PAPER

Directivity control of a finite cylindrical loudspeaker array based on circular harmonics and longitudinal multipole expression

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Abstract: The directivity control of a finite cylindrical loudspeaker array can be applied to various systems such as personal audio and smart speaker systems. As a conventional directivity control method, the filter coefficients of a cylindrical array may be derived analytically on the basis of cylindrical harmonic expansion when desired directivity patterns are set as the control points in the cylindrical coordinate system. However, it is more convenient to set the control points in the spherical coordinate system when considering practical use. In this paper, we propose a filter design method to control the directivity patterns on a spherical surface for a finite cylindrical array. Although the proposed method is similar to the mode-matching method based on spherical harmonic expansion, it uses a combination of circular harmonics and longitudinal multipoles to express the directivity patterns and filter coefficients. To validate the proposed method, we evaluated the directivity patterns using a prototype 24-element cylindrical loudspeaker array in an anechoic chamber. Consequently, we validated that the three-dimensional directivity pattern formed was the same as that obtained by computer simulations.

Keywords: Cylindrical loudspeaker array, Directivity control, Spherical harmonics, Circular harmonics, Longitudinal multipole

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1. INTRODUCTION

A smart speaker has been recently developed and is becoming popular. This device provides much information regarding a user's schedule and news reports, and plays music with omnidirectional sound radiation. However, these pieces of information often contain personal information, which might annoy other users. To overcome this problem, a smart speaker with directivity is conceivable. Since the shape of a smart speaker is typically cylindrical, we propose a novel directivity control method for a finite cylindrical loudspeaker array with a number of loudspeakers arranged on a finite cylindrical surface. This method is very important because it can be used for various applications, e.g., surround sound systems and direction-of-arrival estimation [1–3].

Filter design methods for linear, cylindrical, or spherical arrays in the wavenumber domain using the spatial Fourier transform [4] have been studied. One of the filter design methods in the wavenumber domain is the mode-matching method [5,6], by which the filter coefficients are obtained analytically as a linear weighted sum of the basis

[7,8]. Because this method uses the radiation gain of each mode, called the mode strength [7], it becomes easy to predict the magnitudes of the filter coefficients [9].

The mode-matching method for cylindrical arrays is derived strictly when we assume that both the directivity control points and the loudspeaker units of the arrays are on infinite cylindrical surfaces [4]. However, this assumption is unreasonable for practical use. Therefore, many researchers study the filter design method in the wavenumber domain for circular arrays when assuming uniform sound radiation along the z -axis instead of for cylindrical arrays [10–13].

In this paper, we propose a filter design method for a finite cylindrical array that allows the directivity to be controlled on a spherical surface using the mode-matching method. Although conventional mode-matching methods use the same basis to expand the directivity and filter coefficients, our proposed method applies a different basis. The proposed method introduces multipoles [4] to express the modes along the z -axis of both the finite cylindrical array and the spherical control points. In our method, the directivity pattern and filter coefficients are composed of

combinations of circular harmonics [14] and longitudinal multipoles. Furthermore, we indicate the mode strength from the cylindrical array to the control points on the spherical surface.

The rest of the paper is organized as follows. In Sect. 2, we explain the mode-matching methods for cylindrical and spherical arrays and mention some problems. In Sect. 3, we propose a filter design method for a finite cylindrical array on the basis of the circular harmonics and longitudinal multipole expression. In Sect. 4, we describe the simulation and experiment conditions. Computer simulations of the directivity patterns, mode strength, and *directivity basis* forming the directivity are discussed in Sect. 5. In this section, we also provide their measurements using a real finite cylindrical array composed of 24 loudspeaker units. Finally, our conclusions are presented in Sect. 6.

2. MODE-MATCHING METHOD

In this section, we explain the filter design method in the wavenumber domain called the mode-matching method in the cylindrical or spherical coordinate system [5,6].

2.1. Cylindrical Harmonic Domain

Initially, we derive the mode-matching method in the cylindrical coordinate system because it is useful for expressing the cylindrical array. Figure 1 shows the relationship between the cylindrical and spherical coordinate systems. R is the radius on the x - y plane and ϕ is the horizontal angle.

Although the directivity factor is generally defined as the radiation power at a sufficiently far position, we set the distribution of a desired directivity pattern $d(R, \phi, z, \omega)$ at the control points (R, ϕ, z) . Its spatial Fourier transform into the cylindrical harmonic domain [4,15] can be described as

Fig. 1 Cylindrical and spherical coordinate systems. Note that the definitions of R and r are different. R and r are the distances from the z -axis and the origin, respectively.

$$\tilde{d}_m(R, k_z, \omega) = \frac{1}{2\pi} \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d(R, \phi, z, \omega) e^{-jm\phi} e^{-jk_z z} dz, \quad (1)$$

where $\tilde{d}_m(R, k_z, \omega)$ is the expansion coefficient of mode order m , $\omega = 2\pi f$ is the angular frequency corresponding to frequency f , and k_z is the wavenumber along the z -axis. $e^{jm\phi} e^{jk_z z}$ is the cylindrical harmonics [4]. The cylindrical harmonics have an orthogonal basis; therefore, we can define the inverse transform of Eq. (1) as

$$d(R, \phi, z, \omega) = \sum_{m=-\infty}^{\infty} e^{jm\phi} \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{d}_m(R, k_z, \omega) e^{jk_z z} dk_z. \quad (2)$$

