{w1} is constant at 180 deg and decreases abruptly to 0 deg at $L = 340$ mm. Then, ϕ_{w1} remains constant at 0 deg up to $L = 500$ mm and increases

gradually with L until reaching 180 deg at $L = 1,020$ mm with an increase in the rate of change in the vicinity of $L = 810$ mm. Beyond $L = 1,020$ mm, ϕ_{w1} is constant at 180 deg up to $L = 1,310$ mm, and this behavior follows the same trend as L increases to 2,000 mm. On the other hand, ϕ_{w2} exhibits gradual and abrupt changes, which are similar to the changes in ϕ_{w1} . However, these changes occur in the opposite range to those of ϕ_{w1} . For instance, a gradual decrease occurs between $L = 100$ and 510 mm and an abrupt increase occurs in the vicinity of $L = 810$ mm. Note that ϕ_{w1} and ϕ_{w2} shift between 0 and 180 deg with changing L and intersect at around 90 deg and near the length where L_{pv} shows a peak in Fig. 2. Here, ϕ_{w1} and ϕ_{w2} are very different from ϕ_{max} in the vicinity of L , where L_{pv} shows a peak, but are similar to ϕ_{max} with respect to the shifts between the in-phase and out-of-phase states, and the shifts of ϕ_{w1} and ϕ_{w2} are also slightly different from the shifts in the acoustic modes.

4. EXPERIMENT

4.1. Experimental Apparatus and Method

Figure 6 shows the experimental apparatus used in the present study. The cylindrical structure consists of a steel cylinder with circular aluminum end plates that are 3 mm thick. The cylinder has an inner diameter of 153 mm and a length that can be varied from 500 to 2,000 mm in order to emulate the analytical model. The harmonic point forces excite the end plates on both sides. These forces are applied to the respective plates via small vibrators, and their amplitudes are controlled to be 1 N. The positions of the point forces r_1 and r_2 are normalized by radius a and are set to $r_1/a = r_2/a = 0.4$. In this investigation, the (0,0) mode is used, and the main characteristic of the plate vibration under consideration is the phase difference between the plate vibrations. Therefore, acceleration sensors are installed on both plates to measure this phase difference. In order to estimate the internal acoustic characteristics, the sound pressure level in the cavity is measured using a condenser microphone with a probe tube, the tip of which is located in the vicinity of plate 2 and at the cylinder wall, since the sound pressure level is maximum at both end when the sound field resonates in the (0,0, q) mode.

Before conducting the excitation experiment, the natural frequency of the plate is measured by experimental modal analysis using an impulse hammer. The theoretical and experimental natural frequencies are confirmed to have the highest degree of similarity when rotational stiffness $R_n = 10^1$. Therefore, we can assume that the experimental support conditions are such that the translational and rotational stiffnesses are $T_n = 10^8$ and $R_n = 10^1$.

On the other hand, because electricity generation is considered as the application of this investigation, the

Fig. 6 Configuration of the experimental apparatus.

experiment is performed with piezoelectric elements that are installed at each center on not only both plates of the above structure but also both plates as the cylinder is removed from the above structure. The electric power generated by the piezoelectric elements is estimated from the relationship between the voltage of the electricity and the acceleration of the plate vibration. The effect of vibroacoustic coupling phenomena is expressed in the voltage with coupling normalized by the voltage without coupling that is extracted only from plate vibration excited by a point force.

4.2. Demonstration of Coupling Phenomena

Figure 7 shows the variations in L_{pv} corresponding to ϕ_{max} in the analysis and the variations in the sound pressure level L_p that is measured in the experiment and is maximum when the phase difference between the two point forces ranges from 0 to 180 deg. Peaks in L_{pv} appear at $L = 610, 1,230$, and $1,840$ mm. These peaks are known to be caused by the (0,0,1), (0,0,2), and (0,0,3) modes, respectively. Note that L_p increases greatly at 625, 1,250, and 1,850 mm and shows peaks at values similar to the peaks of L_{pv} . Figures 8(a) and 8(b) show L_{pv} and L_p as functions of ϕ . Also, L_{pv} and L_p of the principal and second peaks and in the middle range of their peaks are considered and occur at $L = 610, 810$, and $1,230$ mm and $L = 625, 950$, and $1,250$ mm, respectively. In the theoretical results shown in Fig. 8(a), L_{pv} of $L = 610$ and $1,230$ mm is maximized in the vicinities of $\phi = 70$ and 110 deg, i.e., ϕ_{max} of the principal and second peaks shifts to the in-phase and out-of-phase sides, respectively. Since the

Fig. 7 Variations in theoretical and experimental sound pressure levels with cylinder length at $F_R = 1.0$.

acoustic modes that dominate the sound field are in the process of shifting and do not exist in the vicinity of $L = 810$ mm, L_{pv} remains approximately constant for all values of ϕ . Figure 8(b) shows the experimental results for L_p corresponding to the above-mentioned L_{pv} . The changes in L_p with ϕ are distinguished by shifts to the in-phase and out-of-phase sides at $L = 625$ and 1,250 mm, and L_p is approximately constant for all values of ϕ at $L = 950$ mm. The behavior of L_p supports the validity of the theoretical results.

