Cross spectral method for the estimation of ground-reflection characteristics and free-field spectrum of noise

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A new method for the estimation of reflection characteristics of a ground and a free-field spectrum of noise is presented. The method utilizes informations of differences of ripples created in power and cross spectra of the sound pressure signals that are simultaneously observed by two microphones closely spaced along an axis normal to the ground surface. Ground effects will change due to various factors in measuring conditions. Therefore it will be desirable to estimate ground effects simultaneously with the measurement of sound pressure signals. The proposed method enables one to make an evaluation of ground effects using noise signals themselves even though their frequency characteristics are not known. Results of several practical performances in cases of an aircraft fly-over noise and others are also described.

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1. INTRODUCTION

In acoustic measurements over a ground plane, the sound pressure signal received by a microphone consists of a direct sound wave and a reflected or scattered one from the ground surface. This multipath propagation causes signal reinforcements and cancellations, and consequently creates anomalous peaks and dips in the observed acoustic spectrum, which is in a quite different form from that in a free-field condition.

Ground effects will change due to various factors such as the geometry between a source, receivers and a ground plane, reflection characteristics of the ground and meteorological conditions. Therefore, in any noise propagation measurements over ground, it will be desirable to make an estimation of ground effects simultaneously with the measurement of sound pressure signals and then to correct the measured spectrum to an invariant form, i.e., a free-field spectrum, if necessary.

A number of different methods to evaluate the ground impedance have been reported,¹⁾ such as interferometric techniques (impedance tube,²⁾ free-field,³⁾ and inclined truck⁴⁾), pressure gradient measurements⁵⁾ and synchronized correlation method.^{6,7)} But in order to correct ground effects in measured noise spectra, a special experiment for the estimation of the ground impedance by making use of test sounds with well-known frequency characteristics has to be carried out. Moreover, interferometric techniques are time consuming because of multiple single-frequency measurements required. In contrast to them, there is another method, sometimes called a curve fitting technique.^{8,9)} This method requires no extra experiment for measuring the impedance, but involves a considerable amount of trial and error computation, besides the unknown source spectrum affects the results.

This paper describes a new method for the estimation of reflection characteristics of a ground and a

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free-field spectrum of noise. The method is essentially an interferometric technique and utilizes informations of differences of ripples created in power and cross spectra of the sound pressure signals, which are simultaneously observed by two microphones closely spaced along an axis normal to the ground surface. Sound signals do not have to be definitely known about their frequency characteristics if they have some acoustic power at frequencies over the range of interest. Therefore, it is possible to do estimations using noise signals themselves. Computational procedures and some of results of practical performances in cases of an aircraft fly-over noise and others are also described. The noise source is assumed to be so far from the receiver position that the sound waves at microphones are considered to be plane waves. A special case of spherical waves is discussed in Appendix II.

2. THEORETICAL BACKGROUND

2.1 Ripples in the Power Spectrum

Ground reflections of sound waves cause multipath interferences, and consequently anomalous ripples in an observed noise spectrum are created. These ripples and their effects on the spectrum will change due to variations in measuring conditions. Especially in such cases as an aircraft fly-over noise measurement, changes will continuously occur as the noise source moves. Figure 1 shows an example of a time pattern of aircraft fly-over noise spectra, which were processed by a real time spectrum analyzer.

Fig. 1 Time pattern of sound pressure spectra of an aircraft fly-over noise measured above a grass-covered ground. Spectra are expressed in linear scale of sound intensity.

Fig. 2 Geometry of receivers and acoustic ray paths.

2.2 Formulation

In practical cases, noise signals are often radiated from a number of sources and may include several echoes. For simplicity, however, the analysis in the present work is restricted to the case of a single source far from a receiver enough for plane wave propagation and of a single echo. Furthermore, a flat ground surface, no background noise and a perfect coherency between the direct and reflected waves are assumed.

