Noise shielding efficiency of edge-effect suppression barriers

Yasuhito Kawai*

Department of architecture, Faculty of environmental and Urban Engineering, Kansai University

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1. Introduction

The charts proposed by Maekawa and by Yamashita are widely used to obtain the insertion loss of a noise barrier for shielding such as road traffic noise and railway noise [1,2]. Though noise barriers are broadly used as a noise prevention measure for the sake of cost performance and the ease of installation, it sometimes is necessary to build a very high barrier to achieve the environmental standards. As a result, it causes problems such as landscape, inhibition of sunshine and rising cost due to reinforcement for wind resistance. To alleviate this situation, many kinds of edge-modified noise barriers are proposed [3,4]. They include sound absorption type, which traps sound propagating along the surface of the absorber, phase interference type, which cancels sound pressure at some frequency bands and their hybrid. The theoretical foothold of the edge-modified noise barrier is considered to be the representation of diffracted sound field by the line-integral along the barrier edge obtained from Kirchhoff's diffraction theory, in which Kirchhoff's boundary conditions [5] are assumed, via Maggi-Rubinowicz transformation [6]. Therefore, those barriers are designed with the idea of lowering velocity potential at the barrier edge, the integrand of the line integral, by means of sound absorption or phase interference equipment attached at the top of the noise barrier. In this paper, the author shows that the region of extremely large particle velocity amplitude, which is omitted in Kirchhoff's diffraction theory, appears in the vicinity of the barrier edge [7] and that suppressing such a large particle velocity yields lowering sound level in the diffracted field through the theoretical analysis using boundary integral equations.

2. Sound field by a semi-infinite thin rigid barrier

To begin with, we consider the sound field by a semi-infinite thin rigid barrier. Let S be the barrier and F the virtual surface which is the extension of S upwardly, as shown in Fig. 1. Then the space is divided into two semi-infinite regions Ω_1 and Ω_2 , one of which (Ω_1) contains a point sound source P_s . Let Φ_1 and Φ_2 be velocity potentials of sound wave in the two regions Ω_1 and Ω_2 due to the point source. In order to express the velocity potential Φ_1 at P , we introduce the principal solution of 3-D Helmholtz's operator,

$$G(P, Q) = \frac{\exp(ikr)}{4\pi r} + \frac{\exp(ikr_i)}{4\pi r_i}, \quad (1)$$

where P_i is the image point of P with respect to the plane $S + F$, points P and Q are located inside Ω_1 or on its boundary, $r = |\mathbf{r}| = |\overrightarrow{PQ}|$, $r_i = |\mathbf{r}_i| = |\overrightarrow{P_iQ}|$ (see Fig. 2). The image method is introduced here so that Φ_1 is expressed using only $\frac{\partial \Phi_1}{\partial n}$ distributing on the boundary F .

If we apply Green's identity to the region $\Omega_1 - \sigma - \sigma_s$ bounded externally by S , F and Σ of center P and infinite radius, and internally by the small spheres σ and σ_s , of centers P and P_s respectively and small radius ϵ , we obtain

$$\begin{aligned} \Phi_1(P) &= \Phi_D(P) + \Phi_D(P_i) - \iint_F \frac{\partial \Phi_1(Q)}{\partial n} G(P, Q) dS \\ &= \Phi_D(P) + \Phi_D(P_i) - \frac{1}{2\pi} \iint_F \frac{\partial \Phi_1(Q)}{\partial n} \frac{\exp(ikr)}{r} dS, \\ P &\in \Omega_1, S, F, \end{aligned} \quad (2)$$

where $\frac{\partial}{\partial n}$ denotes differentiation along the inward normal $\mathbf{n}$ to the boundary, $k = \frac{\omega}{c}$ wave number, ω angular frequency, c phase velocity and the time factor $\exp(-i\omega t)$ is omitted here. Equation (2) holds even where the point P lies on the boundary $S + F$ [8]. The image point P_i is introduced to counteract the integral over S and to restrict the integral over F . Also, the integral over Σ is omitted on the assumption of Sommerfeld's radiation condition [9].