When the filter coefficients of the cylindrical array are set to $w(\phi_l, z_l, \omega)$, the observed sound pressure $p(R, \phi, z, \omega)$ is expressed as

$$p(R, \phi, z, \omega) = \int_0^{2\pi} \int_{-\infty}^{\infty} G(R, \phi, z | A, \phi_l, z_l, \omega) \cdot w(\phi_l, z_l, \omega) A dz_l d\phi_l, \quad (3)$$

where $G(R, \phi, z | A, \phi_l, z_l, \omega)$ is the transfer function between the l th loudspeaker at (A, ϕ_l, z_l) and the control (observation) point at (R, ϕ, z) . A is the radius of the cylindrical array. When applying the spatial Fourier transform to the right-hand side in Eq. (3), we have

$$p(R, \phi, z, \omega) = A \sum_{m=-\infty}^{\infty} e^{jm\phi} \int_{-\infty}^{\infty} \tilde{G}_m(A, R, k_z, \omega) \cdot \tilde{w}_m(k_z, \omega) e^{jk_z z} dk_z. \quad (4)$$

The orthogonality relation of $e^{jm\phi_l} e^{jk_z z_l}$ is used [4]. $\tilde{G}_m(A, R, k_z, \omega)$ and $\tilde{w}_m(k_z, \omega)$ are the expansion coefficients of the transfer function and filter coefficients, respectively. Since $\tilde{G}_m(A, R, k_z, \omega)$ represents the radiation gain for each order m , it is called the mode strength [7]. By comparing the values of the mode in Eqs. (2) and (4), we obtain

$$\tilde{w}_m(k_z, \omega) = \frac{\tilde{d}_m(R, k_z, \omega)}{2\pi A \tilde{G}_m(A, R, k_z, \omega)}. \quad (5)$$

This method is called the mode-matching method [5,6]. When applying the inverse spatial Fourier transform to Eq. (5), we calculate the filter coefficients in the frequency domain.

$$w(\phi_l, z_l, \omega) = \sum_{m=-\infty}^{\infty} e^{jm\phi_l} \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{w}_m(k_z, \omega) e^{jk_z z_l} dk_z. \quad (6)$$

The mode-matching method in the cylindrical harmonic domain derives the filter coefficients analytically. However, this method is not practical because of the infinite integration with respect to z in Eq. (1). Moreover, in the cylindrical coordinate system, we cannot express the position $(0, 0, z)$ if R is fixed. Consequently, directivity control in the z -axis direction is difficult.

2.2. Spherical Harmonic Domain

The mode-matching method can also be used in the spherical harmonic domain with the spherical coordinate system. The spherical coordinate system is shown in Fig. 1. r is the distance from the origin and θ is the elevation angle. Here, we define $\Omega = (\theta, \phi)$. The spherical harmonic domain is generally used for the directivity control of a spherical array.

If a desired sound pressure at a control point is $d(r, \theta, \phi)$, it can be transformed into the spherical harmonic domain as follows [4]:

$$\tilde{d}_{nm}(r, \omega) = \int_0^{2\pi} \int_0^\pi d(r, \Omega, \omega) Y_n^m(\Omega)^* \sin \theta d\theta d\phi, \quad (7)$$

where $Y_n^m(\Omega)$ is the spherical harmonics [4],

$$Y_n^m(\Omega) = \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} P_n^m(\cos \theta) e^{jm\phi}. \quad (8)$$

$P_n^m(\cdot)$ is the associated Legendre function. n and m are the order and degree, respectively. The superscript $(\cdot)^*$ is the complex conjugate. The inverse transform of Eq. (7) is given by

$$d(r, \Omega, \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^n \tilde{d}_{nm}(r, \omega) Y_n^m(\Omega) \quad (9)$$

because the spherical harmonics are orthonormal [4]. The orthogonality relation of the spherical harmonics is expressed in the following equation:

$$\int_0^{2\pi} d\phi \int_0^\pi Y_n^m(\Omega) Y_{n'}^{m'}(\Omega)^* \sin \theta d\theta = \delta_{nn'} \delta_{mm'}, \quad (10)$$

where δ is the Kronecker delta.

The filter coefficients are derived similarly to those in the cylindrical harmonic domain. First, the observed sound pressure at the control point $p(r, \Omega, \omega)$ is expressed as

$$p(r, \Omega, \omega) = \int_0^{2\pi} \int_0^\pi G(r, \Omega|a, \Omega_l, \omega) w(\Omega_l, \omega) \sin \theta_l d\theta_l d\phi_l. \quad (11)$$

$\Omega_l = (\theta_l, \phi_l)$ is the l th loudspeaker position of the spherical array with radius a . $G(r, \Omega|a, \Omega_l, \omega)$ is the transfer function between the l th loudspeaker and the control point. $w(\Omega_l, \omega)$ is the filter coefficients of the l th loudspeaker. $G(r, \Omega|a, \Omega_l, \omega)$ and $w(\Omega_l, \omega)$ can be expanded respectively by using the spherical harmonics as

$$G(r, \Omega|a, \Omega_l, \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^n \tilde{G}_n(a, r, \omega) Y_n^m(\Omega_l)^* Y_n^m(\Omega). \quad (12)$$

$$w(\Omega_l, \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^n \tilde{w}_{nm}(\omega) Y_n^m(\Omega_l). \quad (13)$$

$\tilde{G}_n(a, r, \omega)$ in Eq. (12) is called the mode strength [7] and represents the radiation gain for each mode n . $\tilde{w}_{nm}(\omega)$ in Eq. (13) is the expansion coefficient of the filter coefficients.

Substituting Eqs. (12) and (13) into Eq. (11), we have

$$p(r, \Omega, \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^n \tilde{G}_n(a, r, \omega) \tilde{w}_{nm}(\omega) Y_n^m(\Omega). \quad (14)$$

The orthogonal relation in Eq. (10) is used in the calculation process.