In the geometry of measurements shown in Fig. 2, two microphones are closely spaced along an axis normal to the ground. The sound pressure signal $y_i(t)$ at a microphone $M_i (i=1,2)$ can be written in the form

$$y_i(t) = s(t - \Delta_{1i}) + \int_0^\infty a(\tau) \cdot s(t - \Delta_{1i} - \Delta_i - \tau) d\tau, \quad (1)$$

where $s(t)$ is an incident sound pressure signal, $a(t)$ is the impulse response of the ground surface. The quantity Δ_i is the acoustic path difference between the direct and reflected sounds arriving at the i -th microphone. The quantity Δ_{1i} is the difference of arrival times of direct sound waves between the two microphones M_1 and M_i . These are given by

$$\Delta_i = \frac{2H_i}{c} \cos(\theta) \quad (2)$$

and

$$\Delta_{1i} = \frac{H_1 - H_i}{c} \cos(\theta) = \frac{\Delta_1 - \Delta_i}{2}, \quad (3)$$

where c is the sound velocity, θ is an angle of incidence and H_i is a height of the i -th microphone from the ground. A power spectrum $P_i(f)$ of $y_i(t)$

is, from Eq. (1), given by

$$P_i(f) = P_s(f) \cdot |1 + A(f) \cdot \exp(-j2\pi f\Delta_i)|^2, \quad (4)$$

where $P_s(f)$ is the power spectrum of $s(t)$, $A(f)$ is Fourier transform of $a(t)$ and $j = \sqrt{-1}$. In a similar way, a cross spectrum $P_{12}(f)$ of $y_1(t)$ and $y_2(t)$ is given by

$$P_{12}(f) = P_s(f) \cdot \{1 + A^*(f) \cdot \exp(j2\pi f\Delta_1)\} \cdot \{1 + A(f) \cdot \exp(-j2\pi f\Delta_2)\} \cdot \exp(-j2\pi f\Delta_{12}), \quad (5)$$

where the asterisk * denotes complex conjugate. $A(f)$ means the complex reflection coefficient of sound pressure signals at the ground on the basis of a plane wave assumption.

We consider a problem to estimate $A(f)$ by solving Eqs. (4) and (5). To deal with this problem, we need to know delay times Δ_1 and Δ_2 , which can be estimated by cepstral technique^{10,11)} or by linear prediction^{12,13)} as being stated in next sub-section 2.3. Using evaluated values of Δ_1 and Δ_2 , and eliminating $P_s(f)$ from Eqs. (4) and (5), we can obtain the following expressions

$$A(f) = \exp(j2\pi f\nu_{12}) \cdot \frac{\frac{P_{12}(f)}{P_1(f)} \cdot \exp(j2\pi f\Delta_{12}) - 1}{\exp(j2\pi f\Delta_{12}) - \frac{P_{12}(f)}{P_1(f)}} \quad (6)$$

and

$$A(f) = \exp(j2\pi f\nu_{12}) \cdot \frac{\frac{P_{12}^*(f)}{P_2(f)} \cdot \exp(-j2\pi f\Delta_{12}) - 1}{\exp(-j2\pi f\Delta_{12}) - \frac{P_{12}^*(f)}{P_2(f)}}, \quad (7)$$

where ν_{12} is a delay time that corresponds to the acoustic path difference between the direct and reflected sound waves at the midpoint of the two microphones M_1 and M_2 , i.e., H_r , and is given by

$$\begin{aligned} \nu_{12} &= \frac{\Delta_1 + \Delta_2}{2} = \frac{H_1 + H_2}{c} \cdot \cos(\theta) \\ &= \frac{2H_r}{c} \cdot \cos(\theta). \end{aligned} \quad (8)$$

2.3 Estimation of Delay Times

The determination of delay times Δ_1 and Δ_2 from a direct measurement of the angle θ and heights of microphones H_1 and H_2 is not always

Fig. 3 An illustrative example of estimating echo delay time using (a) auto-correlation function, (b) cepstrum, and (c) auto-correlation function of a time sequence of prediction errors for linear prediction modeling. (d) shows the measured sound pressure spectrum in decibel and a linear prediction model fit of order $p=1$.

feasible, for instance, in the case of thick grass growing on the ground, because the surface boundary of the ground may not clearly be definable. Moreover, it is not easy to measure θ directly when the noise source moves like an aircraft fly-over.

