Sidelobe suppression by desired directivity pattern optimization for a small circular loudspeaker array

Koya Sato and Yoichi Haneda*

Graduate School of Informatics and Engineering, The University of Electro-Communications, 1–5–1 Chofugaoka, Chofu, 182–8585 Japan

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Abstract: Directivity control using a loudspeaker array is widely studied for various applications. Suppressing sidelobe levels is important for applications such as personal audio systems. In this paper, we propose a filter design method using a window function shape as the desired directivity pattern to reduce the sidelobe levels. The proposed method consists of three steps. The first step defines a cost function with a criterion for the directivity pattern. Next, filter coefficients for each loudspeaker are calculated and stored while changing the window function shape of the desired directivity pattern. Finally, we determine the optimum filter coefficients having the best performance of the cost function by using a full-search algorithm at each frequency. We conducted directivity experiments with a real six-element circular loudspeaker array having a radius of 0.055 m and evaluated its directivity to confirm the performance of the proposed method. The results, which were compared with those obtained from a conventional method, showed that the maximum sidelobe level improved by about 2 dB, although the beam was wide. We verified that using the window function shape as the desired directivity pattern is more effective than using the conventional method for sidelobe suppression.

Keywords: Circular loudspeaker array, Directivity control, Sidelobe suppression, Window function, Least-squares method

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1. INTRODUCTION

Directivity and sound field control for loudspeaker arrays have been developed over a long time to be applied, for instance, in personal audio, public address, and surround sound systems [1–10]. In addition, directivity control, also known as beamforming in the field of microphone array processing, is widely studied for speech enhancement, sound source separation, and direction of arrival estimation [11–15]. Filter design methods that allow filter coefficients to be obtained are important to control the directivity or sound field in these applications.

The filter coefficients are mainly obtained in two domains. First, the pressure-matching method (PMM) based on the least-squares method [16] in the frequency domain is widely used for sound field control [17–20]. The PMM is an extension of inverse filtering [21]. Second, the wavenumber domain has recently attracted attention because the filter coefficients, the radiation patterns of a loudspeaker array, and the corresponding gains can be derived analytically [22–29]. However, the wavenumber domain can currently be used for specific array configurations, namely, linear, cylindrical, circular, and spherical configurations.

Considering personal audio systems, directivity control aims to reduce sound leakage outside the listening area to guarantee privacy protection [30]. That is, it is important to suppress sidelobe levels, which appear in directions besides the look direction [31–33]. To attain this, we focus on a filter design method in the wavenumber domain using either a rectangular or Hanning window [34] as the distribution of the desired sound pressure [27,28]. Although the window function shapes are fixed in this method, we can modify the shape according to the desired directivity pattern in our proposed method. Moreover, we define a cost function with a criterion for the directivity pattern to determine the optimum filter coefficients that suppress the sidelobe levels. Hence, we propose a filter design method with a full-search algorithm based on the cost function. In this paper, we address directivity control with sidelobe suppression by using a small loudspeaker array with a small number of loudspeakers because we
assume that a personal audio system must be small for practical use.

This paper is organized as follows. Section 2 describes the PMM based on the least-squares method as a conventional method. Section 3 presents the window function shape that corresponds to the desired directivity pattern in the PMM. Section 4 explains the three cost functions used to determine the filter coefficients and gives details of our proposed method using a full-search algorithm to suppress the sidelobe levels. Experiments on directivity patterns and their evaluation using a real six-element circular loudspeaker array are described in Sects. 5 and 6. Notes on the proposed method are provided in Sect. 7. Finally, conclusions are given in Sect. 8.

2. PRESSURE-MATCHING METHOD (PMM)

The PMM is a typical filter design method in the frequency domain for sound field control [17–20]. In this section, we introduce the use of the PMM in directivity control. We consider the spherical coordinate system shown in Fig. 1 throughout this paper. A circular loudspeaker array consisting of L elements is placed on the x–y plane. M control points with coordinates \((r, \theta_m, \phi_m)\) are set on the spherical surface, where \(r\) is the distance from the origin to the control points, and \(\theta_m\) and \(\phi_m\) are the elevation and horizontal angles of the \(m\)th control point, respectively. Note that the ranges of \(\theta_m\) and \(\phi_m\) are defined as \(-90^\circ \leq \theta_m \leq 90^\circ\) and \(-180^\circ \leq \phi_m \leq 180^\circ\), respectively.