The expansion coefficients of $w(\Omega_l, \omega)$ are obtained from Eqs. (9) and (14) using the mode-matching method, i.e.,

$$\tilde{w}_{nm}(\omega) = \frac{\tilde{d}_{nm}(r, \omega)}{\tilde{G}_n(a, r, \omega)}. \quad (15)$$

Therefore, the filter coefficients of the spherical array are given by

$$w(\Omega_l, \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^n \frac{\tilde{d}_{nm}(r, \omega)}{\tilde{G}_n(a, r, \omega)} Y_n^m(\Omega_l). \quad (16)$$

The mode-matching method in the spherical harmonic domain can control the directivity in any direction [16–18].

3. FILTER DESIGN OF A FINITE CYLINDRICAL ARRAY

The conventional mode-matching method in the cylindrical or spherical harmonic domain must express the filter coefficients and directivity in the same coordinate system. Additionally, in the directivity control of the cylindrical array, many researchers have only studied the directivity pattern on the x - y plane [3,10–13]. On the other hand, the three-dimensional (3D) directivity control of the cylindrical array must be considered because sound radiates in a 3D space in a real environment. Since our objective is 3D directivity control with a finite cylindrical array, we investigate the relationships between the filter coefficients of the cylindrical array and the 3D directivity on a spherical surface.

Let us consider the L -element cylindrical array with V arrays of an H -element circular array in the vertical direction. Thus, $L = H \cdot V$. The directivity control points are set on the spherical surface and their total number is O . Figure 2 shows the arrangement of the cylindrical loudspeaker array and control points. θ_o and ϕ_o are the elevation and horizontal angles of the o th control point, respectively. The cylindrical array in Fig. 2 shows that three circular arrays are stacked, i.e., $V = 3$. The cylindrical coordinates of the cylindrical array are defined by (A, ϕ_h, z_v) , where

$$\phi_h = \frac{2\pi(h-1)}{H}, \quad (h = 1, 2, \dots, H), \quad (17)$$

$$z_v = \Delta z \cdot v. \quad (18)$$

Δz is the interval of the loudspeakers in the vertical direction. The index of the stacked array in the vertical direction is defined by

Fig. 2 Arrangement of the cylindrical loudspeaker array and control points. (r, θ_o, ϕ_o) is the position of the o th control point. v is the index of the stacked circular array in the vertical direction.

$$v = \left\lfloor -\frac{V-1}{2} \right\rfloor, \left\lfloor -\frac{V-1}{2} + 1 \right\rfloor, \dots, \left\lfloor \frac{V-1}{2} \right\rfloor, \quad (19)$$

where $\lfloor \cdot \rfloor$ is the floor function. The circular array corresponding to $v = 0$ is arranged on the x - y plane.

3.1. Assumptions

Our aim is to control the directivity in the spherical coordinate system using the finite cylindrical array located in the cylindrical coordinate system. Therefore, we consider two bases separately: the expansion basis of the filter coefficients on the cylinder with the cylindrical coordinate system and that of the directivity on the surface with the spherical coordinate system. Specifically, we replace the expansion bases of the directivity patterns and the filter coefficients in Sect. 2.2 as follows:

$$Y_n^m(\Omega) \longrightarrow Y_n^m(\Omega_o), \quad (20)$$

$$Y_n^m(\Omega_l) \longrightarrow E_n^m(\phi_h, z_v). \quad (21)$$

$Y_n^m(\Omega_o)$, which indicates the basis for expanding the directivity on the surface, is the spherical function itself. Here, we use the angular expression Ω_o to clarify that it is a control point. $E_n^m(\phi_h, z_v)$ is the basis used to expand the filter coefficients in the cylindrical coordinate system, which uses ϕ_h and z_v instead of Ω_l . $E_n^m(\phi_h, z_v)$ is called *the filter basis*, herein. Note that the numbers of loudspeakers and control points are finite.

After replacing both of the bases, we obtain the following relations from the analogy of Eqs. (9) and (12)–(14):

$$d(r, \Omega_o, \omega) = \sum_{n=0}^N \sum_{m=-n}^n \tilde{d}_{nm}(r, \omega) Y_n^m(\Omega_o), \quad (22)$$

$$G(r, \Omega_o | A, \phi_h, z_v, \omega) = \sum_{n=0}^N \sum_{m=-n}^n \tilde{G}_n(A, r, \omega) E_n^m(\phi_h, z_v)^* \cdot Y_n^m(\Omega_o), \quad (23)$$

$$w(\phi_h, z_v, \omega) = \sum_{n=0}^N \sum_{m=-n}^n \tilde{w}_{nm}(\omega) E_n^m(\phi_h, z_v), \quad (24)$$

$$p(r, \Omega_o, \omega) = \sum_{n=0}^N \sum_{m=-n}^n \tilde{G}_n(A, r, \omega) \tilde{w}_{nm}(\omega) Y_n^m(\Omega_o), \quad (25)$$

where N is the maximum order. $G(r, \Omega_o | A, \phi_h, z_v, \omega)$ is the transfer function between the loudspeaker at (A, ϕ_h, z_v) and the o th control point at (r, Ω_o) . $w(\phi_h, z_v, \omega)$ is the filter coefficients of the finite cylindrical loudspeaker array. The expansion coefficients $\tilde{w}_{nm}(\omega)$ become

$$\tilde{w}_{nm}(\omega) = \frac{\tilde{d}_{nm}(r, \omega)}{\tilde{G}_n(A, r, \omega)} \quad (26)$$

from Eqs. (22) and (25). Thus, the filter coefficients of the cylindrical array are obtained from Eq. (24) as

$$w(\phi_h, z_v, \omega) = \sum_{n=0}^N \sum_{m=-n}^n \frac{\tilde{d}_{nm}(r, \omega)}{\tilde{G}_n(A, r, \omega)} E_n^m(\phi_h, z_v). \quad (27)$$

3.2. Longitudinal Multipole Expression

Let us consider *the filter basis* $E_n^m(\phi_h, z_v)$, which depends on the horizontal angle ϕ and z coordinate. Because the general solution of the wave equation in the horizontal angle typically becomes the circular harmonics $e^{jm\phi}$, $E_n^m(\phi_h, z_v)$ can be decomposed into

$$E_n^m(\phi_h, z_v) = e^{jm\phi_h} \cdot F_n(z_v). \quad (28)$$

If we consider the directivity in the spherical coordinates, $F_n(z_v)$ has to be a function of θ . However, our proposed method requires $F_n(z_v)$, which is a function of z . Therefore, to transform θ to z , we introduce the longitudinal multipole expression [19,20] as $F_n(z_v)$.