Computing a correlation function is one way to estimate delay times. But the noise spectrum is not white in general, and its auto-correlation function may have a long response. Then it is often difficult to separate the direct and ground-reflected waves in the correlation function (see Fig. 3(a)). Even in this case, cepstral technique and linear prediction can be applied successfully.

In the present work a cepstrum is defined as Fourier transform of a logarithmic function of a power spectrum. A ripple component due to an echo is transformed to a train of impulses, whose quefrequencies correspond to integral multiples of the expected echo delay time (see Fig. 3(b)). Here the

quefrequency is an independent variable of the cepstrum and has the dimension of time.

By applying linear prediction to measured sound pressure signals, one can also estimate an echo delay time. That is, in the auto-correlation function of a time sequence of prediction errors, the echo appears as an impulsive component at a time that corresponds to the echo delay time (see Fig. 3(c)). This method is superior to cepstral technique when the data length being utilized is very short.

In the above two methods, reflection characteristics of the ground are assumed to be independent of a frequency. In practical procedure, not only for a hard perfectly reflecting surface such as asphalt, but also for a soft, finite-impedance surface like a grass-covered ground, clear ripples can be observed in the spectrum as long as the acoustic signal is incident on the ground at a non grazing angle. The echo delay times obtained by the above methods may include estimational errors in some degree, because reflection characteristics are often dependent on frequency and the resolvability of delay time estimation is limited.

2.4 III Conditions

Equations (4) and (5) cannot be solved when time delays Δ_1 and Δ_2 satisfy the relation

$$2\pi f \cdot (\Delta_1 - \Delta_2) = 4\pi f \cdot \frac{H_1 - H_2}{c} \cdot \cos(\theta) = 2n\pi, \quad (9)$$

where n is any integer. In this case power and cross spectra, i.e., $P_1(f)$, $P_2(f)$ and $P_{12}(f)$, take the same value at frequencies where Eq. (9) is satisfied. In other words, the accuracy in estimation of reflection characteristics using Eqs. (6) and (7) will be strongly reduced in the neighborhood of frequencies that satisfy Eq. (9). These frequencies decrease in number as the difference of heights of microphones, i.e., $\Delta H = H_1 - H_2$, becomes narrower. However, an extremely narrow difference of heights also reduces the accuracy in estimation, because effects of scattering of sounds by microphones themselves can never be neglected at higher frequencies. Therefore, it will be desirable to select some appropriate ΔH corresponding to both the angle of sound incidence θ and the frequency range to be evaluated. Note that Eq. (9) is always satisfied at a frequency of zero, where one can never obtain any good estimation of reflection characteristics regardless of ΔH , as can

be seen in examples of practical performances in a later section 3.

2.5 Ground Impedance and Free-field Spectrum

The sound source is assumed to be so far from the observer that the acoustic signal at the receiver position is considered to be a plane wave. Then the quantity $A(f)$ derived in the sub-section 2.2 is equivalent to the complex reflection coefficient of a plane wave. The ground is assumed to behave as a locally reacting surface,¹⁴⁾ which will be satisfied in most cases. Therefore, a normalized impedance of the ground $Z(f)/\rho c$ is computed from $A(f)$ as follows:

$$\frac{Z(f)}{\rho c} = \frac{1}{\cos(\theta)} \cdot \frac{1 + A(f)}{1 - A(f)}, \quad (10)$$

where ρ is the density of the air and θ is an angle of incidence and is given by

$$\theta = \arccos\left(\frac{\Delta_{12}}{\Delta H} \cdot c\right). \quad (11)$$

