Effect of the dimension of a tone hole on the fundamental frequency of flutes

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1. Introduction

In flute design, the size and position of tone holes are important parameters that affect the fundamental frequency of the flute and the ease of playing. The clarification of the effects of the size and position of tone holes on the fundamental frequency of notes quantitatively will be useful for flute design. Coltman discussed the important acoustical parameters that affect the pitch of the Boehm flute [1]. In Ref. [1], Coltman showed how much the fundamental frequency of notes changes on moving the tone hole 1 mm. Hence, we investigated the effects of the size and position of a tone hole on the fundamental frequency.

The flutes used in this study were constructed in accordance with the model published by Boehm in the 19th century and have 12 or 13 large tone holes and three small tone holes [2]. In the flute design, the size and position of all tone holes are related to the fundamental frequency of the flute. In particular, small tone holes serve as register holes. The small tone hole of the C key is related to the fundamental frequencies of D6, G#6, A6, and A#6, and serves an important role in producing sounds in the high-frequency region of the flute. Although the small tone hole of the C key is also related to the fundamental frequencies of C#5, D5, D#5, and C#6, this tone hole does not serve as the register hole for these notes. As mentioned above, the C key has different functions depending on the note. In these two cases above, we consider that changes in the size and position of the tone hole of the C key also lead to different effects on the fundamental frequency. Hence, we examined the small tone hole of the C key in this study. In addition, we treated C#6 and D6 as examples of the two above-described cases, respectively. We did not treat the remaining two small tone holes, called D# and D trill keys in this study, because these are less frequently used.

In this study, we clarify how changing the size and position of the small tone hole of the C key affects the fundamental frequencies of two notes, C#6 and D6, by playing the instrument using an artificial blowing system and through transmission-line matrix modeling to provide aide in the design of the flute. We use an artificial blowing system to eliminate the instability of human blowing and to reduce the calculation cost compared with numerical simulations such as a computational fluid analysis using the finite-element method.

2. Sound measurement using artificial blowing system

2.1. Measurement method

We conduct sound measurements using an artificial blowing system to evaluate the effects of the size and position of a tone hole on the fundamental frequency of flutes. We constructed six experimental flutes using the following materials. The head joint of the experimental flutes is plastic and manufactured by Nuvo. The mid- and foot joints of the experimental flutes are made of polyvinyl chloride with an inner diameter of 20 mm.

To investigate the change in fundamental frequency, different sizes and positions of the tone hole were set as follows. We set the sizes of the tone hole of the C key to 8 mm for an original tone hole and 15.3 mm for an enlarged tone hole. The sizes of the original and enlarged tone holes of the C key were based on the sizes of the C key and a large tone hole of a Nuvo flute, respectively. Although the size of each tone hole of a Nuvo flute differs, we set the same size for three small tone holes and also for 13 large tone holes for simplicity. To compare the effect of tone hole size on the fundamental frequency, we examined two positions of the enlarged tone hole, as shown in Fig. 1.

Figure 2 shows the experimental configuration. The artificial blowing system stores air using a compressor and sends the air to the experimental flute at a constant wind speed monitored with a flow rate meter. We attached a recorder-like mouthpiece to the embouchure hole to blow air into the flute at the same angle. A microphone was installed near the embouchure hole to record the sounds. The parameters of the experiment using the artificial blowing system are the air pressure of 0.6×10^6 Pa, the flow rate of 25 l/min, the measurement time of 15 s, the number of measurements of 16 and the sampling frequency of 48 kHz.

2.2. Analysis method

A STFT (short-time Fourier transform) with a window length of 4,096 points and a shift length of 256 points started 3 s after exceeding 90% of the peak amplitude, which is the reference level indicating sound to have risen. The duration of the analysis was 10 s from the start point of STFT. The Hamming window was applied to each frame. The frequency at which the power was maximized by interpolating the peak of the power spectrum of each frame with a quadratic function was taken to be the fundamental frequency.

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Fig. 1 Dimensions of three types of tone hole of the C key. “N” is the original tone hole. “LC” is a larger tone hole with its center position aligned with the center of “N.” “LT” is a larger tone hole with its top position aligned with the top of “N.”

Fig. 2 Experimental configuration.

Fig. 3 Dimensions of tone hole and tube.

3. Resonance frequency obtained using calculation model

3.1. Two-port matrix model

The input impedance of a flute must be determined to obtain the resonance frequency. We employed the calculation model proposed by Caussé *et al.* [3]. When a cylindrical tube is composed of N short cylindrical tubes, the sound pressure and volume flow of the input end of a cylindrical tube denoted by p_{in} and U_{in} , respectively, can be obtained as the product of N transmission matrices of the short cylindrical tubes $T_n = \begin{bmatrix} A_n(\omega) & B_n(\omega) \\ C_n(\omega) & D_n(\omega) \end{bmatrix}$, as shown as

$$\begin{bmatrix} p_{in}(\omega) \\ U_{in}(\omega) \end{bmatrix} = T_N T_{N-1} \cdots T_2 T_1 \begin{bmatrix} p_{out}(\omega) \\ U_{out}(\omega) \end{bmatrix}. \quad (1)$$

Here, ω is the angular frequency and p_{out} and U_{out} are the sound pressure and volume velocity of the output end, respectively.