When the circular array is driven by the filter coefficients in the vector \(\mathbf{w}(\omega) = \begin{bmatrix} w_1(\omega), w_2(\omega), \ldots, w_L(\omega) \end{bmatrix}^T\), the observed sound pressures at each control point \(p(\omega) = \begin{bmatrix} p(\theta_1, \phi_1, \omega), p(\theta_2, \phi_2, \omega), \ldots, p(\theta_M, \phi_M, \omega) \end{bmatrix}^T\) are determined by

\[
p(\omega) = G(\omega)w(\omega),
\]

with \(G(\omega)\) defined as

\[
G(\omega) = \begin{pmatrix}
G_{11}(\omega) & G_{12}(\omega) & \cdots & G_{1L}(\omega) \\
G_{21}(\omega) & G_{22}(\omega) & \cdots & G_{2L}(\omega) \\
\vdots & \vdots & \ddots & \vdots \\
G_{M1}(\omega) & G_{M2}(\omega) & \cdots & G_{ML}(\omega)
\end{pmatrix},
\]

where \(G(\omega)\) is the transfer function matrix, \(G_m(\omega)\) is the transfer function between the \(m\)th loudspeaker and the \(m\)th control point, and \(\omega = 2\pi f\) is the angular frequency corresponding to frequency \(f\). Let \(d(\omega) = \begin{bmatrix} d(\theta_1, \phi_1, \omega), d(\theta_2, \phi_2, \omega), \ldots, d(\theta_M, \phi_M, \omega) \end{bmatrix}^T\) denote a desired directivity pattern defined at each control point. From the PMM, we obtain the filter coefficients that minimize the sum of the square errors between the observed and desired directivity patterns [17,19]. This is expressed as the following minimization problem:

\[
\min_{\mathbf{w}(\omega)} \|d(\omega) - G(\omega)\mathbf{w}(\omega)\|_2^2,
\]

where \(\|x\|_2^2\) is defined as \(\sum |x|^2\). Then, the optimum filter coefficients are given by

\[
\mathbf{w}(\omega) = \frac{G(\omega)^{H}d(\omega)}{G(\omega)^{H}G(\omega)},
\]

where the superscript \((\cdot)^{H}\) denotes the complex conjugate transposition. Moreover, Eq. (4) can include a regularization parameter [16] to obtain the stable filter gains or white noise gain [19,20,35].

3. DESIRED DIRECTIVITY PATTERN WITH WINDOW FUNCTION SHAPE

When using the conventional PMM for sound field control, the desired sound pressure is usually set to 1 in the look direction and 0 at the other control points [18,20]. In research to generate acoustically bright and dark zones for personal sound systems [36,37], filter design methods in which the desired sound pressure is modeled by a rectangular or Hanning window were recently proposed [27,28]. These methods achieved a high contrast between the bright and dark zones, and were analytically derived in the wavenumber domain.

If we use the conventional PMM to control directivity, sidelobes appear because this method cannot completely reproduce the desired directivity pattern. In order to mitigate this problem, we apply a window function shape as the desired directivity pattern in consideration of the results in [27,28]. Although there are several window functions, we use the Hanning and rectangular windows in this paper.

We describe the relation between the control points shown in Fig. 1 and the window function shape. When
using a Hanning window, the desired directivity pattern at the horizontal angle \( \phi_m \) is defined as follows:

\[
d(\phi_m, \phi_w, \omega) = \begin{cases} 
1 & (\phi_m = 0^\circ, \phi_w = 0^\circ), \\
0.5 - 0.5 \cos \frac{\phi_m + 2\phi_w}{\phi_w} \pi & (-\frac{\phi_w}{2} \leq \phi_m \leq \frac{\phi_w}{2}), \\
0 & \text{(otherwise)},
\end{cases}
\]