Multipoles are constructed by point sources that are located infinitesimally close to the origin and have equal amplitudes, but with opposite phases [4]. By combining the multipoles along the z -axis, the spherical harmonics of $Y_n^0(\Omega_o)$ ($m = 0$) can be expressed. $Y_n^0(\Omega_o)$ only depends on θ_o .

First, we define the sound pressure of a monopole at the origin,

$$p(r, \Omega_o, \omega) = Q \frac{e^{-jkr}}{4\pi r}, \quad (29)$$

where Q is the source strength [4]. Next, we consider a dipole along the z -axis that consists of two point sources located at an interval Δz . In this case, the sound pressure of the dipole can be obtained using the space differential of the point source,

$$\frac{\partial}{\partial z} p(r, \Omega_o, \omega) = -jk\Delta z \left(1 - \frac{j}{kr}\right) \cos \theta_o \cdot Q \frac{e^{-jkr}}{4\pi r}. \quad (30)$$

$r = \sqrt{x^2 + y^2 + z^2}$ and $z = \cos \theta$ are used in the calculation process. Let the source strength be $Q(\Delta z)^2$ in Eq. (30) [4]. We further differentiate Eq. (30) with respect to z and obtain the so-called longitudinal quadrupole given by

$$\frac{\partial^2}{\partial z^2} p(r, \Omega_o, \omega) = (-jk\Delta z)^2 \left(1 - \frac{2j}{kr} - \frac{2}{k^2 r^2}\right) \cdot \cos^2 \theta_o \cdot Q \frac{e^{-jkr}}{4\pi r}. \quad (31)$$

Hereafter, we introduce the longitudinal multipoles into the finite cylindrical array. Therefore, each circular array of the cylindrical array is regarded as one point source, as shown in Fig. 3. Figure 4 shows an illustration of the multipoles corresponding to Fig. 3. In Fig. 4(b), the interval of the point source is set to $2\Delta z$ to express the dipole with the three sources. In Fig. 4(c), the two point sources located at the origin describe the quadrupole with three sources [19,20]. When normalizing Eqs. (29)–(31) using $Qe^{-jkr}/(4\pi r)$, we can express $Y_n^0(\Omega_o)$ by driving the cylindrical array as follows [19,20]:

$$F_0(z_v, \omega) = \left[0, \sqrt{\frac{1}{4\pi}}, 0\right]^T, \quad (32)$$

$$F_1(z_v, \omega) = \sqrt{\frac{3}{4\pi}} \left[\frac{1}{\xi_1(\omega)}, 0, \frac{-1}{\xi_1(\omega)}\right]^T, \quad (33)$$

Fig. 3 Each circular array is regarded as one point source.

Fig. 4 Illustration of the multipoles along the z -axis. This figure shows (a) monopole, (b) dipole, and (c) longitudinal quadrupole.

$$F_2(z_v, \omega) = \sqrt{\frac{5}{16\pi}} \left[\frac{3}{\xi_2(\omega)}, \frac{-6}{\xi_2(\omega)} - 1, \frac{3}{\xi_2(\omega)}\right]^T, \quad (34)$$

where $\xi_1(\omega)$ and $\xi_2(\omega)$ are respectively defined as

$$\xi_1(\omega) = -jk2\Delta z \left(1 - \frac{j}{kr}\right), \quad (35)$$

$$\xi_2(\omega) = (-jk\Delta z)^2 \left(1 - \frac{2j}{kr} - \frac{2}{k^2 r^2}\right). \quad (36)$$

The superscript $(\cdot)^T$ is the transposition. It is noteworthy that the elements in Eqs. (32)–(34) correspond to $v = 1$ from the left. It turns out that $F_n(z_v, \omega)$ depends on frequency owing to the multipoles derived from the monopole source. Contrary to the assumption in Sect. 3.1 and Eq. (28) in Sect. 3.2, $E_n^m(\phi_h, z_v)$ becomes $E_n^m(\phi_h, z_v, \omega)$.

3.3. Directivity Basis and Mode Strength

We discuss the basis of the directivity patterns here. In the spherical harmonic domain, we can obtain the following relationship by using Eqs. (10) and (12):

$$Y_n^m(\Omega) = \int_0^{2\pi} \int_0^\pi \frac{G(r, \Omega|a, \Omega_l, \omega)}{\tilde{G}_n(a, r, \omega)} Y_n^m(\Omega_l) \sin \theta_l d\theta_l d\phi_l. \quad (37)$$

