An evaluation method of L_{eq} for noise fluctuation of non-Gaussian and nonstationary types

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An L_{eq} is usually calculated through an averaging operation of anti-logarithm of the measured instantaneous level value in a decibel scale. Considering the nonlinear transformation of level observation, there seems to remain a principal problem that the average energy can not be exactly obtained owing to the inequality of quantization width in an energy scale differing from the equality of quantization width in a level scale. In this paper, the averaged sound energy is estimated on the basis of level statistics with help of the moment generating function of sound level by considering the fundamental relationship between the sound level and its energy. Then, an evaluation method of L_{eq} is derived in the series expansion form reflecting the stationary and Gaussian properties as the first expansion term and the correction term for the deviation from these statistical properties as the higher order expansion terms. Finally, the proposed estimation method is experimentally confirmed by applying it to the actual noise environment inside and outside a room.

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1. INTRODUCTION

The equivalent continuous A-weighted sound pressure level L_{eq} is adopted as a basic measure of environmental noise in ISO 1996 Part 1. This L_{eq} is usually measured based on the continuous integral form or the arithmetic mean of sound energy values calculated by an anti-logarithm of the discrete level observation data according to its definition. Accordingly, there seems to remain a principal problem that the average energy can not be exactly obtained owing to the inequality of quantization width in an energy scale differing from the equality of quantization width in a decibel scale. On the other hand, it is well-known that an L_{eq} value given by discrete level observation is influenced by its sampling number and/or sampling frequency.¹⁾ To evaluate exactly a mean value of the sound energy under the actual level observation, it seems to be more reason-

able to estimate it indirectly by use of theoretical relationship between energy statistics and level statistics than to apply directly an anti-logarithmic type transformation to instantaneous decibel data. As is well-known, some equations for evaluating L_{eq} of environmental noise were reported in connection with mean value, variance and/or L_{50} for a typical case when the sound level can be described by the Gaussian distribution. In these cases, of course, the non-Gaussian property and/or the nonstationary property were not reflected in those estimation methods. However, for actual cases like an inner city situation, the noise level distribution shapes are far from Gaussian in shape and the assumption of a Gaussian distribution may be invalid.²⁾ Recently, Ohta *et al.* published an evaluation method of L_{eq} in connection with the mean, variance and L_x .³⁾ Maruyama *et al.* derived a general way of estimating L_{Aeq} from the level of observed noise in terms of

cumulants.⁴⁾

In this study, first, considering the fundamental relationship between the sound level and its energy, the averaged sound energy is estimated on the basis of level statistics with help of the moment generating function of sound level. An evaluation method of L_{eq} is derived in the series expansion form taking a typical case of Gaussian distribution type as the first expansion term and the correction term for the deviation from Gaussian type as the higher expansion terms, which reflect the non-Gaussian and/or nonstationary properties of level moment statistics. Finally, the proposed estimation method is applied to the actual noise environment inside and outside a room. Thus, the theoretical estimates of L_{eq} show good agreement with experimentally sampled values in the actual case showing a remarkable deviation from Gaussian type in the probability distribution shape of actual data.

2. THEORETICAL CONSIDERATION

2.1 L_{eq} Estimation of Stationary Non-Gaussian Noise

A sound level observation, L , of arbitrarily fluctuating environmental noise is written in terms of sound energy, E , as:

$$L \triangleq 10 \log_{10} (E/E_0) = M \log_{10} (E/E_0), \quad (1)$$

where

$$M \triangleq 10/\log_{10} 10, \quad E_0 \triangleq 10^{-12} \text{ [W/m}^2\text{]}.$$

Considering Eq. (1), the moment generating function, $M_L(\theta)$, of level is given by the expectation of θM power of sound energy as (see Ref. 1) or 3):

$$M_L(\theta) \triangleq \langle e^{\theta L} \rangle = \langle (E/E_0)^{\theta M} \rangle. \quad (2)$$

The other expression of, $M_L(\theta)$, can be given in terms of the cumulant statistics, κ_n , as:

$$M_L(\theta) = \exp \left\{ \sum_{n=1}^{\infty} \frac{\kappa_n}{n!} \theta^n \right\}. \quad (3)$$