Next, the relational expression in another semi-infinite region Ω_2 is derived in a similar way. Taking into account that no sound source exists and outward normal is taken, we have

$$\begin{aligned} \Phi_2(P) &= \iint_F \frac{\partial \Phi_2(Q)}{\partial n} G(P, Q) dS \\ &= \frac{1}{2\pi} \iint_F \frac{\partial \Phi_2(Q)}{\partial n} \frac{\exp(ikr)}{r} dS, \quad P \in \Omega_2, S, F. \end{aligned} \quad (3)$$

In Eqs. (2) and (3), when the point P lies on the boundary F , then P_i overlaps P and also the following relations $\Phi_1 = \Phi_2 \stackrel{\text{def}}{=} \Phi$ and $\frac{\partial \Phi_1}{\partial n} = \frac{\partial \Phi_2}{\partial n} \stackrel{\text{def}}{=} \frac{\partial \Phi}{\partial n}$ hold on F . Consequently, subtracting Eq. (3) from Eq. (2) yields

$$\frac{1}{\pi} \iint_F \frac{\partial \Phi(Q)}{\partial n} \frac{\exp(ikr)}{r} dS = 2\Phi_D(P), \quad P \in F. \quad (4)$$

Equation (4) is the boundary integral equation of the first kind with unknown function of $\frac{\partial \Phi}{\partial n}$ on F . Solving the integral equation (4) and substituting the solution into Eq. (3), we can obtain the diffracted sound field behind the barrier. Note that

*e-mail: kawai@kansai-u.ac.jp

Fig. 1 Sound diffraction by a semi-infinite thin rigid barrier.

Fig. 2 Derivation of the boundary integral equation for the semi-infinite region Ω_1 : Ω_1 is bounded by infinite sphere Σ of center P and plane boundary $S + F$; n denotes inward normal; a point source is located at P_s and a receiving point at P ; P_i denotes the image point of P with respect to the plane $S + F$; Q is a point inside Ω_1 or on $S + F$; σ_s and σ denote small spheres of centers P_s and P respectively.

from Eq. (3) the diffracted sound field is expressed by the normal derivative of velocity potential $\frac{\partial \Phi}{\partial n}$, which denotes particle velocity with the opposite sign, distributing on F .

As an example, under the condition as shown in Fig. 3, the distribution of particle velocity amplitude along the x -axis solved by the boundary element method (BEM) is compared with Kirchhoff's approximation of boundary value in Fig. 4.

In the implementation of numerical calculation, since the sound field is symmetrical to the x - z plane and infinite region cannot be treated numerically, the area $0 \leq x \leq 10\lambda$, $0 \leq y \leq 10\lambda$ (λ denotes wavelength) is taken into account. An omnidirectional point sound source with a unit velocity potential amplitude at 1 m from the source is also used. Figure 4 shows

Fig. 3 Calculation of the particle velocity amplitude above the barrier.

Fig. 4 Particle velocity amplitude along x -axis under the condition shown in Fig. 3; 125 Hz.

that the region of very large particle velocity amplitude exists in the vicinity of the barrier edge. The phenomenon that the incident sound yields very large particle velocity amplitude in the vicinity of the edge of a thin rigid plate is referred to as “edge effect” in this paper. The edge effect is caused by the sudden change of the sound pressure between the illuminated side and the other one at the edge of the plate. As mentioned previously, since the diffracted field is expressed by the contribution of particle velocity distributing on F above the barrier, it is suggested that suppressing the prominent particle velocity near the barrier edge using such as cloth or thin porous absorbing material having an appropriate flow resistance and a surface density will reduce the diffracted field.

3. Impedance of a thin porous absorbing material [10]

In the following, thin porous absorbing material is

Fig. 5 Impedance of cloth or thin absorbing layer.

referred to as cloth. As shown in Fig. 5, supposing that flow resistance and particle velocity relative to the cloth are denoted by r_s and v_s respectively, and sound pressures in front of and behind the cloth are denoted by p_1 and p_2 , then one obtains the following relation

$$v_s = -\frac{p_1 - p_2}{r_s}. \quad (5)$$

The cloth is also excited by the pressure difference $p_1 - p_2$. If the surface density of the cloth is denoted by M_s and the vibration velocity by v_m , then we have

$$-(p_1 - p_2) = M_s \frac{dv_m}{dt} = -i\omega M_s v_m. \quad (6)$$

Since the particle velocity at the surface v is expressed by the summation of v_s and v_m , we have

$$v = v_s + v_m = -\left(\frac{1}{r_s} - \frac{1}{i\omega M_s}\right)(p_1 - p_2). \quad (7)$$

If we write $-(p_1 - p_2)/v = Z_r$, Eq. (7) gives

$$Z_r = -\frac{p_1 - p_2}{v} = \left(\frac{1}{r_s} - \frac{1}{i\omega M_s}\right)^{-1}. \quad (8)$$

Therefore, we have

$$v = -\frac{p_1 - p_2}{Z_r} = \frac{i\omega\rho(\Phi_1 - \Phi_2)}{Z_r}, \quad (9)$$

where ρ denotes air density.