We assume that a flute is composed of short cylindrical tubes and open tone holes. Figure 3 shows the dimensions of the tone holes and the tube for the calculation model of the flute. The transmission matrix of a tone hole is inserted between two adjacent transmission matrices of the cylindrical tube. We describe the transmission matrix of a tone hole T_{hole} and a cylindrical tube T_{tube} in the following subsection. We set the values of the model parameters of the flute shown in Fig. 3 as follows: length of tube $L = 592$ mm, inner radius of tube

$a = 10$ mm, radius of large hole $b_l = 7.65$ mm, radius of small hole $b_s = 4$ mm, thickness of tube wall $t = 3$ mm, radius of curvature at either end of hole $r_c = 0.3$ mm, and area of embouchure hole $A_{em} = 94.2$ mm². The area of the embouchure hole was approximated from that of an ellipse with a semi-major axis of 6 mm and a semi-minor axis of 5 mm. The inner radius of the actual flute is 9.5 mm, but we set it to 10 mm on the basis of the measurement result of our experimental flute made of polyvinyl chloride. We measured the inner radius using calipers.

We set the values of various parameters in air as follows [4]: sound velocity $c = 346.3$ m/s, density $\rho = 1.17$ kg/m³, viscosity $\mu = 1.8 \times 10^{-5}$ kg/(m·s), thermal conductivity $\lambda = 6.2 \times 10^{-3}$ cal/(m·s·K) when the temperature is 25°C, specific heat at constant pressure $C_p = 240$ cal/(kg·K), and specific heat ratio $\gamma = 1.4$.

3.2. Transmission matrix of tone hole

The transmission matrix of a tone hole T_{hole} can be represented by the shunt impedance Z_s and the series impedance Z_a as follows [5]:

$$T_{hole}(\omega) \approx \begin{bmatrix} 1 & Z_a(\omega) \\ \frac{1}{Z_s(\omega)} & 1 \end{bmatrix}. \quad (2)$$

The shunt impedance can be represented by [6–8]

$$Z_s(\omega) = \frac{\rho c}{\pi b^2} (ikt_e + \xi), \quad (3)$$

where $k (= \omega/c)$ is the wave number, c is the sound velocity, and b is the radius of the tone hole. The effective length t_e can be represented by the sum of the thickness of the tube wall t and the three corrections:

$$t_e = t + t_m + t_i + t_r. \quad (4)$$

The three corrections are based on Ref. [5]. The series impedance Z_a is given by

$$Z_a = i \frac{\rho c}{\pi b^2} kt_a, \quad (5)$$

where the effective length t_a is

$$t_a = -b\delta^2 \{0.35 - 0.06 \tanh(2.7t/b)\}. \quad (6)$$

3.3. Transmission matrix of cylindrical tube

We modeled the transmission matrix of a cylindrical tube T_{tube} by considering the visco-thermal loss [3]. The transmission matrix of a short cylindrical tube is expressed as

$$T_{tube} = \begin{bmatrix} \cosh(\Gamma L_n) & \frac{\zeta}{A} \sinh(\Gamma L_n) \\ \frac{A}{\zeta} \sinh(\Gamma L_n) & \cosh(\Gamma L_n) \end{bmatrix}, \quad (7)$$

where A is the area of the bore of the tube and L_n is the length of the n -th short cylindrical tube. Here, the propagation constant Γ and the wave impedance ζ are given by

$$\Gamma = \sqrt{Z_v Y_t}, \quad \zeta = \sqrt{\frac{Z_v}{Y_t}}. \quad (8)$$

The series impedance per unit length Z_v and the shunt admittance per unit length Y_t are modeled with the following equations:

$$Z_v = i\omega\rho\left(1 + \frac{2}{r_v}(1-i) - \frac{3i}{r_v^2}\right), \quad (9)$$

$$Y_t = \frac{i\omega}{\rho c^2} \left[1 + (\gamma - 1) \left(\frac{\sqrt{2}}{r_t}(1-i) + \frac{i}{r_t^2} \right) \right]. \quad (10)$$

Here, $r_v = a\sqrt{\omega\rho/\mu}$ and $r_t = a\sqrt{\omega\rho C_p/\lambda}$ [4]. In Ref. [4], the pipe radius a was regarded as the arithmetic mean of the radii of the two ends. Hence, in cylindrical sections, we treated both ends of the tube as having the same radius.

