Doppler frequency estimation using maximum likelihood function for low ultrasonic velocity profile

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1. Introduction

In general, the circulating water system with accurate flowrate measurement is a necessary part in nuclear power plants because it can guarantee the stability of the flow-loop control. The important affects of flowrate measurement often reduce accuracy due to pipeline characters, for examples, very large diameter, bent pipes, and double bent pipes. These limitations lead to very complicated water flow after passing, and the conventional flowmeters cannot satisfy for this case. Ultrasonic velocity profile (UVP) method can provide accurate flowrate under the complicated flow as non-symmetrical flow by multiple sensors [1]. UVP is a method to obtain velocity profiles using ultrasonic wave without any installation of sensors inside the pipeline, and it does not need any calibration while the usage. Pulse ultrasound waves are sent from a transducer with a basic frequency f_0 passing through the streamline with an incident angle θ_w along a measurement line as shown in Fig. 1. Then, the ultrasonic pulses with shifted frequency f_d based on the Doppler principle are reflected back from the surface of particles flowing in the fluid, and they are returned to the original transducer installed on the pipeline's surface. The Doppler-shifted frequency is proportional to a component V_{TX} of the velocity V_{axial} along the measurement line of the particles, where c_f is a sound velocity in liquids.

$$V_{TX} = \frac{c_f}{2f_0} f_d \quad (1)$$

Thus, UVP can be obtained by only component along ultrasonic path. By assuming a one-directional flow parallel to the streamline direction, the velocity in the axial direction V_{axial} can be computed as Eq. (2):

$$V_{axial} = \frac{V_{TX}}{\sin \theta_f} \quad (2)$$

Takeda is the first person to apply the ultrasonic pulsed Doppler method into UVP [2]. The ultrasonic pulsed Doppler method can be computed from the multiple echo signals of pulse repetitions. The several analysis methods including Fast Fourier transform (FFT), the wavelet transform (WT), and the autocorrelation (AC) disturbed with effects of pulse repetitions and noise [3] are evaluated experimentally to achieve the Doppler frequency. The ultrasonic pulsed Doppler method based on the maximum likelihood estimation (MLE) is originally developed for use in blood velocity estimation [4]. However, the maximum likelihood estimation method is far from applying it to UVP. Therefore, this report is the first to apply this technique to the UVP method, and the experimental results can guarantee that MLE is possible on the low velocity. Not only the Doppler frequency is estimated using MLE, but an amplitude's Doppler echo can be obtained together as well. This ability is superiority when comparing with others which give us only the Doppler frequency measurement. The amplitude estimator leads to further study, for examples, bubble sizes moving in the pipeline, turbidity, and so on. Thus, MLE is an interesting choice of development involving UPV.

2. The maximum likelihood estimation inside a channel width of UPV measurement

An ultrasonic transducer transmits a short emission of ultrasound, which is moving along the measurement line, and then switches over to listening as shown in Fig. 2. While the ultrasonic pulse hits a tiny particle, a part of the ultrasonic energy scatters on particle and echo back, and then other parts of the ultrasonic energy enter into the next measuring volume of particle. Usually, the reflected echoes are recorded at every channel width of the measurement line. We can consider the observed data set $s[n]$ form the frequency demodulation referred in Fig. 2, including three unknown parameters as shown in Eq. (3):

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Fig. 1 Measurement scheme of the UVP measurement.

Fig. 2 Channel width of UVP on a flow with free surface on the several signal processing method including Fast Fourier transform, the autocorrelation, and the maximum likelihood estimation.

$$s[n] = A \sin(2\pi f_d n + \phi) + w[n] \quad n = 0, 1, \dots, N-1 \quad (3)$$

where A is amplitude, f_d is Doppler-shifted frequency, and ϕ is phase. If $w[n]$ is White Gaussian Noise with unknown variance, we first determine the Cramer-Rao Lower Bound (CRLB) to satisfy with equality. The probability density function (PDF) is

$$p(s; A, f_d, \phi) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp \left[-\frac{1}{2\sigma^2} \sum_{n=0}^{N-1} (s[n] - A \sin(2\pi f_d n + \phi))^2 \right] \quad (4)$$

where σ^2 is variance. To achieve the maximum likelihood of the PDF, we should have to minimize $\sum_{n=0}^{N-1} (s[n] - A \sin(2\pi f_d n + \phi))^2$. If assuming that $F(A, f_d, \phi) = \sum_{n=0}^{N-1} (s[n] - A \sin(2\pi f_d n + \phi))^2$, we reform

$$F(A, f_d, \phi) = \sum_{n=0}^{N-1} (s[n] - A(\cos \phi \sin(2\pi f_d n) + \sin \phi \cos(2\pi f_d n)))^2 \quad (5)$$

$$F(A, f_d, \phi) = \sum_{n=0}^{N-1} (s[n] - C_1 \sin(2\pi f_d n) - C_2 \cos(2\pi f_d n))^2 \quad (6)$$

where $C_1 = A \cos \phi$ and $C_2 = A \sin \phi$. Then, we take a square $C_1^2 = A^2 \cos^2 \phi$ and $C_2^2 = A^2 \sin^2 \phi$. We can get two unknown parameters A and ϕ

$$A = \sqrt{C_1^2 + C_2^2} \quad (7)$$

$$\phi = \tan^{-1} \frac{C_2}{C_1} \quad (8)$$

Again, we let this definition.

