Evaluation of numerical simulation of acoustic wave propagation using method of characteristics-based constrained interpolation profile (CIP-MOC) method with non-uniform grids

Yuta Matsumura1, Kan Okubo1*, Norio Tagawa1, Takao Tsuchiya2 and Takashi Ishizuka3

1Faculty of System Design, Tokyo Metropolitan University, 6–6 Asahigaoka, Hino, 191–0065 Japan
2Faculty of Information Systems Design, Doshisha University, 1–3 Tatara Miyakodani, Kyotanabe, 610–0394 Japan
3Shimizu Institute of Technology, 3–4–17 Etchujima, Koto-ku, Tokyo, 135–8530 Japan

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1. Introduction

As a result of computer development, numerical analysis of sound wave propagation in the time domain has been investigated widely. For high-performance sound field imaging and/or prediction, the development of accurate numerical schemes is an important issue [1–4]. A method of characteristics (MOC) [5] is used as a time domain numerical analysis method, examples of which are the constrained interpolation profile (CIP) method, the LAX method, and the QUICKEST method.

In this study, we examine MOCs using a collocated grid. These methods have an advantage that the treatment of the interface between different media is simpler than in the staggered-grid-based methods (e.g., finite difference time-domain method using Yee’s leapfrog algorithm [6,7]).

The CIP method [8–13], one of the MOCs, is a novel low-dispersive numerical scheme. In our past studies, we have applied the CIP method to numerical analyses of sound wave propagation. New grid systems, however, are required for large-scale simulations of wave propagation using the CIP method, because this method generates some numerical dissipation error [8].

In previous studies, subgrid techniques [14,15] were proposed for the CIP method to overcome these problems. However, handling the derivatives in the perpendicular directions at the interface between grids of different sizes is complicated in these techniques. Therefore, this study introduces a non-uniform grid system for the CIP method. We report the validity and advantages of the non-uniform grid system.

2. CIP-MOC method [8–13]

The governing equations for linear acoustic fields are given as

\[ \nabla \cdot \vec{u} = -\frac{1}{K} \frac{\partial p}{\partial t}, \]  
\[ \rho \frac{\partial \vec{u}}{\partial t} = -\nabla \cdot p. \]

where \( \rho \) denotes the density of the medium, \( K \) represents the bulk modulus, \( p \) is the sound pressure, and \( \vec{u} \) is the particle velocity. We assume a lossless medium for calculations. When we consider the \( x \)-direction in these formulas, we obtain

\[ \frac{\partial}{\partial t} p + c \frac{\partial}{\partial x} Z u_x = 0, \]  
\[ \frac{\partial}{\partial t} Z u_x + c \frac{\partial}{\partial x} p = 0. \]

In these equations, \( Z \) is the characteristic impedance (i.e., \( Z = \sqrt{\rho K} \)), \( c \) is the sound velocity in the medium (i.e., \( c = \sqrt{K/\rho} \)), and \( u_x \) is the \( x \)-direction component of the particle velocity.

In CIP analysis, these equations are transformed into the advection forms

\[ \frac{\partial}{\partial t} (p \pm Z u_x) \pm c \frac{\partial}{\partial x} (p \pm Z u_x) = 0. \]

This transformation process is used for the calculation of \( x \)-direction advection. In addition, through simple spatial differentiation of the equations, the equations of the derivatives are given as

\[ \frac{\partial}{\partial t} (\partial_x p \pm Z \partial_x u_x) \pm c \frac{\partial}{\partial x} (\partial_x p \pm Z \partial_x u_x) = 0. \]

We can calculate sound wave propagation by applying the CIP method to these equations. Note that we can calculate sound wave propagation in the \( y \)-direction in a similar manner [11,13,14].

In this study, the type-M and type-C CIP methods are applied. The difference between the type-M and type-C CIP
methods is the handling of the second-order spatial derivative [8,14]. The type-M CIP method is a simple technique with smaller memory use and less calculation time required than the type-C CIP method.

3. Non-uniform grids

Figure 1 shows a schematic of the non-uniform grid and sub-grid system [15]. The memory required for the non-uniform grid system is slightly larger than that for the sub-grid shown in the figure. However, handling the interface between grids of different sizes is more complicated in the sub-grid system owing to the treatment of the derivatives in the perpendicular direction. Here, $\Delta x$ and $\Delta y$ are the sizes in the fine grid, while $\Delta x_c$ and $\Delta y_c$ are those in the coarse grid.