4. A semi-infinite barrier with a zonal cloth installed at the top

Let us consider a semi-infinite thin rigid barrier, along the top of which a zonal cloth is set to suppress the large particle velocity amplitude, as shown in Fig. 6. In the following, the author refers this type of barrier to as “edge-effect suppression barrier.” Supposing that the surface of the cloth is denoted by A , and since $\Phi_1 = \Phi_2 \stackrel{\text{def}}{=} \Phi$ on F and $\frac{\partial\Phi_1}{\partial n} = \frac{\partial\Phi_2}{\partial n} \stackrel{\text{def}}{=} \frac{\partial\Phi}{\partial n}$ on $F + A$, we obtain the following relation as before when P is located on F

$$\frac{1}{\pi} \iint_{F+A} \frac{\partial\Phi(Q)}{\partial n} \frac{\exp(ikr)}{r} dS = 2\Phi_D(P), \quad P \in F. \quad (10)$$

If P is located on A , using Eq. (9), we have

Fig. 6 A semi-infinite thin rigid barrier with a zonal cloth at the top.

Fig. 7 1/10 scale model experiment.

$$\begin{aligned} \Phi_1(P) - \Phi_2(P) + \frac{1}{\pi} \iint_{F+A} \frac{\partial\Phi(Q)}{\partial n} \frac{\exp(ikr)}{r} dS \\ = -\frac{Z_r}{i\omega\rho} \frac{\partial\Phi(P)}{\partial n} + \frac{1}{\pi} \iint_{F+A} \frac{\partial\Phi(Q)}{\partial n} \frac{\exp(ikr)}{r} dS \\ = 2\Phi_D(P), \quad P \in A. \end{aligned} \quad (11)$$

Solving Eqs. (10) and (11) simultaneously, one can obtain $\frac{\partial\Phi}{\partial n}$ on $A + F$. The diffracted sound field is obtained by using the following relation same as Eq. (3).

$$\Phi_2(P) = \frac{1}{2\pi} \iint_{F+A} \frac{\partial\Phi(Q)}{\partial n} \frac{\exp(ikr)}{r} dS, \quad P \in \Omega_2, S, F, A. \quad (12)$$

Figure 8 shows the result of 1/10 scale model experiment compared to that of theoretical calculation under the condition shown in Fig. 7. In the scale model experiment, the barrier is made of wooden panel with thickness of 9 mm at the top of which a zonal cloth with 50 mm wide is

Fig. 8 The results of experimental values compared with theoretical ones (converted to full-scale); for comparison, theoretical values calculated by BEM for semi-infinite barrier (no cloth) having the same height of the barrier with cloth is shown by the dashed line.

attached. Therefore, the total height of the barrier with cloth is 50 mm higher than that without cloth. The surface density and the flow resistance of the cloth used in the experiment, which is overlapped two pieces of curtain fabrics (overall thickness of about 3 mm), are $M_s = 0.66 \text{ kg/m}^2$ and $r_s = 789 \text{ Ns/m}^3$ respectively, and a time stretched pulse (TSP) signal is also used as a sound source. Figure 8 shows that the experimental and theoretical values are in good agreement. These results show that the effectiveness of cloth installed is considerably large.

5. Conclusions

Diffacted sound field by a semi-infinite thin rigid barrier is determined by the particle velocity distribution on a virtual plane that is the extension upward of the barrier, and that the region of large particle velocity amplitude appears near the top of the barrier, which is called the edge effect in this paper, is shown. By suppressing the particle velocity in the region where edge effects occur using porous materials such as a cloth with suitable flow resistance and surface density, that the diffracted sound is reduced is indicated by model experiments and numerical analysis.

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