(5)

where \( \phi_w \) is the window width at the horizontal angle, whose range is from \( 0^\circ \) to \( 360^\circ \). The desired directivity pattern should be set at control points on a spherical surface because sound waves are radiated to the three-dimensional (3D) space in a real environment. The desired 3D directivity pattern is obtained by applying the Hanning window to the control points arranged in the horizontal plane in an analogous way to how it was obtained for the horizontal angle, i.e.,

\[
d(\theta_m, \theta_w, \omega) = \begin{cases} 
1 & (\theta_m = 0^\circ, \theta_w = 0^\circ), \\
0.5 - 0.5 \cos \frac{\theta_m + 2\theta_w}{\theta_w} \pi & (-\frac{\theta_w}{2} \leq \theta_m \leq \frac{\theta_w}{2}), \\
0 & \text{(otherwise)},
\end{cases}
\]

(6)

where \( \theta_w \) is the window width at the elevation angle, whose range is from \( 0^\circ \) to \( 180^\circ \). Therefore, we obtain the distribution of the desired 3D directivity pattern, \( d(\theta_m, \phi_m, \omega) \), by multiplying the elements calculated from Eqs. (5) and (6).

When using a rectangular window as the desired directivity pattern, the definitions are given as follows:

\[
d(\phi_m, \phi_w, \omega) = \begin{cases} 
1 & (\phi_m = 0^\circ, \phi_w = 0^\circ), \\
1 & (-\frac{\phi_w}{2} \leq \phi_m \leq \frac{\phi_w}{2}), \\
0 & \text{(otherwise)},
\end{cases}
\]

(7)

\[
d(\theta_m, \theta_w, \omega) = \begin{cases} 
1 & (\theta_m = 0^\circ, \theta_w = 0^\circ), \\
1 & (-\frac{\theta_w}{2} \leq \theta_m \leq \frac{\theta_w}{2}), \\
0 & \text{(otherwise)},
\end{cases}
\]

(8)

The ranges of \( \phi_w \) and \( \theta_w \) are the same as those when using the Hanning window.

Figure 2 shows examples of the desired directivity patterns in the horizontal plane using the conventional method, Hanning, and rectangular window shapes. In Figs. 2(b) and 2(c), the solid lines indicate that the Hanning and rectangular windows are set at the control points arranged at horizontal angles from \(-90^\circ\) to \(90^\circ\), whereas the dashed lines indicate that the window functions are set at the control points from \(-30^\circ\) to \(30^\circ\), respectively. The corresponding window widths, \( \phi_w \), are \(180^\circ\) and \(60^\circ\). The main lobe of the conventional desired directivity pattern is narrow, as shown in Fig. 2(a), whereas that of the desired directivity patterns using the window functions is wide, as shown in Figs. 2(b) and 2(c). Hence, we can form a variety of desired directivity patterns, with narrow to wide main lobes, by changing the window widths.

### 4. FILTER DESIGN METHOD WITH FULL SEARCH

To obtain the filter coefficients, \( w(\omega) \), that can suppress the sidelobe levels from Eq. (4), we must determine the window widths at the elevation and horizontal angles for the desired directivity pattern, \( d(\omega) \). Modifying the window widths causes a variation of the directivity pattern and filter coefficients. Therefore, the window widths that result in the desired directivity pattern to suppress the sidelobe levels are unknown. For this reason, we need to define the best directivity pattern. Consequently, we prepare a cost function that defines a criterion for this directivity pattern. In addition, we propose a full-search algorithm method for
the filter coefficients to satisfy the cost function. The search is performed over all the possible window widths.

Although there are many ways of describing cost functions, we experimentally prepare three cost functions in our study. These cost functions are described as

\[
J_1(\theta_w, \phi_w, \omega) = \frac{M_S}{M_T} \sum_{\phi_1} \sum_{\phi_3} |p(\theta_m, \phi_m, \omega)|^2,
\]

(9)

\[
J_2(\theta_w, \phi_w, \omega) = \frac{1}{M_S} \sum_{\phi_1} |p(\theta_m, \phi_m, \omega)|^2,
\]

(10)

\[
J_3(\theta_w, \phi_w, \omega) = \max |p(\theta_m, \phi_m, \omega)|, (\theta_m, \phi_m \in \theta_S, \phi_S),
\]

(11)

where \(\theta_T\) and \(\phi_T\) are the angles specifying the target area, \(\theta_S\) and \(\phi_S\) denote the sidelobe area whose levels we aim to suppress, and \(M_T\) and \(M_S\) are the numbers of control points in the target and sidelobe areas, respectively. In the proposed method, we consider the maximization of Eq. (9), that is, the maximization of the energy ratio between the target and sidelobe areas. On the other hand, we minimize Eqs. (10) and (11). These operations aim to minimize the energy and the maximum sidelobe in the sidelobe area, respectively.