The measured power spectrum, from which reflection characteristics of the ground are estimated, can be corrected to a spectrum in a free-field condition. This free-field spectrum is equivalent to the power spectrum of the incident signal, i.e., $P_s(f)$. It can be computed from Eq. (4) using the above estimated quantities $A(f)$ and Δ_i ($i=1, 2$) as

$$P_s(f) = \frac{P_i(f)}{|1 + A(f) \cdot \exp(-j2\pi f \Delta_i)|^2}. \quad (12)$$

We have to take care that the accuracy in estimation of the free-field spectrum becomes extremely reduced at frequencies where the following relations are satisfied:

$$|A(f)| \simeq 1 \quad \text{and} \quad 2\pi f \Delta_i \simeq (2n+1) \cdot \pi + \angle A(f), \quad (13)$$

where $|A(f)|$ is the amplitude of $A(f)$ and $\angle A(f)$ is the phase angle of $A(f)$.

When reflection characteristics of the ground are independent of a frequency, we can estimate the free-field spectrum from the cepstrum by applying a comb-filtering and taking inverse Fourier transform (see Appendix I).

2.6 Estimation of Reflection Characteristics Using only Power Spectra

In the sub-section 2.2, we obtained Eqs. (6) and (7) as solutions of linear equations using both power

and cross spectra. There is another way to derive equations that evaluate reflection characteristics of the ground, $A(f)$. That is, by making use of Eq. (4) and power spectra measured at three distinct heights of microphones along an axis normal to the ground, one can obtain $A(f)$ as a solution of simultaneous quadratic equations. This method has an advantage that an estimation using only one microphone can be possible in the case of a stationary noise. But it remains a serious problem that the existence of a solution cannot necessarily be guaranteed when the presence of background noise is not neglected, so Eqs. (6) and (7) are utilized in later sections. A similar method was used by Watanabe and Kotake¹⁵⁾ in single-frequency measurements of reflection characteristics.

3. PRACTICAL PROCEDURE

3.1 Computational Steps

The steps necessary to obtain reflection characteristics of a ground and a free-field spectrum of noise are as follows: (i) calculate power and cross spectra, (ii) estimate delay times by cepstral technique or by linear prediction, (iii) calculate the coherency $\gamma_{12}(f)$ between signals of the two microphones as

$$\gamma_{12}(f) = |P_{12}(f)| / \sqrt{P_1(f) \cdot P_2(f)}, \quad (14)$$

(iv) calculate reflection characteristics, (v) calculate the ground impedance and the free-field spectrum, and (vi) adjust values of delay times and make trial and error until an optimal estimation will be obtained. In the last step, $P_3(f)$ and Δ_3 are utilized (see the description in a later section 3.3). These steps are schematically diagrammed in Fig. 4.

3.2 Coherency

In the sub-section 2.2, effects of the atmospheric turbulence, ground surface roughness and background noise are assumed to be so small that the direct and reflected waves incident to microphones are considered to be fully coherent each other. To make sure of this assumption, it is necessary to compute the coherency. If none of the atmospheric turbulence, surface roughness and background noise exists, all signals, i.e., the direct and reflected waves at each microphone become perfectly coherent and $\gamma_{12}(f) = 1$.

Equations (6) and (7) that express reflection characteristics are equivalent each other. But when

Fig. 4 Schematic diagram of computational steps to obtain reflection characteristics and free-field spectrum.

the presence of background noise can never be neglected, it is better to use either equation of the two corresponding to the larger frequency component of $P_1(f)$ or $P_2(f)$, in order to reduce effects of background noise. For the computation of a free-field spectrum by Eq. (12), it is desired to do the same treatment.