Applying this transformation to our assumptions in Sect. 3.1, we can obtain

$$B_n^m(\Omega_o, \omega) = \sum_{h=1}^H \sum_{v=\lfloor \frac{V-1}{2} \rfloor}^{\lfloor \frac{V-1}{2} \rfloor} \frac{G(r, \Omega_o|A, \phi_h, z_v, \omega)}{\tilde{G}_{nm}(A, r, \omega)} \cdot \underbrace{e^{jm\phi_h} F_n(z_v, \omega)}_{E_n^m(\phi_h, z_v, \omega)}, \quad (38)$$

where we introduce the new variables

$$Y_n^m(\Omega_o) \longrightarrow B_n^m(\Omega_o, \omega), \quad (39)$$

$$\tilde{G}_n(A, r, \omega) \longrightarrow \tilde{G}_{nm}(A, r, \omega), \quad (40)$$

because we decompose $E_n^m(\phi_h, z_v, \omega)$ into the frequency-independent basis of $e^{jm\phi_h}$ and the frequency-dependent function $F_n(z_v, \omega)$, while the degree m of the spherical harmonics depends on the order n . Note that the summation range of m in Eqs. (22)–(27) needs to be modified to

$$\sum_{m=-n}^n \longrightarrow \sum_{m=-M}^M, \quad (41)$$

where M is the maximum degree. According to Eq. (37), the basis function on the left-hand side might be the same as that on the right-hand side in Eq. (38). However, we try to express the filter and directivity bases using different coordinate systems. As a result, we introduce the new basis $B_n^m(\Omega_o, \omega)$ instead of the basis appearing on the right-hand side, which depends on the angular frequency ω . In this paper, we call $B_n^m(\Omega_o, \omega)$ the *directivity basis*.

Here, we expect that $B_n^m(\Omega_o, \omega)$ is similar to $Y_{n+|m|}^m(\Omega_o)$ according to Eqs. (37) and (38) [6]. This is also because

$E_n^m(\phi_h, z_v, \omega)$ is based on the circular harmonics $e^{jm\phi}$ and the longitudinal multipole expression corresponding to $Y_n^0(\Omega_o)$. This implies that we can determine the mode strength $\tilde{G}_{nm}(A, r, \omega)$ in Eq. (38) using the least-squares method [21] by replacing $B_n^m(\Omega_o, \omega)$ with $Y_{n+|m|}^m(\Omega_o)$ if we set a theoretical transfer function or a measured transfer function of a prototype system as $G(r, \Omega_o|A, \phi_h, z_v, \omega)$. When $\tilde{G}_{nm}(A, r, \omega)$ is obtained, we can instantly decide $B_n^m(\Omega_o, \omega)$ from Eq. (38).

As mentioned in Sect. 2 regarding the conventional mode-matching method, the mode strength represents the radiation gain of each mode when the loudspeaker array and the control points are in the same coordinate system. In our proposed method, the mode strength between the cylindrical and spherical coordinate systems also corresponds to the radiation gain of each filter basis mode. Therefore, the gain of filter coefficients with the finite cylindrical array for controlling the 3D directivity can be predicted by using the mode strength. It becomes clear how much each mode contributes to the filter coefficients as shown in Eq. (27).

3.4. Filter Design

Substituting Eq. (38) into Eq. (22) applying Eqs. (39) and (41) and changing the calculation order, we have

$$d(r, \Omega_o, \omega) = \sum_{h=1}^H \sum_{v=\lfloor -\frac{V-1}{2} \rfloor}^{\lfloor \frac{V-1}{2} \rfloor} G(r, \Omega_o|A, \phi_h, z_v, \omega) \cdot \sum_{n=0}^N \sum_{m=-M}^M \frac{\tilde{d}_{nm}(r, \omega)}{\tilde{G}_{nm}(A, r, \omega)} E_n^m(\phi_h, z_v, \omega). \quad (42)$$

Consequently, the filter coefficients become

$$w(\phi_h, z_v, \omega) = \sum_{n=0}^N \sum_{m=-M}^M \frac{\tilde{d}_{nm}(r, \omega)}{\tilde{G}_{nm}(A, r, \omega)} E_n^m(\phi_h, z_v, \omega), \quad (43)$$

which is the same as Eq. (27) modified by Eqs. (39)–(41). In general, the maximum degree M depends on the number of array elements in the horizontal direction [3,10], which is restricted to $M = \lfloor (H-1)/2 \rfloor$.

The expansion coefficients $\tilde{d}_{nm}(r, \omega)$ are given by

$$\tilde{\mathbf{d}}(\omega) = \frac{\mathbf{B}(\omega)^H \mathbf{d}(\omega)}{\mathbf{B}(\omega)^H \mathbf{B}(\omega) + \lambda(\omega) \mathbf{I}} \quad (44)$$

from the least-squares method in Eq. (22) and applying Eqs. (39) and (41), where $(\cdot)^H$ is the complex conjugate transposition. $\tilde{\mathbf{d}}(\omega)$ and $\mathbf{d}(\omega)$ are defined as

$$\begin{aligned} \tilde{\mathbf{d}}(\omega) &= [\tilde{d}_{0,-M}(r, \omega), \tilde{d}_{0,-M+1}(r, \omega), \dots, \tilde{d}_{N,M}(r, \omega)]^T, \\ \mathbf{d}(\omega) &= [d(r, \Omega_1, \omega), d(r, \Omega_2, \omega), \dots, d(r, \Omega_o, \omega)]^T. \end{aligned}$$

$\mathbf{B}(\omega)$ is a matrix given by

$$\mathbf{B}(\omega) = [\mathbf{B}_0^{-M}(\omega), \mathbf{B}_0^{-M+1}(\omega), \dots, \mathbf{B}_N^M(\omega)]$$

with $\mathbf{B}_n^m(\omega)$ defined as

$$\mathbf{B}_n^m(\omega) = [B_n^m(\Omega_1, \omega), B_n^m(\Omega_2, \omega), \dots, B_n^m(\Omega_o, \omega)]^T.$$

$\mathbf{I}$ is the identity matrix and $\lambda(\omega)$ is the regularization parameter [21]. $\lambda(\omega)$ constrains the l_2 -norm of $\tilde{\mathbf{d}}_{nm}(\omega)$.