The proposed method is summarized in Algorithm 1 and consists of three main steps. In the first step, the window function is selected in line 1 of Algorithm 1. We use either a Hanning or rectangular window. Next, the cost function is determined from Eq. (9), Eq. (10), or Eq. (11) in line 2. \(\epsilon_\theta\) and \(\epsilon_\phi\) are the corresponding angle increments in the loop of the algorithm. In the second step, which comprises line 4 to line 14 of Algorithm 1, the filter coefficients and cost values are calculated while changing the window function shape for the desired directivity pattern. Finally, we retrieve the optimum filter coefficients from the cost value that satisfies the corresponding criterion in line 15. Line 4 to line 15 of this algorithm are executed for each angular frequency \(\omega\).

5. DIRECTIVITY EXPERIMENTS

5.1. Measurements and Directivity Experiments

To confirm the directivity of our proposed method using a real loudspeaker array, we measured the transfer functions between the loudspeaker array and the control points. Figure 3 shows the circular loudspeaker array and the measuring device. The circular array was composed of six loudspeaker units. The radius and height of the array were 0.055 m and 0.25 m, respectively. For the measuring device, the microphones were arranged at 5° intervals in the elevation angle, and the table at the center was rotated at 10° intervals to vary the horizontal angle. Note that the transfer functions at \((\theta_m, \phi_m) = (90^\circ, \phi_m)\), which was a point located below the array, could not be measured owing to the presence of the rotation shaft. The transfer functions between the loudspeaker unit and the control points were measured in an anechoic chamber. In total, we obtained the transfer functions at 1,296 points on the spherical surface with a radius of 0.5 m.

After measuring the transfer functions, we conducted directivity experiments to evaluate the proposed method under the conditions listed in Table 1. In addition, we conducted the experiments using the conventional PMM to compare its results with those of the proposed method. In the conventional method, we set the desired directivity pattern as

![Fig. 3 Six-element circular loudspeaker array and measuring device.](image)
5.2. Evaluation Indexes

We defined three evaluation indexes to assess the directivity performance. One index is the target to sidelobe ratio (TSR) defined as follows:

\[
\text{TSR} = 10 \log_{10} \frac{M_S}{M_T} \sum_{\theta_T} \sum_{\phi_T} \frac{|p(\theta_T, \phi_T)|^2}{M_T} \sum_{\theta_S} \sum_{\phi_S} |p(\theta_S, \phi_S)|^2.
\]  

The TSR determines the difference in sound pressure level between the target and sidelobe areas. The other indexes are the maximum sidelobe level (MSL) and the beam width (BW). The MSL is the maximum sound pressure level in the sidelobe area, and the BW is defined as the angular width that is suppressed by not less than \(-3\) dB with respect to the sound pressure level in the look direction.

6. RESULTS

Figure 4 shows the results of the directivity experiments for the proposed method and for the conventional method. We set the sound pressure level in the look direction to 0 dB as a reference. Sidelobes for the conventional method appeared in several directions, as shown in the right column of Fig. 4, owing to the highly narrow desired directivity pattern. Nevertheless, the sound pressure levels for the conventional method around \((-90^\circ, 0^\circ)\) and \((90^\circ, 0^\circ)\) were suppressed slightly more than those for the proposed method as shown in the left column of Fig. 4. The proposed method with \(J_1\) and \(J_3\) showed similar directivity patterns and suppressed the sidelobe level by about 20 dB in the opposite direction to the look direction, as shown in Figs. 4(a) and 4(c). The proposed method with \(J_2\) showed less suppression of the sidelobe level at \((0^\circ, \pm 180^\circ)\) than the proposed method with the other cost functions from the results on the \(x-y\) plane. Overall, the proposed method showed wider main lobes than the conventional method. When we tested the directivity of the proposed method using the rectangular window, the results of the directivity patterns were similar to those using the Hanning window. From the directivity experiments, we concluded that the proposed method with \(J_1\) and \(J_3\) was superior to that with \(J_2\) for sidelobe suppression in the opposite direction. Moreover, there were no significant