On the other hand, L_{eq} is originally defined as:

$$L_{eq} \triangleq 10 \log_{10} \langle E/E_0 \rangle. \quad (4)$$

Since the average sound energy can be easily estimated from Eq. (2) by substituting $1/M$ for θ , the following estimation algorithm of L_{eq} can be derived as^{3,4)}:

$$L_{eq} = M \log_{10} \{ M_L(\theta) |_{\theta=1/M} \} = M \sum_{n=1}^{\infty} \frac{\kappa_n}{n!} (1/M)^n. \quad (5)$$

In general, it is difficult to observe directly the above

cumulant statistics in the usual level observation. Now, considering the definition of cumulant and moment statistics, we can obtain a general form of cumulant statistics in terms of the moment statistics as follows:

$$\kappa_n = \mu_n + \sum_{l=1}^{n-1} \binom{n-1}{l} \mu_{n-l} \sum_{t=1}^l \frac{(-1)^t}{t!} \left[\sum_{s=1}^t \binom{t}{s} (-1)^{t-s} \cdot \sum_{i_1+\dots+i_s=l} \frac{l!}{i_1! \dots i_s!} \mu_{i_1} \dots \mu_{i_s} \right]. \quad (6)$$

Thus, the L_{eq} estimation can be given as:

$$L_{eq} = \mu + 0.115\sigma^2 + \sum_{n=3}^{\infty} \frac{(1/M)^{n-1}}{n!} \left[\mu_n + \sum_{l=1}^{n-1} \binom{n-1}{l} \mu_{n-l} \sum_{t=1}^l \frac{(-1)^t}{t!} \left\{ \sum_{s=1}^t \binom{t}{s} (-1)^{t-s} \cdot \sum_{i_1+\dots+i_s=l} \frac{l!}{i_1! \dots i_s!} \mu_{i_1} \dots \mu_{i_s} \right\} \right], \quad (7)$$

where μ and σ^2 are respectively mean value and variance of the noise level. Since L_{50} is equal to a mean value μ in a typical case with Gaussian type, the first two expansion terms of Eq. (7) correspond to the well-known estimation formula ($L_{eq} = L_{50} + 0.115\sigma^2$) of L_{eq} under the assumption of a standard Gaussian distribution of level fluctuation. Accordingly, the non-Gaussian property of noise level fluctuation is reflected in the third and higher expansion terms as correction factors in relation to the higher order moment statistics.

2.2 L_{eq} Estimation of Nonstationary Noise

Let us consider the actual nonstationary environmental noise having a temporal change of moment statistics. For this noise, it is usual to show a locally stationary property within a short time interval and a nonstationary property over a long time interval. The moment generating function of this nonstationary noise level fluctuation with a temporal change of every moments can be given as⁵⁾:

$$m_L(\theta) \triangleq \langle e^{\theta L} \rangle = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} e^{\theta L} P(L|\mu) dL \right] P(\mu) d\mu. \quad (8)$$

Here, $P(\mu)$ is the probability distribution for a temporal fluctuation of the moment statistics $\mu = [\mu_1, \mu_2, \dots, \mu_n, \dots, \mu_r]$: the r -th order moment, which causes the non-stationary property of L and $P(L|\mu)$ denotes the conditional probability distribution defined within a short time interval. By introducing the conditional moment generating function, $m_0(\theta|\mu_0)$, $m_L(\theta)$ is given in the series ex-

pansion form taking the stationary case as the first expansion term.

$$\begin{aligned} m_L(\theta) &= m_0(\theta) |\mu_0| \sum_{r=0}^{\infty} \frac{\langle B_r \rangle_{\mu}}{r!} \theta^r \\ &= \sum_{n=0}^{\infty} \frac{\mu_{0n}}{n!} \theta^n \sum_{r=0}^{\infty} \frac{\langle B_r \rangle_{\mu}}{r!} \theta^r, \end{aligned} \quad (9)$$

where $\langle \rangle_{\mu}$ denotes an average operation concerned with the moment statistics. The expansion coefficient B_r reflects the nonstationary property of level fluctuation as a temporal change of moment statistics as follows:

$$\begin{aligned} B_r &= \mu_r - \mu_{0r} + \sum_{l=1}^r \binom{r}{l} (\mu_{r-l} - \mu_{0r-l}) \\ &\quad \cdot \sum_{t=1}^l \frac{(-1)^t}{t!} \sum_{s=1}^t \binom{t}{s} (-1)^{t-s} \\ &\quad \cdot \sum_{i_1+\dots+i_s=l} \frac{l!}{i_1! \dots i_s!} \mu_{0i_1} \dots \mu_{0i_s}, \end{aligned} \quad (10)$$

where μ_{0r} denotes the r -th order moment statistics in a locally stationary case. On the other hand, as shown in the previous section, L_{eq} can be calculated easily from this moment generating function by substituting $1/M$ for θ :

$$\begin{aligned} L_{eq} &= M \log_e \{ m_L(\theta) |_{\theta=1/M} \} \\ &= M \log_e \left\{ \sum_{n=0}^{\infty} \frac{\mu_{0n}}{n!} (1/M)^n \right. \\ &\quad \left. + M \log_e \left\{ \sum_{r=0}^{\infty} \frac{\langle B_r \rangle_{\mu}}{r!} (1/M)^r \right\} \right\}. \end{aligned} \quad (11)$$

Here, the first expansion term in the right-hand side is the L_{eq} estimation in the stationary noise process. Considering Eq. (7), we can obtain:

$$\begin{aligned} L_{eq} &= \mu_0 + 0.115 \sigma_0^2 + \sum_{n=3}^{\infty} \frac{1}{n!} (1/M)^{n-1} \left[\mu_{0n} \right. \\ &\quad + \sum_{l=1}^{n-1} \binom{n-1}{l} \mu_{0n-l} \sum_{t=1}^l \frac{(-1)^t}{t!} \left\{ \sum_{s=1}^t \binom{t}{s} (-1)^{t-s} \right. \\ &\quad \cdot \sum_{i_1+\dots+i_s=l} \frac{l!}{i_1! \dots i_s!} \mu_{0i_1} \dots \mu_{0i_s} \left. \right\} \left. \right] \\ &\quad + M \log_e \left\{ \sum_{n=0}^{\infty} \frac{\langle B_n \rangle_{\mu}}{n!} (1/M)^n \right\}. \end{aligned} \quad (12)$$

In the above equation, the first two expansion terms correspond to an estimation of L_{eq} directly related to the Gaussian property and the third expansion term reflects the non-Gaussian property within the locally stationary case. The fourth expansion term reflects how the nonstationary fluctuation of moment statistics over a long time interval affects the estimation of L_{eq} .

Consequently, from Eqs. (7) and (12), the estimation of L_{eq} connected with non-Gaussian and/or nonstationary properties can be given in a general form containing the well-known simplified relationship of the Gaussian distribution.

$$\begin{aligned} L_{eq} &= \left[\begin{array}{c} L_{eq} \text{ in a} \\ \text{Gaussian} \\ \text{case} \end{array} \right] + \left[\begin{array}{c} \text{correction term of} \\ L_{eq} \text{ in a non-Gaussian} \\ \text{case of stationary type} \end{array} \right] \\ &\quad + \left[\begin{array}{c} \text{correction term of} \\ L_{eq} \text{ in a nonstationary} \\ \text{case} \end{array} \right]. \end{aligned} \quad (13)$$

3. EXPERIMENTAL CONSIDERATION

3.1 Measurement of Environmental Noise and Its Statistical Property

The theoretical method has been experimentally confirmed by applying it to non-Gaussian and nonstationary type level fluctuations inside and outside noise environments observed at a discrete time interval by a sound level meter. Figure 1 shows the actual situation of noise measurement—Case A: a street with the light traffic flow (about 500 to 600 vehicles per hour), Case B: a quiet laboratory room with rare ring of phone or chattering voices. Figure 2 shows the schematic diagram of the noise measure-

A) Outside a room (road traffic noise)

B) Inside a room (laboratory noise)

Fig. 1 Plan views of noise measurement.

Fig. 2 Schematic diagram of the noise measurement.

ment. Sound pressure levels have been observed every one second and L_{eq} 's are observed simultaneously, by using a sound level and an integral sound level meters (NL-10 and NL-11, RION). The total observation time interval was 3,600 s, and was divided into 60 short time intervals of locally stationary type and the moment statistics have been obtained in each interval. Figure 3 shows the cumulative distribution of observed noise within some locally stationary time interval and over a whole time period of observation. The observed level fluctuation shows obviously the non-Gaussian and nonstationary properties. In case A, the whole distribution shape shows slightly a non-Gaussian property with a fluctuation range (about 10 dB) on L_{50} . On the other hand, in case B, the whole distribution shape shows extremely a non-Gaussian property and a nonstationary property with a fluctuation range (about 20 dB) on L_{50} .