$$x[n] = [1 \quad \cos(2\pi f_d[1]) \quad \cos(2\pi f_d[2]) \dots \cos(2\pi f_d[N-1])]^T$$

$$y[n] = [0 \quad \sin(2\pi f_d[1]) \quad \sin(2\pi f_d[2]) \dots \sin(2\pi f_d[N-1])]^T$$

Replace C_1 , C_2 , $x[n]$ and $y[n]$ into Eq. (5).

$$F'(C_1, C_2, f_d) = \sum_{n=0}^{N-1} (s[n] - C_1 x[n] - C_2 y[n])^2$$

$$F'(C_1, C_2, f_d) = (s - H\Lambda)^T (s - H\Lambda) \quad (9)$$

where $s = [s[0] \quad s[1] \quad \dots \quad s[N-1]]^T$, $H = [y[n] \quad x[n]]$, and $\Lambda = [C_1 \quad C_2]^T$. We can estimate $\hat{\Lambda}$ by minimizing Eq. (9).

$$\hat{\Lambda} = [\hat{C}_1 \quad \hat{C}_2]^T = (H^T H)^{-1} H^T s[n] \quad (10)$$

Plugging Eq. (10) into Eq. (9).

$$F'(\hat{C}_1, \hat{C}_2, f_d) = s^T (I - H(H^T H)^{-1} H^T) s \quad (11)$$

I is an identity matrix and $I - H(H^T H)^{-1} H^T$ is an idempotent matrix. To find $\hat{f}_d$, we need minimizing F' over f_d or maximizing $s^T H(H^T H)^{-1} H^T s$. Therefore, the Doppler frequency is estimated according to Eq. (12).

$$\hat{f}_d = \max_{f_d \in f} (s^T H(H^T H)^{-1} H^T s) \quad (12)$$

The estimated frequency in Eq. (12) is replaced into Eq. (10) to estimate amplitude $\hat{A}$ and phase $\hat{\phi}$.

$$\hat{A} = \sqrt{\hat{C}_1^2 + \hat{C}_2^2} \quad (13)$$

$$\hat{\phi} = \tan^{-1} \frac{\hat{C}_2}{\hat{C}_1} \quad (14)$$

3. Performance analysis with the CRLB

The lowest possible variance of an unbiased estimator can be obtained using the CRLB. The CRLB is definitely equal to the inverse of the Fisher matrix defined as:

$$\text{var}(\hat{f}_d) \geq -\frac{1}{E \left[\frac{\partial^2 \ln p(s; A, f_d, \phi)}{\partial f_d^2} \right]} = \frac{12\sigma^2}{2\pi^2 A^2 N(N^2 - 1)} \quad (15)$$

$$\text{var}(\hat{A}) \geq -\frac{1}{E \left[\frac{\partial^2 \ln p(s; A, f_d, \phi)}{\partial A^2} \right]} = \frac{2\sigma^2}{N} \quad (16)$$

$$\text{var}(\hat{\phi}) \geq -\frac{1}{E \left[\frac{\partial^2 \ln p(s; A, f_d, \phi)}{\partial \phi^2} \right]} = \frac{8\sigma^2}{A^2 N} \quad (17)$$

where $E[\cdot]$ is the expectation. The CRLB evaluates intuitive

Table 1 Comparison of the CRLB and variance of estimators for 20.00 dB SNR.

Estimation	$\hat{f}_d$ (kHz)	$\hat{A}$ (V)	$\hat{\phi}$ (degree)
Actual parameter	5	1	0
20.00 dB SNR			
Mean	5.0027	0.9995	0.0198
Variance	1.5157×10^{-6}	5.7998×10^{-6}	0.0783
CRLB	3.0286×10^{-7}	4.9680×10^{-6}	1.987×10^{-5}

insights on the unknown parameters. In this case, the estimation focuses on only the Doppler-shifted frequency for the UVP. The variance of any reflected echo per a channel width depends on the SNR and sampling number. The higher SNR and sampling frequency improve the accuracy of parameter estimation. To assess the performance of estimators, a Monte-Carlo simulation was utilized to observe the means and the variances against the CRLB as shown in Table 1. 5 kHz of Doppler-shifted frequency added to a carrier frequency is equal to 0.8489 m/s of the particle's velocity. The Monte-Carlo simulation results were obtained using 100 trials of repeating with 100 MHz of sampling. Then, the searching method, because of no divergence, was applied to maximize the Doppler-shifted frequency from the minimum frequency 0 kHz up to 10 kHz with 1 Hz of resolution. The simulation results have been observed that the estimated Doppler-shifted frequency, and amplitude can reach a high degree of consistency with the actual parameters. In the case of the phase parameter, the estimator had much more variance than CRLB since the Doppler-shifted frequency and the amplitude parameters were estimated beforehand, and the variances from both the Doppler-shifted frequency and the amplitude were accumulated.