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**Table 1 Experimental conditions.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sound velocity, (c) [m/s]</td>
<td>340</td>
</tr>
<tr>
<td>Sampling frequency [Hz]</td>
<td>48,000</td>
</tr>
<tr>
<td>Frequency band [Hz]</td>
<td>800–3,000</td>
</tr>
<tr>
<td>Filter length</td>
<td>2,400</td>
</tr>
<tr>
<td>Number of elements, (L)</td>
<td>6</td>
</tr>
<tr>
<td>Number of control points, (M)</td>
<td>1,296</td>
</tr>
<tr>
<td>Look direction</td>
<td>((0^\circ, 0^\circ))</td>
</tr>
<tr>
<td>Target area (\theta_T = -90^\circ–85^\circ), (\phi_T = -80^\circ–80^\circ)</td>
<td></td>
</tr>
<tr>
<td>Sidelobe area (\theta_S = -90^\circ–85^\circ), (\phi_S = -170^\circ–90^\circ), (\phi_S = 90^\circ–180^\circ)</td>
<td></td>
</tr>
<tr>
<td>(M_T) in Eqs. (9), (10), and (13)</td>
<td>612</td>
</tr>
<tr>
<td>(M_S) in Eqs. (9), (10), and (13)</td>
<td>684</td>
</tr>
<tr>
<td>Increment angles</td>
<td>(\epsilon_\theta = 10^\circ, \epsilon_\phi = 20^\circ)</td>
</tr>
</tbody>
</table>

\[
d(\theta_m, \phi_m, \omega) = \begin{cases} 
1 & (\theta_m, \phi_m) = (0^\circ, 0^\circ), \\
0 & \text{(otherwise)}, 
\end{cases}
\]  

Owing to the small loudspeaker array, no regularization parameter was applied in the proposed and conventional methods because the pseudoinverse matrices of \(G(\omega)\) were stable.

Fig. 4 Results of directivity patterns using the proposed method and the conventional method. The left and right figures show the directivity patterns on the \(z-x\) and \(x-y\) planes, respectively (Fig. 1 shows the planes).
Table 2 Evaluation results of the conventional and proposed methods.

<table>
<thead>
<tr>
<th></th>
<th>Hanning window</th>
<th>Rectangular window</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conventional</td>
<td>$J_1$</td>
</tr>
<tr>
<td>TSR [dB]</td>
<td>5.7</td>
<td>9.2</td>
</tr>
<tr>
<td>MSL [dB]</td>
<td>−7.9</td>
<td>−9.3</td>
</tr>
<tr>
<td>BW [°]</td>
<td>±20</td>
<td>±40</td>
</tr>
</tbody>
</table>

differences between the two window functions in the results of the directivity patterns.

Table 2 shows the evaluation results. The TSR values of the proposed method with the window functions improved by over 3 dB in comparison with the values obtained from the conventional method. These TSR results imply that more energy in the sidelobe area was suppressed. In the MSL, the greatest improvement of the proposed method was 2.2 dB when using $J_3$ with the Hanning window. It seems that the suppression of the sidelobe level is small when referring only to the MSL results. However, the effectiveness of the proposed method is clear when considering the TSR results. When using the cost functions $J_1$ and $J_3$, we obtained the largest TSR values from $J_1$ and the lowest MSL values from $J_3$ owing to the full-search algorithm. The wide BW was because we defined the desired directivity patterns having wide main lobes using the window function shapes.