3.3 Errors in Delay Times

As stated in 2.3, delay times which are estimated by cepstral technique or by linear prediction may include some estimation errors because of the frequency dependence of reflection characteristics and of the limited time resolution in the delay time

Fig. 5 Computer simulation for the evaluation of effects of errors in the delay time Δ_{12} on reflection characteristics: (a) phase and (b) amplitude.

Reflection characteristics were computed from Eqs. (6) and (7) by the use of a delay time Δ_{12} including an error of 0 to $-20.0 \mu\text{s}$. True curves of reflection characteristics are those that are depicted in straight lines.

estimation. These errors affect the above procedure, and consequently the results obtained in the steps (iv) and (v) may be deteriorated. Therefore, the repetition of the procedure described in the step (vi) is necessary.

Figure 5 is an example of computer simulation of effects of errors in the delay time Δ_{12} on reflection characteristics $A(f)$ computed from Eqs. (6) and (7). One can see that only several tens of micro-seconds of errors in Δ_{12} produce anomalous ripples of high frequency around the true curve of reflection characteristics. To correct these ripples, we utilize one more microphone at some distinct height H_3 and do trial and error so that the measured power spectrum $P_3(f)$ of the third microphone coincide with the predicted one $\hat{P}_3(f)$ that is computed from the results in the steps (iv) and (v). For the criterion of fitness the following normalized mean square error averaged over some appropriate frequency range is utilized:

$$\bar{E} = \frac{1}{N} \cdot \sum \left\{ \frac{P_3(f) - \hat{P}_3(f)}{P_3(f)} \right\}^2. \quad (15)$$

From the prediction error E_{final} of final estimations, the accuracy in estimation can be expressed as follows:

$$\overline{\hat{P}_3(f)/P_3(f)} = 1 \pm \sqrt{E_{\text{final}}}. \quad (16)$$

Errors of ν_{12} also have influences on estimation.

Fig. 6 Computer simulation for the evaluation of effects of errors in the delay time ν_{12} on reflection characteristics: (a) phase and (b) amplitude.

Reflection characteristics were computed from Eqs. (6) and (7) by the use of a delay time ν_{12} including an error of 0 to $-15.0 \mu\text{s}$. All amplitude curves including some errors in the delay times coincide with the true curve, and have resulted in a single straight line.

An example of computer simulation is shown in Fig. 6. But errors of ν_{12} give no influences on the absolute values of reflection characteristics and the free-field spectrum. For this reason, errors of ν_{12} cannot be minimized by the above procedure. To do with this minimization, we have to make another noise measurement at a distinct angle of sound incidence at the same time, and then to determine a final estimation so that the two curves of the computed ground impedance coincide with each other. However, in examples being described in next section, we only made manual adjustments of ν_{12} slightly so as to obtain a smooth curve of the impedance.

3.4 Effects of Sound Scattering

There is another factor to give influences on estimation, i.e., scattering of sound by microphones themselves. We carried out a measurement in an anechoic room and confirmed that effects of scattering on the observed power spectra were very small at frequencies below 5 kHz as long as microphones less than 1 inch in diameter were used and a difference of heights of two microphones was 10 to 15 cm.

4. APPLICATIONS OF THE METHOD

4.1 Aircraft Fly-over Noise

Field measurements were made on a large rela-

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tively flat ground (sideline of a runway end at Haneda Airport, Tokyo). The ground was almost bare and soft. But it was very wet due to heavy rain fall during the measurements. Therefore, the estimation of reflection characteristics of the ground resulted in nearly perfect reflection as shown in Figs. 7(a)–(f)

Sound pressure signals were received by microphones, 1 inch in diameter, of sound level meters and were recorded on a 2-channels tape recorder. Heights of microphones from the ground were 1.05 and 0.95 m, and the horizontal distance from microphones to the runway was about 200 m. The reproduced signals from the tape recorder were passed through a bandpass filter (pass-band: 1.8–4,500 Hz) so that aliasing in the spectrum was not a problem,

Fig. 7 Results of the estimation for an aircraft fly-over noise: (a) and (b) power spectra, (c) coherency, (d) and (e) reflection characteristics, and (f) free-field spectrum.

and then they were sampled at every 100 μ s. Afterwards, power and cross spectra were computed by averaging 15 independent periodograms, each of which was obtained by Fourier transform of $2^9 = 512$ sample data points. Total data length was only 0.768 s. The reason is that as an aircraft fly over with a high speed, sound pressure signals over a very long time interval cannot be considered to be sampled from a stationary random process, as can be seen from the example of a time pattern of spectral changes in Fig. 1.