Incidentally, $B_n^m(\Omega_o, \omega)$ is hypothesized to be similar to $Y_{n+|m|}^m(\Omega_o)$. Hence, we can replace $\mathbf{B}(\omega)$ in Eq. (44) with $\mathbf{Y}$. $\mathbf{Y}$ is defined as

$$\mathbf{Y} = [\mathbf{Y}_{0+|-M|}^{-M}, \mathbf{Y}_{0+|-M|}^{-M+1}, \dots, \mathbf{Y}_{N+|M|}^M],$$

where

$$\mathbf{Y}_{n+|m|}^m = [Y_{n+|m|}^m(\Omega_1), Y_{n+|m|}^m(\Omega_2), \dots, Y_{n+|m|}^m(\Omega_o)]^T.$$

The clear difference between $\mathbf{B}(\omega)$ and $\mathbf{Y}$ is whether it depends on the angular frequency ω .

We can easily change the direction of the directivity owing to the circular harmonics [13] in the proposed filter design method. By contrast, it is difficult to change the directivity direction in the elevation angle because the rotated cylindrical array is oblique to the x - y plane unlike the spherical array.

4. SIMULATIONS AND EXPERIMENTS

In the study of the directivity control of loudspeaker arrays, it is important to achieve a narrow directivity and to reproduce the sound without distortion. Distortion of the reproduced sound with directivity is related to the filter gain, and the filter gain is derived from the mode strength. Thus, we analyzed the directivity pattern and mode strength in our proposed method. Moreover, to discuss the relationship between our proposed directivity basis and spherical harmonics described in Sect. 3.3, we confirmed both their directivity patterns. These analyses were carried out using the theoretical and measured transfer functions.

4.1. Transfer Functions

Although the theoretical transfer function of an infinite rigid cylindrical array has been derived [13], this function cannot be used for a finite rigid cylindrical array. For simplicity, we use the 3D free-field Green's function [4] defined by

$$G(r, \Omega_o|A, \phi_h, z_v, \omega) = \frac{e^{-jk|r-r_{h,v}|}}{4\pi|r-r_{h,v}|} \quad (45)$$

as the theoretical transfer function for the computer simulation. Here, $r_{h,v}$ is the distance between the origin and the coordinates determined by (A, ϕ_h, z_v) in the cylindrical coordinate system.

For experiments in a real environment, we measured the impulse responses with a prototype finite cylindrical loudspeaker array as shown in Fig. 5. This array was constructed by stacking three elements of an eight-element circular array. The radius and height of the array were 0.07

Fig. 5 24-element cylindrical array and measuring device.

and 0.28 m, respectively. The loudspeaker unit was an HDR9350-010020 unit from Hosiden Corporation. The large microphone array in Fig. 5 was used for the measurement of impulse responses around the loudspeaker array. The radius of the circular microphone array was 0.5 m. The microphones were arranged at 5° intervals in the elevation angle. The array at the center was rotated at 5° intervals to vary the horizontal angle. The microphone was omnidirectional EM172Z1 from PRIMO Corporation. We measured the transfer functions on the spherical surface using the measuring device and time-stretched pulse (TSP) signals in an anechoic chamber. Note that the transfer functions at $(\theta_o, \phi_o) = (180^\circ, \phi_o)$, which was a point located below the array, could not be measured owing to the rotation of the shaft. We measured the transfer functions at 2,592 points on the spherical surface of radius 0.5 m. We also used the measured transfer functions for evaluations of directivity in the experiments. No calibrations of the loudspeaker units and microphones were carried out because their performance variations were small.

4.2. Simulation and Experimental Conditions

The simulation and experimental conditions are listed in Table 1. In the frequency band, the upper-bound frequency was determined to avoid spatial aliasing [11,22], whereas the lower-bound frequency was set from the frequency responses of the loudspeaker unit. We heuristically determined the regularization parameter $\lambda(\omega)$.

We set three desired sound pressures in the directivity simulation and experiment. The patterns indicate that the directivity is formed just above or in the diagonal upper or horizontal direction of the array. Each pattern of the desired sound pressures was set to the control points as

$$d(\theta_o, \phi_o, \omega) = \begin{cases} 1 & (\theta_T, \phi_T), \\ 0 & (\text{otherwise}). \end{cases} \quad (46)$$

Table 1 Simulation and measurement conditions.

# of elements in horizontal direction, H	8
# of stacks in vertical direction, V	3
# of elements, L	24
Element interval in vertical direction, Δz [m]	0.055
Height of the array [m]	0.28
Radius of the array, A [m]	0.07
# of control points, O	2,592
Radius of the measuring device, r [m]	0.5
Sound velocity, c [m/s]	340
Sampling frequency [Hz]	48,000
Frequency band [Hz]	800–3,000
Filter length	2,400
Maximum order, N	2
Maximum degree, M	3
Look direction, (θ_T, ϕ_T)	$(0^\circ, 0^\circ)$ $(45^\circ, 0^\circ)$ $(90^\circ, 0^\circ)$
Regularization parameter, $\lambda(\omega)$ in Eq. (44)	10

(θ_T, ϕ_T) is the look direction that forms the directivity. To confirm the effectiveness of our proposed method in the directivity simulation, we also conducted the directivity simulation using end-fire and circular arrays. The end-fire array consisted of three loudspeakers with a spacing of 0.055 m along the z -axis. The circular array was composed of eight loudspeakers and its radius was 0.07 m. The look direction of the end-fire array was set to $\theta_T = 0^\circ$ and that of the circular array was $\theta_T = 90^\circ$. We used the pressure-matching method with the 3D free-field Green's function for these simulations [23]. For the directivity experiments, we compared the performance of the real cylindrical array and circular array located in the middle of the prototype. To make the frequency responses flat in each look direction, the filter coefficients were modified in the directivity simulation and measurement.