4. Experimental result in UVP

To evaluate MLE for UVP measurement, this section examines the accuracy of the Doppler-shifted frequency estimator, only Eq. (12), in a term of a low constant flowrate 15 liters per minute measured by an electromagnetic flowmeter of Yokogawa shown in Fig. 3, 50 millimeters of the pipe diameter, 45° of the incident angle, and 100 channels along to the measurement line. The velocity profile from MLE was compared with other signal processing methods (FFT and autocorrelation). The FFT and autocorrelation were used widely in the conventional Doppler frequency measurement [3]. The power spectrum of the frequency, $P(f_k)$, is demonstrated as follows:

$$P_f = (\text{Re}[X_L] - \text{Im}[X_Q])^2 + (\text{Re}[X_Q] + \text{Im}[X_L])^2 \quad (18)$$

$$P_b = (\text{Re}[X_L] + \text{Im}[X_Q])^2 + (\text{Re}[X_Q] - \text{Im}[X_L])^2 \quad (19)$$

where $X_L(f_k)$ and $X_Q(f_k)$ are the discrete Fourier transforms of the frequency demodulation, and P_f and P_b are power spectra of forward direction and backward direction, respectively. The value of f_d can be computed by averaging the spectrum in Eq. (20).

Fig. 3 Ultrasonic velocity profile measurement system and a standard electromagnetic flowmeter.

$$f_d = \frac{\sum_{k=0}^{N_{\text{pulse}}} f_k (P_f(f_k) - P_b(f_k))}{\sum_{k=0}^{N_{\text{pulse}}} (P_f(f_k) + P_b(f_k))} \quad (20)$$

N_{pulse} is a number of pulse repetitions. Next, the autocorrelation method is based on the time domain analysis to compute the phase difference. The autocorrelation function R is expressed in Eq. (21).

$$R[n] = \sum_{n=0}^{N_{\text{pulse}}} s[n] \cdot \overline{s[n]} \quad (21)$$

The Doppler frequency according to the autocorrelation method can be written in Eq. (22).

$$f_d = \frac{1}{2\pi T_{\text{prf}}} \tan^{-1} \frac{\text{Im}[R[n]]}{\text{Re}[R[n]]} \quad (22)$$

T_{prf} is a period of pulse repetitions. Figure 3 depicts the measurement system constructed for this report. It consists of four hardware packets, an ultrasonic prob of Imasonic, a pulser/receiver of PUL2 Honda Electronics, a 8 bit, 100 MS/s digitizer of NI USB-5133 and a personal computer. The flow condition was set up with 15 liters per minute of the low flowrate. Number of pulse repetitions was 32, the pulse repetition frequency was 2 kHz, and 4 MHz of the basic frequency was adjusted on the pulser/receiver. Figure 4 is the UVP series at 15 liters per minute of flowrate relied on the Fast Fourier transform method, the autocorrelation method, and the maximum likelihood estimation method. The ultrasonic velocity profile of these techniques was converted mathematically to flowrate in as follows

$$Q = \pi \sum_{r=-R}^R V(r) \cdot r \quad (23)$$

where R is defined as a radius of the pipeline, and $V(r)$ is the velocity profile on any radius, r , measured from the Doppler frequency methods. Table 2 shows the flowrate measurement derived using UPV of the Fast Fourier transform method, the autocorrelation method, and the maximum likelihood estimation, comparing with the standard flowmeter. The flowrate result solved using UVP of the maximum likelihood estimation is sufficient for accuracy.

Fig. 4 A comparison of the low ultrasonic velocity profile at 15 liters per minute of flowrate. (a) The Fast Fourier transform method. (b) The autocorrelation method. (c) The maximum likelihood estimation method.

Table 2 Experimental results of the Doppler frequency methods at 15 l/min of flowrate.

Doppler frequency method	Flowrate l/min	Error l/min
Fast Fourier transform	15.12	+0.12
Autocorrelation	12.16	-2.84
Maximum likelihood estimation	15.20	+0.20

5. Conclusion

This paper developed a statistical solution for estimating the UVP measurement relied on the maximum likelihood function from a low constant flowrate. The solution method introduces the Doppler-shifted frequency estimator that allow the velocity profile to be solved efficiently, comparing with the low flowrate. The maximum likelihood estimation is the algorithm of choice for measuring velocity distributions.

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