In this case, no sound pressure signals to estimate the power spectrum $P_3(f)$ for the correction of effects of errors in delay times were obtained because of 2-channels tape-recording. So, in place of $P_3(f)$, we utilized either of the two power spectra $P_1(f)$ and $P_2(f)$, which was not used in the computation of reflection characteristics for each frequency (see the description in the sub-section 3.2).

In spite of a large spectral dynamic range, the estimation of both reflection characteristics and the free-field spectrum was done successfully at frequencies so high as about 3 kHz. The reflection characteristics obtained exhibit that the ground was a surface of nearly perfect reflection, i.e., $A(f) \approx 1$ for each frequency. In this case, the free-field spectrum was also computed from the cepstrum by applying a comb-filtering and then taking inverse

Fig. 8 (a) Free-field spectrum computed using cepstral technique. (b) Cepstrum level of the power spectrum $P_1(f)$ in Fig. 7(a).

Fourier transform (see Appendix I). The result in Fig. 8(a) shows a good agreement with that of Fig. 7(f).

In other noise measurements, a surface that was paved with asphalt and a sea surface were also proved to be perfectly reflecting.

4.2 Experiments on a Finite Impedance Ground

The method was applied to field measurements on a grass-covered ground as an example of a finite impedance boundary. The measurements were made in June. White noise through a loudspeaker with unknown frequency characteristics was used as a noise source. The loudspeaker was set up on the roof of a building. The geometry was as follows: the height of the loudspeaker from the ground 12

m, heights of microphones from the ground 0.96 and 0.81 m, and the horizontal distance between the loudspeaker and microphones 29.0 m. An extra microphone was set up at a height 1.11 m along the same axis in order to correct effects of errors in delay times. The ground was not wet and the grass was short. The ground surface where experiments were made was almost flat except small-scale roughness and gentle slope on the whole. The data were processed in the same way as that in 4.1.

Results of the estimation are shown in Figs. 9(a)–(f). Despite the high coherency, the estimation could not be made successfully at frequencies higher than about 3 kHz, where anomalous ripples in reflection characteristics were formed. Also an incorrect peak appears at a very low frequency part of the esti-

Fig. 9 Results of the estimation for field measurements over a grass-covered ground: (a) and (b) power spectra, (c) coherency, (d) and (e) reflection characteristics, and (f) free-field spectrum.

Fig. 10 Ground impedance computed from the results in Fig. 9.

The field measurement was made in June.

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mated free-field spectrum. The ground impedance, shown in Fig. 10, was calculated using the reflection characteristics of Figs. 9(d) and (e) after being taken of a moving average on frequencies so as to obtain a smooth curve. The dashed curves in Fig. 10 were computed using the following simple relations which were derived by Delany and Bazley¹⁸⁾ in order to represent the characteristic impedance $Z(f)$ of fibrous absorbent materials:

$$R(f)/\rho c = 1 + 9.08 \cdot (f/\sigma)^{-0.75} \quad (17)$$

and

$$X(f)/\rho c = -11.9 \cdot (f/\sigma)^{-0.73}, \quad (18)$$

where σ is the specific flow resistance and the im-

pedance $Z(f)$ is

$$Z(f) = R(f) + j \cdot X(f). \quad (19)$$

The value of the specific flow resistance $\sigma = 400$ was used in this example. A good agreement of these curves to those of the estimated impedance shows that the above relations can be applied successfully to the case of the ground impedance.