4.3. Evaluation Indexes for Directivity Patterns

To evaluate the directivity, we prepared two evaluation indexes, namely, the beam width (BW) and directivity index (DI) [16]. The BW is defined as the angular width for which there is suppression by not less than -3 dB with respect to the sound pressure level on the z - x plane in the look direction. The DI is defined as

$$\text{DI} = 10 \log_{10} \frac{|p(\theta_T, \phi_T)|^2}{\frac{1}{O} \sum_{\theta_o} \sum_{\phi_o} |p(\theta_o, \phi_o)|^2}, \quad (47)$$

(a) Using $\tilde{d}_{nm}(r, \omega)$ expanded by $B_n^m(\Omega_o, \omega)$. (b) Using $\tilde{d}_{nm}(r, \omega)$ expanded by $Y_{n+|m|}^m(\Omega_o)$.**Fig. 6** Results of the directivity simulations. The left and right figures show the results on the x - y and z - x planes, respectively.

where $p(\theta_o, \phi_o)$ is the sound pressure in the time domain when $p(\theta_T, \phi_T)$ is normalized to 1. O is the number of control points listed in Table 1. As the BW decreases and DI increases, the directivity performance improves.

5. RESULTS

5.1. Directivity Patterns

First, we show the results of directivity patterns obtained with our proposed method. The results of the directivity simulations are shown in Fig. 6. Figure 6(a) shows the results when the desired sound pressure was expanded by the directivity basis $B_n^m(\Omega_o, \omega)$ in Eq. (44). On the other hand, Fig. 6(b) shows the results when using $Y_{n+|m|}^m(\Omega_o)$. We normalized the sound pressure level in each look direction to 0 dB. From Fig. 6, the proposed method using $B_n^m(\Omega_o, \omega)$ had a narrower directivity overall than that using $Y_{n+|m|}^m(\Omega_o)$. The results of the directivity experiment with the prototype are shown in Fig. 7. Similarly to the simulation, it was found that the results using $B_n^m(\Omega_o, \omega)$ formed narrow directivities. From the results at $\theta_T = 90^\circ$ shown in Figs. 6 and 7, the measured directivity pattern was better than the simulation pattern from the viewpoint of the sidelobe level [24]. The reason for this better measured directivity pattern was the difference between the conditions of the loudspeaker array in the simulation and measurement. In the computer simulation, we assumed that an open cylindrical array was used, which has no rigid

(a) Using $\tilde{d}_{nm}(r, \omega)$ expanded by $B_n^m(\Omega_o, \omega)$. (b) Using $\tilde{d}_{nm}(r, \omega)$ expanded by $Y_{n+|m|}^m(\Omega_o)$.**Fig. 7** Results of the directivity experiments. The left and right figures show the results on the x - y and z - x planes, respectively.**Table 2** Evaluation results of the proposed method. Figures 6 and 7 correspond to the directivity simulation and measurement, respectively. (a) and (b) indicate two patterns obtained using $B_n^m(\Omega_o, \omega)$ and $Y_{n+|m|}^m(\Omega_o)$, respectively.

Evaluation results of Fig. 6(a).			
Look direction	$\theta_T = 0^\circ$	$\theta_T = 45^\circ$	$\theta_T = 90^\circ$
BW [°]	± 25	± 25	± 20
DI [dB]	7.9	13.9	13.9
Evaluation results of Fig. 6(b).			
BW [°]	± 25	± 25	± 30
DI [dB]	7.5	12.6	11.9
Evaluation results of Fig. 7(a).			
BW [°]	± 20	± 20	± 25
DI [dB]	8.2	12.0	13.7
Evaluation results of Fig. 7(b).			
BW [°]	± 25	± 20	± 35
DI [dB]	6.3	10.9	11.4

surface on the cylinder, that is, the loudspeaker units are located in the free field. On the other hand, the prototype loudspeaker array had a rigid surface, as shown in Fig. 5.

The evaluation results are listed in Table 2. For BW when the look direction was set to $\theta_T = 90^\circ$, the results of using $B_n^m(\Omega_o, \omega)$ were 10° sharper than those of using

Fig. 8 Results of the directivity simulations with the end-fire array and the proposed method. The proposed method used $\tilde{d}_{nm}(r, \omega)$ expanded by $B_n^m(\Omega_o, \omega)$. The left and right figures show the results on the x - y and z - x planes, respectively.

$Y_{n+|m|}^m(\Omega_o)$ for both the directivity simulation and measurement. From the results of DI, the evaluation values obtained using $B_n^m(\Omega_o, \omega)$ were better than those obtained using $Y_{n+|m|}^m(\Omega_o)$ in all angular results. The values were improved by 0.4 dB to 2.3 dB. Taken together, we found that the performance of the directivity using $B_n^m(\Omega_o, \omega)$ depending on the angular frequency is better than that of the directivity using $Y_{n+|m|}^m(\Omega_o)$.

Figure 8 shows the directivities of the end-fire array and the prototype with the proposed method. The results for the end-fire array showed a large beam width, whereas the prototype with the proposed method had a small beam width. Furthermore, the results for the circular array are shown in Fig. 9. The proposed method controlled the directivity in three dimensions, as shown by these results. Table 3 shows the evaluation indexes in Figs. 8 and 9. The cylindrical array obtained with the proposed method was narrower than other arrays by over 15° in terms of BW and was improved by over 2.2 dB in terms of DI.

We confirmed the directivity by listening to the directivity reproduction with a female speech using the prototype in our proposed method. However, the reproduced speech was slightly distorted in utterances with high amplitudes. This distortion can be prevented by using an appropriate regularization parameter.

5.2. Mode Strength

Figure 10 shows the mode strengths obtained by the simulation and measurement with the prototype. This figure represents $20 \log_{10} |\tilde{G}_{nm}(A, r, \omega)|$. The mode strengths $\tilde{G}_{00}(A, r, \omega)$ at 800 Hz in Fig. 10 were set to 0 dB as a reference. The mode strength indicates that the radiation efficiency decreases with decreasing gain.