3.4. Input impedance of flute

The input impedance of the flute can be represented by the following equation [4]:

$$Z_{in}(\omega) = \frac{p_{in}(\omega)}{U_{in}(\omega)} = \frac{A(\omega)Z_r(\omega) + B(\omega)}{C(\omega)Z_r(\omega) + D(\omega)}. \quad (11)$$

Here, $Z_r(\omega)$ is the radiation impedance that can be modeled using the radiation from a round piston head surrounded by an infinite baffle. $Z_r(\omega)$ is approximated as

$$Z_r(\omega) = \frac{p_{out}(\omega)}{U_{out}(\omega)} = \frac{\zeta}{A} \frac{i\omega R_r L_r}{R_r + i\omega L_r}, \quad (12)$$

where

$$R_r = \frac{128}{9\pi^2}, \quad L_r = \frac{8a}{3\pi c}. \quad (13)$$

3.5. Calculation of input impedance

We assume that a flute is composed of cylindrical tubes and open tone holes. Figure 4 shows an example of the dimensions of the flute and the fingering patterns of notes C#6 and D6. The size and position of the tone hole of the C key shown in Fig. 4 correspond to “N” shown in Fig. 1. Although the actual length of the tube is 592 mm, to conduct a simulation, we should consider the end correction produced at the embouchure hole. The end correction produced at the embouchure hole has been defined as [9]

$$\Delta L = \frac{c}{2\pi f} \arctan\left[\frac{cf}{2\pi r(2f_r^2 - f^2)}\right], \quad (14)$$

Fig. 4 Position of tone holes (top) and fingerings for notes C#6 (middle) and D6 (bottom). The top figure shows the distance from the output end of the tube to the center of the tone holes and their sizes. In addition, ○ and ● denote open and closed tone holes, respectively. The bottom figure shows the tone hole numbers. Tone hole No. 14 corresponds to the tone hole of the C key in this study. The sizes of tone holes Nos. 1–13 are the same. In addition, the sizes of tone hole Nos. 14–16 are the same.

where r is the length between the center of the embouchure hole and the cork, and f_r is the Helmholtz resonator frequency. The length r is typically set to 17 mm for the Boehm flute and the frequency f_{17} is approximately 1,450 Hz for the normal playing condition [9]. When r is 17 mm and f_{17} is 1,450 Hz, ΔL is approximately 42 mm. If the player sufficiently covers the embouchure hole, f_r decreases. Hence, we also consider the following conditions: $r = 17$ mm and $f_{17} = 1,200$ Hz. Under these conditions, ΔL is approximately 58 mm. We inserted a 42 or 58 mm cylindrical tube as the end correction between the embouchure hole and tone hole No. 16 shown in Fig. 4.

4. Result

4.1. Fundamental frequency obtained experimentally

In the case of the experimental flutes for C#6, the average fundamental frequencies were 1,120.3 Hz for N, 1,154.3 Hz for LC, and 1,142.1 Hz for LT. If the size of the C key is set to 15.3 mm, the fundamental frequency will be 30 Hz higher for LC and 20 Hz higher for LT compared with the fundamental frequency of N. It was observed that the fundamental frequency decreased from LC to LT, because the position of the tone hole of the C key for LT was closer to the output end of the tube than that for LC, and the effective length of LT became larger than that of LC.

In the case of the experimental flutes for D6, the average fundamental frequencies were 1,161.3 Hz for N, 1,169.0 Hz for LC, and 1,170.0 Hz for LT. When the size of the C key tone hole was set to 15.3 mm, the fundamental frequency increased by approximately 5 Hz for both LC and LT compared with that of N, and there was no significant change in fundamental frequency regardless of the tone hole conditions.

4.2. Resonance frequency obtained using calculation model

The resonance frequency corresponds to the admittance peak frequency. The admittance is defined as the reciprocal of the input impedance. Each peak of the admittance represents a resonance mode of the flutes. Table 1 shows the resonance frequencies of the experimental flutes for C#6 and D6 when $\Delta L = 42$ and 58 mm.

We focused on the resonance frequencies of the f_2 mode of the experimental flutes for C#6, and the f_3 mode of the experimental flutes for D6. A comparison of the tone hole conditions N and LC showed that the relative error of the resonance frequency of C#6 was approximately 5% for $\Delta L =$

Table 1 Resonance frequency f_n obtained using calculation models of the experimental flutes. $\Delta L = 42$ mm (left) and 58 mm (right).

		f_2	f_3	f_2	f_3 [Hz]
C#6	N	1,169	1,736	1,109	1,651
	LC	1,216	1,815	1,151	1,720
	LT	1,203	1,798	1,140	1,704
D6	N	909	1,231	890	1,183
	LC	1,006	1,243	993	1,188
	LT	1,020	1,236	1,006	1,184

42 and 58 mm. In contrast, the relative error of the resonance frequency of D6 was below 1% for $\Delta L = 42$ and 58 mm. Here, the relative error was calculated using the ratio of the difference between the resonance frequencies for LC and N to the resonance frequency for N. Although the resonance frequency was a semitone higher than the actual fundamental frequencies of C#6 and D6 when ΔL was set assuming a normal playing condition, the tendency for the resonance frequency to change depending on the tone hole conditions was the same for $\Delta L = 42$ and 58 mm.