Figure 11 shows another result of the ground impedance that was estimated using almost the same ground surface and geometry as Fig. 10. However, fitting the relations of Eqs. (17) and (18) resulted in a different value $\sigma = 200$. This difference may be ascribed to a seasonal condition of field measurements. Figure 11 was obtained from the measurement in February, when the grass was withered and the ground was dry as compared to the condition for Fig. 10 in June.

From these results together with the last example, one can see that reflection characteristics of the ground can easily vary according to the ground condition, i.e., local and temporal moisture contents of the ground, as was already examined by Dickinson and Doak.³⁾ Therefore, it is indispensable to make simultaneous measurements of sound pressure signals and ground effects.

Note that estimation errors computed from Eq. (16) resulted in $+1.48 \text{ dB} \sim -2.25 \text{ dB}$ for Fig. 10 and in $+2.1 \text{ dB} \sim -4.2 \text{ dB}$ for Fig. 11 (frequency range: $390 \sim 2,930 \text{ Hz}$). These values are quite reasonable by taking account of the tolerance of microphones of sound level meters used in the field measurements.

5. CONCLUSION

A new method for the estimation of a free-field spectrum of noise received on a reflecting ground and reflection characteristics of the ground, or the ground impedance is proposed. The method utilizes both power and cross spectra of sound pressure signals received at microphones, which are closely spaced along an axis normal to the ground. It is able to make an estimation of ground effects simultaneously with noise measurements, and no information about frequency characteristics of the noise source is required. This is quite important because reflection characteristics of the ground easily change their values according to ground conditions. Therefore, it is not possible to make a proper estimation of noise spectra without taking account of ground

Fig. 11 Ground impedance of almost the same ground surface and geometry as those in the case of Fig. 9. But the field measurement was made in February.

effects by simultaneous measurements. Field measurements of an aircraft fly-over noise and others were carried out in order to confirm the validity of the proposed method. The estimation of reflection characteristics of the ground and the free-field spectrum of noise was done successfully up to frequencies about 3 kHz.

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APPENDIX I: CEPSTRUM

A logarithmic function of a measured acoustic power spectrum can be expressed by Fourier series expansion as follows:

$$\ln[P_i(f)] = \ln[P_s(f)] - 2 \cdot \sum_{n=1}^{\infty} \frac{(-|A|)^n}{n} \cdot \cos[n(2\pi f \Delta_i - \angle A)], \quad (\text{A.1})$$

where $|A|$ and $\angle A$ are the amplitude and phase components of reflection characteristics respectively and they are assumed to be independent of a frequency. Therefore, the cepstrum is obtained by Fourier transform of Eq. (A.1) as

$$F\{\ln[P_i(f)]\} = F\{\ln[P_s(f)]\} - \sum_{n=1}^{\infty} \frac{(-|A|)^n}{n} \cdot \{\exp(jn \angle A) \cdot \delta(\tau - n \Delta_i) + \exp(-jn \angle A) \cdot \delta(\tau + n \Delta_i)\}, \quad (\text{A.2})$$

where $F\{\}$ denotes Fourier transform, $\delta(\)$ is the Dirac's delta function, and τ is an independent variable, i.e., the quefrequency.

The free-field spectrum, which corresponds to the spectrum $P_s(f)$ of the direct sound wave, can be obtained from the cepstrum. That is, from Eq. (A.2), the cepstrum can be reduced to the free-field spectrum by taking inverse Fourier transform after delta functions due to the echo having been removed by applying a comb-filtering.

Miles *et al.*⁽¹¹⁾ explained that a spectrum due to intensity addition only of the direct and reflected signals was to be obtained by the above procedure.

Fig. 12 Geometry of a source, receivers and acoustic ray paths in a case of spherical propagation.

It may have been originated from confusion of Fourier series expansion with the Taylor series expansion.