In the simulation results shown in Fig. 10, the mode strength decreased as the degree m increased at low frequencies. This behavior was similar to that of the mode strength of the spherical array [7,25]; however, differences were also present. Despite using the 3D free-field Green's

Fig. 9 Results of the directivity patterns with the prototype and the circular array. The proposed method used $d_{nm}(r, \omega)$ expanded by $B_n^m(\Omega_o, \omega)$. The left and right figures show the results on the x - y and z - x planes, respectively. In Fig. 9(b), the blue and orange lines indicate the results of the 24-element cylindrical array and the 8-element circular array in the middle of the prototype.

Table 3 Evaluation results of the end-fire and circular arrays, and the cylindrical array with the proposed method.

	Fig. 8.		Fig. 9(a).		Fig. 9(b).	
	Prop.	End-fire	Prop.	Circular	Prop.	Circular
BW [$^\circ$]	± 25	± 50	± 20	± 45	± 25	± 40
DI [dB]	7.9	5.0	13.9	10.6	13.7	11.5

function, no notches in the mode strength appearing at the forbidden frequencies [4] were found. Instead, the peak appeared in $\tilde{G}_{10}(A, r, \omega)$ at 3,000 Hz. This is because the mode strength of the cylindrical array is derived from the cylindrical coordinate system, as mentioned in Sect. 2; however, the mode strength was forcibly obtained on the spherical surface herein.

The measurement results in the right figures were significantly different from the simulation results. In Fig. 10(a), the mode strength of the degree $m = 3$ at 800 Hz was the highest. Compared with the simulation results in Figs. 10(b) and 10(c), the measured mode strengths have considerably higher gains. Moreover, the measured mode strengths at $n = 2$ showed the opposite

Fig. 10 Results of $20 \log_{10} |\tilde{G}_{nm}(A, r, \omega)|$. The left and right figures show the results of the simulation and measurement with the prototype, respectively.

behavior from those in Fig. 10(c). It appears that the measured mode strength was higher than the simulation mode strength because of the high gain. However, this is due to the overfitting [21] when the mode strength was determined using the spherical harmonics in Sect. 3.3. As shown in Fig. 5, the prototype was connected to many cables and each loudspeaker protruded from the cylindrical surface. The prototype had many reflections. Therefore, the measured mode strength showed more complicated behaviors than the theoretical simulation mode strength. If the radiating surface of the loudspeaker in the prototype is only located on the rigid cylindrical surface, the measured mode strength can be improved.

For mode strengths of the cylindrical and spherical [3,7] arrays, the number of parameters in the proposed mode strength increased. However, the filter coefficients depend on the mode strength in Eq. (43); therefore, the mode strength is important. By clarifying the mode strength, filter coefficients that do not use mode strengths with low radiation efficiency can be designed, which is an advantage. With this contrivance, the distortion in the directivity reproduction can be suppressed owing to the stable filter coefficients [9].

5.3. Directivity Basis

The results of *the directivity basis* are shown in Fig. 11. Figure 11(a) shows the spherical harmonics $Y_{n+|m|}^m(\Omega_o)$. The simulated and measured typical *directivity bases* at 1,000 Hz are shown in Figs. 11(b) and 11(c), respectively. As the spherical harmonics and *the directivity basis* are complex numbers, Fig. 11 shows the amplitudes of the real parts. Figure 11 implies that *the directivity basis* closer to Fig. 11(a) is better.

Comparing Figs. 11(a) and 11(b), the results in these figures show nearly the same shape except for the simulated $B_0^0(\Omega_o, \omega)$. The simulated $B_0^0(\Omega_o, \omega)$ was extended in the vertical direction because we used the cylindrical array. Regarding *the directivity basis* of the prototype shown in Fig. 11(c), the forms were not similar to those of the spherical harmonics. In particular, the bottom side of the measured $B_0^0(\Omega_o, \omega)$ became rather unnatural in form. This is greatly affected by the rotating table and the cable shown in Fig. 5. It appears that the reflections in the prototype were a considerable problem. Nevertheless, the directions of *the directivity basis* were correct when compared with those in Fig. 11(a). $B_2^0(\Omega_o, \omega)$ was obtained similarly to $Y_2^0(\Omega_o)$ even in the prototype. Although omitted because of space limitations herein, in the mode strengths of the prototype with the high gains at low frequencies as shown in Fig. 10(c), the measured $B_n^m(\Omega_o, \omega)$ could not form the spherical harmonics. Nevertheless, the directivity on the spherical surface for the cylindrical array is expressed by a combination of *the directivity basis* as shown in Figs. 11(b) and 11(c).

6. CONCLUSIONS

We proposed that the directivity patterns on a spherical surface and the filter coefficients of a finite cylindrical array can be expressed by combinations of the circular harmonics and longitudinal multipoles. The mode strength representing the radiation efficiency in each mode was obtained using the relationships among the spherical harmonics, circular harmonics, and longitudinal multipoles. To confirm the performance of the proposed method, we simulated the directivity patterns, mode strength, and *directivity basis* using a theoretical transfer function. In addition, we measured them using an actual 24-element cylindrical loudspeaker array. From the results of the directivities of the cylindrical array determined using the proposed method, the end-fire array, and the circular array, we concluded that the proposed method is useful for a finite cylindrical array. Although our proposed method requires the measurement of the mode strength, we can control the directivity of the finite cylindrical array while avoiding a large filter gain. Even if the number of loudspeakers in the vertical direction increases, the proposed method can handle the increase owing to the longitudinal multipole

Fig. 11 Results of the real parts in the directivity basis. (a) is the spherical harmonics $Y_{n+|m|}^m(\Omega_o)$. (b) and (c) show the typical $B_n^m(\Omega_o, \omega)$ at 1,000 Hz obtained by simulation and measurement, respectively. The color differences are positive (magenta) and negative (cyan).

expression. A filter design considering the mode strength to reduce the distortion in the directivity reproduction using the prototype will be future work.

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