Here, in order to investigate the effect of the phase difference on the acoustic characteristics, the theoretical phase differences ϕ_{w1} and ϕ_{w2} , at which w_1 and w_2 , respectively, are maximized, are considered since the actual level L_p becomes maximum in the vicinity $\phi = 0$ and 180 deg, differing from the behavior of L_{pv} near those phase differences, and ϕ_{w1} and ϕ_{w2} shift between the in-phase and out-of-phase states, as shown in Fig. 5(b). Figure 9 shows the experimental phase difference ϕ_{exp} , at which L_p is maximum, is compared with ϕ_{w1} and ϕ_{w2} as functions of L . The shifts in the $(0,0,q)$ modes are also represented by the changing colors. ϕ_{w1} and ϕ_{w2} are also fixed at 0 and 180 deg and vary abruptly and gradually, respectively, between 0 and 180 deg, in the same manner as the theoretical phase differences at $R_n = 10^8$ shown in Fig. 5(b). However, the range of L , over which ϕ_{w1} or ϕ_{w2} behaves uniformly and changes gradually, and at which ϕ_{w1} or ϕ_{w2} changes abruptly, are different from the range and length at $R_n = 10^8$ shown in Fig. 5(b) because the natural frequency decreases with R_n . In addition, ϕ_{exp} shifts between 0 and 180 deg with changing L and corresponds approximately with ϕ_{w1} and ϕ_{w2} that behave uniformly in the vicinity of the in-phase side or the out-of-phase side. Being slightly different from ϕ_{w1} and ϕ_{w2} between $L = 900$ and 1,100 mm and between $L = 1,550$ and 1,700 mm, the abrupt changes in ϕ_{exp} correspond well with the shifts of

(a) Theoretical results.

(b) Experimental results.

Fig. 8 Sound pressure level as a function of phase difference for various cylinder lengths at $F_R = 1.0$.

Fig. 9 Theoretical and experimental phase differences at $F_R = 1.0$.

the $(0,0,q)$ modes. Moreover, small deviations in ϕ_{exp} from the in-phase side, which occur at $L = 625$ and 1,850 mm, at which L_p showed a peak in Fig. 7, reflects the theoretical results in Fig. 5, in which ϕ_{max} was close to 90 deg at $F_R = 1$ when L_{pv} showed a peak.

(a) Acceleration as a function of point force.

(b) Voltage as a function of acceleration.

Fig. 10 Relationship between voltage of electricity generation and plate vibration.

4.3. Effect of Coupling on Electricity Generation

The mechanical power supplied to the respective plates by each vibration can be estimated from the relationship between the point force and the flexural displacement at the excitation point. However, when measured at the respective centers of both plates and if closely connected with the flexural displacement, the acceleration is compared with the point force. Figure 10(a) shows variations in the acceleration α_1 and α_2 with the point force F_1 and F_2 in plates 1 and 2, respectively, as the cylinder is removed; the plate vibrations do not couple with the internal sound field. The acceleration should be closely related to the power supplied to each plate, being directly proportional to the respective point forces, whereas the difference between α_1 and α_2 is caused by the actual support conditions that depend on the precision of mechanical processing on the cylinder ends. In order to understand the circumstances of electricity generation by piezoelectric elements, Fig. 10(b) shows the voltages E_1 and E_2 generated by the respective plate vibrations as functions of acceleration. E_1 and E_2 are also directly proportional to α_1 and α_2 , respectively. These

Fig. 11 Effect of vibroacoustic coupling on electricity generation at $F_R = 1.0$.

results imply that these voltages are correlated with not only the mechanical power but also the electric power if both powers have a linear relationship. The effect of vibroacoustic coupling on electricity generation is represented by E_1 and E_2 measured in the experimental apparatus with cylinders shown in Fig. 6. These voltages are normalized by those generated by only the plate vibrations, and they are denoted by E_{R1} and E_{R2} , respectively. Figure 11 shows variations in E_{R1} and E_{R2} with L . Both E_{R1} and E_{R2} increase greatly at 650, 1,250, and 1,850 mm, at which the sound pressure level L_p showed a peak in Fig. 7, whereas they change around 1.0 in the wide range of L except the vicinities of the above L at which they peak, becoming lower than 1.0 in the range of length slightly shorter than the above L . These increases in E_{R1} and E_{R2} demonstrate the effect of vibroacoustic coupling on electricity generation. In other words, these results show that vibroacoustic coupling is ineffective in electricity generation and also obstructs, rather than promotes, electricity generation at shorter length than that at which generation is maximized.

5. CONCLUSIONS

In order to apply vibroacoustic coupling to electricity generation, as one means of extracting energy from vibration systems, vibroacoustic coupling between plate vibrations and a sound field was investigated theoretically and experimentally for a cylindrical structure with circular end plates excited by harmonic point forces. Moreover, the effect of vibroacoustic coupling on the extraction of energy was estimated from the experiment of electricity generation. The present study focused on the excitation ratio between the two point forces and the phase difference between the vibrations of the two plates in terms of the effect of coupling on the vibration characteristics of the plate.

A theoretical study revealed that the excitation of both ends causes the acoustic energy to increase efficiently, activating energy transfer between the vibration and acoustic systems, in comparison with the excitation of one end. If the effect of the phase difference is focused on the flexural displacements of both plates, the respective phase differences that maximize the flexural displacements shift between the in-phase state and the out-of-phase state upon changing the cylinder length. The experimental results demonstrate the above theoretical estimation in terms of the phase difference and correspond approximately to the theoretical results. When vibroacoustic coupling is promoted, the experiment of electricity generation verifies that this coupling causes the generation efficiency to increase in comparison with electricity generation caused only by the plate vibrations without coupling.

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