The selected window widths for the Hanning window in the proposed method are shown in Fig. 5. There were no large variations in the horizontal angle. For $J_1$ and $J_2$, the corresponding window widths, $\theta_w$, of the elevation angle were extremely narrow around 1.620 Hz, whereas the window width for $J_3$ was highly variable depending on the frequency. The frequency responses in the look direction followed similar trends as shown in Fig. 6. In this figure, we used the conventional method as a reference, and we biased each proposed method by $-5$ dB for ease of visualization and comparison. The amplitude level of the conventional method was within 5 dB and its frequency response was mostly flat. In contrast, the frequency responses for the methods using $J_1$ and $J_2$ showed notches around 1,620 Hz. Furthermore, the frequency response for the method using $J_3$ was highly variable. Therefore, the frequency response corresponding to $J_3$ suggests that the sound quality is degraded when performing a reproduction using directivity. In fact, when we listened to the directivity reproduction for female speech using the array, the reproduced speech signal from the proposed method with $J_3$ had poor sound quality. We were not able to distinguish differences in sound quality among the proposed method with $J_1$ and $J_2$, and the conventional method. From the above results, we affirmed that the proposed method with $J_1$ was the best for providing sidelobe suppression and suitable sound quality.
7. NOTES

From Figs. 5 and 6, we can see that a variation of the desired directivity pattern affects the frequency responses in the look direction. Therefore, a method that sets the same window widths regardless of frequency would be useful to avoid large variations of the frequency response in the look direction. For example, we can fix the window widths to \( \theta_w = 180^\circ \) and \( \phi_w = 180^\circ \). Note that when using the fixed window widths with a rectangular window, the directivity in the look direction is split for window widths at a horizontal angle \( \phi_w \) above 140°. Figure 7 shows examples of this phenomenon. The transfer function of a rigid circular array has been analytically derived [27]; however, it can only be maintained when using a rigid cylindrical array with infinite length. Because of this, we used theoretical transfer functions assuming the 3D free-field Green's function [22] in the theoretical simulation. The circular loudspeaker array and control points had the same configurations as those used in Sect. 5. The directivity was split when \( \phi_w \) was set to above 140° in Fig. 7. Meanwhile, the Hanning window has its maximum value at 1 and is 0 at its edges. Therefore, the desired directivity pattern in the look direction has a gain of 1 and gradually decreases in the other directions. Hence, using a Hanning window can avoid the directivity split in the look direction. We verified that the frequency response without the notches was accomplished by fixing the window widths using the Hanning window from Fig. 8. Furthermore, we can reduce the calculation time by using fixed window widths. In contrast, the proposed method in Sect. 4 required a long calculation time owing to the full-search algorithm in wide frequency bands.

In Sect. 6, the difference in the effectiveness between the rectangular and Hanning windows was not clear because the array had only six loudspeaker units. By increasing the number of loudspeakers, we may find a difference between these windows in the wavenumber domain. This problem has been described in detail using a linear array in [28]. Since this paper dealt with a small loudspeaker array, an investigation of the difference between these windows for a large circular array is a future work.

In this paper, we present successful sidelobe suppression and directivity reproduction using a real six-element circular loudspeaker array. However, when we used a real eight-element circular array, the speeches reproduced with directivity were distorted using the proposed and conventional methods. Owing to the instability of the pseudoinverse matrix of \( G(\omega) \) in Eq. (4), the filter gains [5] were large at low frequencies or the white noise gain [35] was small. To obtain stable inverse matrices, a regularization parameter [16] has to be introduced in Eq. (4) when using a large array. Moreover, it is known that the filter gains at low frequencies are larger with increasing number of loudspeaker units because of the mode strength at high orders [5,7,29]. Consequently, we must pay attention to the filter coefficient magnitudes in the filter design for a loudspeaker array consisting of a large number of loudspeakers.

8. CONCLUSIONS

For the PMM based on the least-squares method for directivity control, we proposed the use of a window function shape to form the desired directivity pattern having
a wide main lobe. Furthermore, we proposed a full-search algorithm with a cost function, which defines a criterion for the suppression of sidelobe levels at each frequency. By using defined cost functions, we maximized the energy ratio between the target and sidelobe areas, minimized the energy in the sidelobe area, and minimized the maximum sidelobe level in the sidelobe area. We obtained the directivity for the proposed method using either a Hanning or rectangular window as the desired directivity pattern. Next, we evaluated the directivity performance in detail. From the results, we confirmed that the proposed method suppressed the maximum sidelobe level by about 2 dB, although the beam widths at 10° to 20° were wider than those for the conventional method. The use of a rectangular window was better than the conventional method for sidelobe suppression. However, there was no significant difference from the results obtained using the Hanning window. Overall, we verified that the desired directivity pattern with a wide main lobe using the window function shape is effective for sidelobe suppression.

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