APPENDIX II: THEORETICAL FORMULATION FOR SPHERICAL PROPAGATION

As far as the noise source is not very far from a receiver position, sound pressure signals have to be treated as spherical waves. We consider the geometry shown in Fig. 12. In this case, power and cross spectra of sound pressure signals arrived at microphones M_1 and M_2 can be expressed as

$$P_i(f) = \frac{P_s(f)}{R_{di}^2} \cdot \left| 1 + \frac{R_{di}}{R_{ri}} \cdot A_i(f) \cdot W(-f, \Delta_i) \right|^2 \quad (\text{A.3})$$

and

$$P_{12}(f) = \frac{P_s(f)}{R_{d1} \cdot R_{d2}} \cdot \left\{ 1 + \frac{R_{d1}}{R_{r1}} \cdot A_1^*(f) \cdot W(f, \Delta_1) \right\} \cdot \left\{ 1 + \frac{R_{d2}}{R_{r2}} \cdot A_2(f) \cdot W(-f, \Delta_2) \right\} \cdot W(-f, \Delta_{12}), \quad (\text{A.4})$$

where R_{di} and R_{ri} ($i=1,2$) are path lengths of the direct and reflected waves from the source to microphones respectively, Δ_i ($i=1,2$) is the acoustic path difference between the direct and reflected waves as follows:

$$\Delta_i = \frac{R_{ri} - R_{di}}{c}, \quad (\text{A.5})$$

the quantity Δ_{12} is the time difference between the direct waves to the two microphones as

$$\Delta_{12} = \frac{R_{d2} - R_{d1}}{c}, \quad (\text{A.6})$$

and the quantity $W(f, \Delta)$ is

$$W(f, \Delta) = \exp(j2\pi f\Delta). \quad (\text{A.7})$$

We assume that the difference between angles of incidence of the two reflected waves is so small that reflection characteristics at each angle of incidence are considered to take the same values, i.e., $A_1(f) = A_2(f) = A(f)$. From this assumption the following expressions are obtained as a first order approximation, where R_{di} , $R_{ri} \gg \Delta H$:

$$A(f) = \frac{R_{r1}}{R_{d1}} \cdot W(f, \nu_{12} + \delta) + \frac{\frac{P_{12}(f)}{P_1(f)} \cdot W(f, \Delta_{12} - \delta) - \frac{R_{d1}}{R_{d2}}}{\frac{R_{r1}}{R_{r2}} W(f, \Delta_{12} + \delta) - \frac{P_{12}(f)}{P_1(f)}} \quad (\text{A.8})$$

and

$$A(f) = \frac{R_{r2}}{R_{d2}} \cdot W(f, \nu_{12} + \delta) + \frac{\frac{P_{12}^*(f)}{P_2(f)} \cdot W(-f, \Delta_{12} - \delta) - \frac{R_{d2}}{R_{d1}}}{\frac{R_{r2}}{R_{r1}} \cdot W(-f, \Delta_{12} + \delta) - \frac{P_{12}^*(f)}{P_2(f)}} \quad (\text{A.9})$$

where δ is defined by

$$\delta = \frac{\Delta H}{2c} \cdot \{\cos(\theta_r) - \cos(\theta_a)\}, \quad (\text{A.10})$$

where $\theta_a \approx \theta_{a1} \approx \theta_{a2}$ and $\theta_r \approx \theta_{r1} \approx \theta_{r2}$.

One has to take care that as the above quantity $A(f)$ corresponds to reflection characteristics for spherical waves, it differs from the complex reflection coefficient of plane waves.¹⁴⁾ Therefore one cannot obtain the ground impedance by taking Fourier transform of Eq. (A.8) or Eq. (A.9). But when the angle of incidence is not very large, the correction for sphericity is small and the difference between Eqs. (A.8) and (6) (or Eqs. (A.9) and (7)) can be ignored so as to obtain a rough estimation of the ground impedance.

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