4. Simulation results and discussions

Figure 2 shows the geometry of the calculation model. The simulation parameters are as follows: Direction of acoustic field propagation, $\pm x$ and $\pm y$ (2-D analysis); fine grid sizes, $\Delta x = 0.05$ m, $\Delta y = 0.05$ m; coarse grid size, $m \Delta x$ and $m \Delta y$, where $m$ is the ratio of coarse grid size to the fine size (i.e., $\Delta x_c = m \Delta x$). The initial spatial distribution of the sound pressure at $t = 0$ is given on discretized grid points by

$$p(x, y) = \exp\left(-\frac{(x-x_0)^2 + (y-y_0)^2}{2\sigma^2}\right).$$

where $x_0$ and $y_0$ are the coordinates of the center point of the sound source ($x_0, y_0 = 40$ m), and $\sigma = 20 \Delta x$. In addition, perfectly matched layer (PML) technique [16,17] is applied at the outer boundary of the analytical domain. Other calculation parameters used in the simulations are summarized in Table 1.

We present numerical results obtained using the non-uniform grid technique for a CIP-MOC analysis. Figure 3 shows the sound pressure distribution obtained by a type-C CIP-MOC simulation with non-uniform grids at $t = 0.0005$ s, $t = 0.05$ s, $t = 0.75$ s, and $t = 0.1$ s. The input pressure is driven from the center region of the fine grid. We can ascertain the propagation behavior including in the non-uniform grid region. Here, we confirm that the PML technique [17] can work efficiently for non-uniform grids of MOCs.

Figure 4 evaluates the error using the type-C and type-M CIP methods with a non-uniform grid by comparison of the absolute pressure at two points $(x, y) = (40$ m, $21$ m), $(53$ m, $27$ m)). Here, the two points are almost the same distance from the input point.

In Fig. 4, the turquoise and pink solid lines indicate the amplitude of the sound pressure against time with a non-uniform grid $(m = 2)$ using the type-C CIP method ($\{P_{C_{(m=2)}}\}$) and type-M CIP method ($\{P_{M_{(m=2)}}\}$), respectively, while the blue dotted line shows the difference between $P_{C_{(m=2)}}$ and $P_{C_{(m=1)}}$ ($\{|P_{C_{(m=2)}} - P_{C_{(m=1)}}|\}$) and the red dashed line shows $|P_{M_{(m=2)}} - P_{M_{(m=1)}}|$, where $P_{C_{(m=1)}}$ is sound pressure calculated using the type-C CIP-MOC method with a uniform fine grid (i.e., $m = 1$). We found that the interface between grids with different sizes in the non-uniform grids has good permeability characteristics with extremely low reflection and that the type-C CIP method has better performance than type-M CIP method.

We also investigated the calculation time required for some non-uniform grid models. Here, we used a PC with an Intel Core i7-980X Extreme Edition 3.33 GHz processor. This processor has 6 cores and 12 hyper-threaded cores, or effectively scales 12 threads. For all analyses, parallel computation using OpenMP was applied.

Figure 5 shows the calculation time of the type-C and type-M CIP methods, as well as the maximum numerical error vs. the ratio $m$. These results illustrate that the non-uniform grid $(m \neq 1)$ requires less calculation time and uses less memory than the fine grid $(m = 1)$. Regarding the ratio of the grid sizes, calculation time and numerical error are in a trade-off relationship.

Table 1 Calculation parameters used in the analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta x$</td>
<td>0.05 m</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>0.05 m</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>$5.0 \times 10^{-2}$ ms</td>
</tr>
<tr>
<td>$c$</td>
<td>340 m/s</td>
</tr>
<tr>
<td>$L$</td>
<td>32</td>
</tr>
<tr>
<td>$Z$</td>
<td>415.03 Pa·kg/m$^3$</td>
</tr>
</tbody>
</table>

5. Conclusions

Using the type-C and type-M CIP-MOC methods, we assessed non-uniform grid systems for the numerical simulation of sound wave propagation. The numerical results obtained by the type-C and type-M CIP methods with non-uniform grid techniques were compared for a two-dimensional acoustic field. It was revealed that the appropriate treatment of the interface between the coarse grid and fine grid in the non-uniform grid system results in extremely low reflection from the boundaries. The use of a suitable non-uniform grid reduces the calculation time and memory required.

References