3.2 L_{eq} Estimation in Stationary Case

Tables 1, 2 and 3 show comparisons between theoretically estimated results and experimentally observed values for L_{eq} in a stationary case. In these tables, "Gauss" denotes the results by using only mean value μ and variance σ^2 of level fluctuation, "Non-Gauss" denotes the results by Eq. (7) to reflect the non-Gaussian property over a whole time interval. The results based on the usual method using L_{50} and σ^2 are also shown as $L_{50} + 0.115\sigma^2$. The whole time period of observation is 60 min in Tables 1 and 2 and is 10 min in Table 3. The results in Tables 2 and 3 are the estimation

Fig. 3 Cumulative probability distributions of observed noise within some locally stationary time intervals and over a whole observation period. Experimentally sampled points are marked by (●); over a whole period, (□, △); within the 39th and 53rd locally stationary time intervals for case A and (□, ○, △); within the 38th, 1st and 6th locally stationary time intervals for case B. The cumulative probability distribution using Gauss approximation with mean value and variance within a whole period is shown by (—).

Table 1 Comparisons between theoretically estimated results and experimentally observed values for L_{eq} over 60 min (with moment observation at every minute).

Cases	(dB(A))			
	Observed value of L_{Aeq}	$L_{50} + 0.115\sigma^2$	L_{Aeq} by Eq. (7)	
			Gauss $\mu + 0.115\sigma^2$	Non-Gaussian
A) Road traffic noise	68.5	67.7	68.7	68.5
B) Inside room	51.9	44.3	46.7	52.0

Table 2 Comparisons between theoretically estimated results and experimentally observed values for L_{eq} over 60 min (Rough observation case).

Cases	Observed value of L_{Aeq}	$L_{50}+0.115\sigma^2$	(dB(A))	
			L_{Aeq} by Eq. (7)	
			Gauss $\mu+0.115\sigma^2$	Non-Gaussian
A) Road traffic noise	68.5	67.6	68.4	68.6
B) Inside room	51.9	45.5	48.5	53.6

Table 3 Comparisons between theoretically estimated results and experimentally observed values for L_{eq} over 10 min (Rough observation case).

Cases	Observed value of L_{Aeq}	$L_{50}+0.115\sigma^2$	(dB(A))	
			L_{Aeq} by Eq. (7)	
			Gauss $\mu+0.115\sigma^2$	non-Gaussian
A) Road traffic noise	68.4	68.8	69.3	68.3
	66.9	64.2	66.0	66.9
B) Inside room	47.3	45.4	46.5	47.4
	52.7	45.9	50.1	52.5

results with a rough time interval of observing the moment statistics while every 3 min. When one uses only a Gaussian approximation for laboratory room's noise data in Table 1 showing originally a remarkable deviation from Gaussian type in the probability distribution shape, there remains in error 5 dB or more. While, by employing the proposed theoretical results with sufficient expansion terms ($n=5$ to 7 in Eq. (7)) reflecting the non-Gaussian properties of level fluctuation, one can estimate L_{eq} accurately with an error less than 1 dB. Especially, in Tables 2 and 3 (except case B in Table 2), the accuracy of L_{eq} estimation can be preserved in spite of a rough moment observation because of the reliability and stability of moment statistics over a long time interval. The estimation result for case B in Table 2 gives some error more than 1 dB because of the nonstationary property of moment statistics. However, it is obvious that the result by proposed method of L_{eq} estimation is more

Table 4 Comparisons between theoretically estimated results by considering the non-stationary property of noise level fluctuation and experimentally observed values for L_{eq} over 60 min (with moment observation at every minute).

Cases	Observed value of L_{Aeq}	$L_{50}+0.115\sigma^2$	(dB(A))	
			L_{Aeq} by Eq. (12)	
			Local	Non-stationary
A) Road traffic noise	68.5	67.7	67.9	68.5
			71.6	68.6
			63.6	69.0
B) Inside room	51.9	44.3	50.3	51.9
			44.9	51.9
			42.5	51.6

Table 5 Comparisons between theoretically estimated results by considering the non-stationary property of noise level fluctuation and experimentally observed values for L_{eq} over 60 min (Rough observation case).