5. Discussion

We investigate the effects of the size and position of the tone hole of the C key on the fundamental frequency by comparing N with LC and LT. For the note C#6, the resonance frequency of the f_2 mode of LC was the highest. In addition, for the note D6, the difference between the resonance frequencies of the f_3 mode was approximately 5 Hz at maximum, as shown in Table 1. These results are in good agreement with the measurement results. That is, we confirmed that the effects of the size and position of the tone holes differed depending on the note. The reason why the effects were different is that the function of the tone hole of the C key differs between C#6 and D6. For note C#6, the tone hole of the C key is the first open tone hole, as shown in Fig. 4, and much radiation occurs from the tone hole of the C key. Hence, the size and position of the tone hole of the C key affect the resonance frequency of C#6. On the other hand, for note D6, the tone hole of the C key is allocated at about two-thirds of the effective length and acts as a register hole when the fingering is as shown in [10]. A pressure node occurs near the tone hole of the C key and less radiation occurs from the tone hole of the C key. Hence, the effects of the size and position of the tone hole of the C key on the resonance frequency of the f_3 mode of D6 are small.

On the basis of the above results, to investigate the effects of the size and position of the tone hole in more detail using the calculation model, the resonance frequency was calculated when the position of the C key tone hole was moved at 1 mm increments. The calculation results are shown in Fig. 5. The resonance frequencies of the f_2 mode for C#6 and the f_3 mode for D6 are shown. In this figure, it is seen that the resonance frequency for C#6 changed linearly with the size and position of the tone hole of the C key. In contrast, the resonance frequency of the f_3 mode for D6 showed a nonlinear relationship with by the position of the tone hole of the C key with a larger tone hole.

6. Conclusion

Although the results of this study are limited to C#6 and D6, we confirmed the effects of the size and position of tone holes on the fundamental frequency of these notes quantitatively. These results indicate that it is possible to change the

Fig. 5 Resonance frequency when the position of the C key tone hole is moved in 1 mm increments. The f_2 mode for C#6 and the f_3 mode for D6 are shown. The sizes of tone holes are 8 mm and 15.3 mm. The positive difference from the original position indicates the C key tone hole is away from the embouchure hole.

fundamental frequency of C#6 by changing the size and position of the tone hole of the C key within a certain range without significantly affecting the fundamental frequency of D6.

By accumulating experimental results, we consider that these findings will contribute to further development of flutes. Hence, we will continue to construct experimental flutes with different sizes and positions of tone holes, and investigate how changing the size and position of the tone holes affects the fundamental frequency.

References

- [1] J. W. Coltman, "Designing the scale of the Boehm flute," *Woodwind Q.*, Issue 4, pp. 24–41 (1994).
- [2] T. Boehm, *The Flute and Flute-playing*, translated by D. C. Miller (Dover, New York, 1964).
- [3] R. Caussé, J. Kergomard and X. Lurton, "Input impedance of brass musical instruments: Comparison between experiment and numerical models," *J. Acoust. Soc. Am.*, **75**, 241–254 (1984).
- [4] S. Adachi and M. Yamada, "An acoustical study of sound production in biphonic singing, Xöömij," *J. Acoust. Soc. Am.*, **105**, 2920–2932 (1999).
- [5] S. Adachi, "Resonance modes of a flute with one open tone hole," *Acoust. Sci. & Tech.*, **38**, 14–22 (2017).
- [6] D. H. Keefe, "Theory of the single woodwind tone hole," *J. Acoust. Soc. Am.*, **72**, 676–687 (1982).
- [7] D. H. Keefe, "Experiments on the single woodwind tone hole," *J. Acoust. Soc. Am.*, **72**, 688–699 (1982).
- [8] D. H. Keefe, "Woodwind air column models," *J. Acoust. Soc. Am.*, **88**, 35–51 (1990).
- [9] A. H. Benade and J. W. French, "Analysis of the flute head joint," *J. Acoust. Soc. Am.*, **37**, 679–691 (1965).
- [10] J. Wofle, J. Smith, J. Tann and N. H. Fletcher, "Acoustic impedance spectra of classical and modern flutes," *J. Sound Vib.*, **243**, 127–144 (2001).