Cases	Observed value of L_{Aeq}	$L_{50}+0.115\sigma^2$	(dB(A))	
			L_{Aeq} by Eq. (12)	
			Local	Non-stationary
A) Road traffic noise	68.5	67.6	65.0	68.5
			71.6	68.4
			68.8	68.6
B) Inside room	51.9	45.5	49.8	51.9
			45.0	52.3
			37.3	51.5

superior than that by usual method of Gaussian type with use of only L_{50} and σ^2

3.3 L_{eq} Estimation in Nonstationary Case

Tables 4 and 5 show comparisons between experimentally observed L_{eq} values and those by theoretically estimated results considering the non-stationary property of moment statistics over 60 min. In these tables, "Local" denotes the results by using only mean value μ_0 and variance σ_0^2 of level fluctuation in a locally stationary case, and "Non-stationary" denotes the results by Eq. (12) calculated by employing sufficient expansion terms. Table 5

shows the estimation results with a rough time interval of observation as described in the previous section. For laboratory room noise data showing a remarkable non-Gaussian shape, the estimation result by usual method of Gaussian type with L_{50} and σ^2 gives an error 5 dB or more. For this case, the proposed theoretical results reflecting the non-stationary property of level fluctuation over a long time interval can estimate L_{eq} accurately with an error less than 1 dB.

4. CONCLUSION

In this paper, first, we have proposed a new trial of reasonably removing the effect of sampling number, sampling frequency of sound level and inequality of amplitude quantization width in an energy scale for the evaluation of L_{eq} . Concretely, by using the moment generating function and the relationship between the sound level and its energy, an evaluation method of L_{eq} for non-Gaussian and/or nonstationary environmental noises has been derived on the basis of the stochastic change of level moment statistics. Consequently, the proposed evaluation formula for L_{eq} reflects the standard case of Gaussian type in the first expansion term and the correction term for the deviation from Gaussian type in the higher order expansion terms especially in close relation to the non-Gaussian and/or nonstationary properties of level moment statistics. Finally, the proposed method has been applied to the actual noise environmental data inside and outside a room and confirmed experimentally with good agreement to observed L_{eq} values. In this paper, our study has been focussed on only a principal part of L_{eq} evaluation method, since it is in the earlier stage of study. Accordingly, the problems on how many terms of series expansion should be considered and how to establish the criterion for selecting the locally stationary time interval still remain as future's

problems.

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REFERENCES

- 1) Y. Mitani, M. Ohta, and Y. Xiao, "A new estimation method of noise evaluation index, L_{eq} , by use of the statistical information on the arbitrary type noise level fluctuation," *J. Acoust. Soc. Jpn. (E)* **9**, 47-51 (1988).
- 2) C. G. Don and I. G. Rees, "Road traffic sound level distributions," *J. Sound Vib.* **100**, 41-53 (1985).
- 3) M. Ohta, Y. Mitani, and T. Sumimoto, "A general theory for the mutual relationship among several type noise evaluation indices connected with L_{eq} and L_x and its experiment," *J. Acoust. Soc. Jpn. (J)* **41**, 598-607 (1985) (in Japanese).
- 4) M. Maruyama and T. Sone, "A study on the estimation of L_{Aeq} in residential areas," *J. Acoust. Soc. Jpn. (E)* **6**, 235-240 (1985).
- 5) M. Ohta, S. Yamaguchi, and K. Nishimura, "A evaluation method for nonstationary random noise in view of temporal change of statistical moments (General theory and its application to probabilistic estimation problem of road traffic noise)," *J. Acoust. Soc. Jpn. (E)* **5**, 223-231 (1984).
- 6) K. Nishimura and M. Ohta, "An evaluation method of L_{eq} for non-stationary sound level on the basis of a temporal change of moment statistics," *Proc. Autumn Meet. Acoust. Soc. Jpn.*, 511-512 (1986).
- 7) K. Nishimura and M. Ohta, "An evaluation method of L_{eq} for non-stationary noises based on the stochastic change of level statistics," *Inter-Noise* **87**, 1129-1132 (1987).