Time-averaged intensity spectral density of acoustic radiation due to time-varied lift force of rotating blades

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(Received 9 November 1988)

The time-averaged intensity spectral density function of the acoustic radiation from rotating blades is theoretically derived by replacing the blades with rotating dipoles. Dimensional analysis reveals two non-dimensional parameters which play an important role in generating the blade-passing frequency tone and its multiples. Increasing the number of rotor blades widens the peak under the condition that the non-dimensional parameter of the rotational speed is fixed. Moreover, the acoustic spectral density function is not stationary even if the inflow turbulence is homogeneous and isotropic. Further, the time variation of the propagation path due to the rotation should be considered in the computation of the spectral density function. For instance, in the rotor specifications of the present study, the rotor radius is approximately 0.3 m and the rotational speed Mach number is approximately 0.2.

PACS number: 43.28.Ra

1. INTRODUCTION

This paper attempts to derive the time-averaged farfield acoustic intensity spectral density function due to the acoustic radiation from low rotational speed rotors starting with Lighthill's acoustic analogy approach, where the rotor blade is assumed to be acoustically compact. For the present analysis, the dipole source due to blade/turbulence interaction is assumed to be predominant over other radiation sources. A rotating dipole model is examined so that the effect of the dipole rotation can be explicitly accounted for in the analysis. Furthermore, there is no blade-to-blade interaction to be included, such as that developed by Kemp *et al.*¹⁾ The assumption of acoustically compact blades permits replacing distributed dipole sources with a single-point dipole whose strength is equal to the distributed dipoles on the blade. Thus, each blade is replaced with a point dipole or multiples of a dipole.

Lowson²⁾ derived the far-field and near-field

acoustic pressure from a moving singular dipole source by using Lighthill's acoustic analogy approach, where the importance of the acceleration of the moving dipole was accounted for in the expression of acoustic pressure, while the dipole strength was assumed to be constant. Sevik³⁾ derived the sound power spectrum from a rotor operating in turbulence. The noise source is the axial component of the blade lift only. The sound power $S_p(\omega)$ is computed based on the general relation $S_p(\omega) = E\{|P(\omega)|^2\}$, where $S_p(\omega)$ is the power spectrum of acoustic pressure $p(t)$ satisfying $\int_{-\infty}^{\infty} \alpha R_{pp}(\alpha) d\alpha < \infty$, where $R_{pp}(\alpha)$ is the correlation function of $p(t)$ which is assumed to be a wide-sense-stationary process, and the correlation function regards the rotor as being stationary. However, because of the use of only the axial component of the lift, the result applies only near the center line axis of the rotor. Unfortunately, the resulting power spectrum does not predict the blade-passing frequency component

and its multiples, due to the assumption of the correlation function of the upwash at the blades, although, as shown by Robbins *et al.*,⁴⁾ the level and general shape of the predicted spectra agree with the experimental data.

This paper attempts to introduce moving dipoles with a time-variant strength into the theory of Lowson²⁾ to consider more practical situations, and to relate the acoustic intensity spectral density to the spectrum of the blade lift taking into account the rotation of the blade to extend Sevik's theory. In addition, discussed is the influence on the spectrum of the time variation of the propagation path due to the rotation.

2. FAR-FIELD ACOUSTIC PRESSURE

The analysis of the acoustic radiation from rotating blades begins with the nonhomogeneous acoustic radiation from the boundary condition imposed by the blades. The Lighthill acoustic analogy approach leads to an integral equation, with Green's function, for an infinite medium where the integration is carried out over three spatial dimensions and one temporal dimension. Here, the time is regarded as a parameter; i.e., the so-called retarded time concept is employed. It is assumed from the discussion of Farassat⁵⁾ that the surface of a moving body can be replaced with a point dipole source whose strength is equal to the total force on the virtual radiation surface providing the following relation is satisfied:

$$\tau_p \ll L/a \quad (2.1)$$

where τ_p , and L are the characteristic time scale of the surface pressure, and the length scale of the virtual surface of an acoustical source.

Thus, the acoustic wave equation to be solved for rotating blades is

$$\nabla^2 \mathcal{P} - \frac{1}{a^2} \frac{\partial^2 \mathcal{P}}{\partial \tau^2} = -\nabla \cdot \vec{f} \quad (2.2)$$

where

$$\vec{f} = \sum_{m=1}^B \sum_{n=1}^N \vec{l}_{mn}(\tau) \delta(\Phi - \Omega\tau - \Phi_m) \delta(Z) \delta(R - R_n) \frac{\Delta R_n}{R} \quad (2.3)$$

Figure 1 shows the configuration of the relation between the rotating dipole and the observation point, where spherical coordinates and cylindrical coordinates are used for the observation point and the rotating dipole, respectively.

Fig. 1 Observer/source coordinates.

Fig. 2 Lift vector components at (m,n) th strip.

With the aid of the fundamental solution of Eq. (2.2), imposing the conditions for the acoustic far-field and geometrical far-field, and introducing $l_{mn}^{\Phi}(\tau) = -|\vec{l}_{mn}(\tau)| \sin \beta_{mn}$ and $l_{mn}^Z(\tau) = |\vec{l}_{mn}(\tau)| \cos \beta_{mn}$ where $\vec{l}_{mn}(\tau)$ is assumed to be normal to the chord line of a thin airfoil (see Fig. 2), then the following equation is obtained,

$$\begin{aligned} \mathcal{P}(r, \phi, t) &= \frac{1}{4\pi ar} \left(\sum_{m=1}^B \sum_{n=1}^N \frac{\Delta R_n}{\left(1 + \frac{R_n \Omega}{a} \sin \phi \sin(\Omega\tau + \Phi_m)\right)^2} \right. \\ &\quad \times \left\{ -\frac{\partial l_{mn}}{\partial \tau} \sin \beta_{mn} \sin \phi \sin(\Omega\tau + \Phi_m) \right. \\ &\quad \left. - \frac{\partial l_{mn}}{\partial \tau} \cos \beta_{mn} \cos \phi - l_{mn} \Omega \sin \beta_{mn} \sin \phi \right. \\ &\quad \left. \times \frac{\cos(\Omega\tau + \Phi_m)}{1 + \frac{R_n \Omega}{a} \sin \phi \sin(\Omega\tau + \Phi_m)} \right\} \end{aligned}$$

$$+l_{mn}\Omega \cos \beta_{mn} \cos \phi \sin \phi \left. \begin{aligned} & \frac{R_n\Omega}{a} \cos(\Omega\tau + \Phi_m) \\ & \times \frac{R_n\Omega}{1 + \frac{R_n\Omega}{a} \sin \phi \sin(\Omega\tau + \Phi_m)} \end{aligned} \right\} : \quad (2.4)$$

where

$$\tau \approx t - D_{mn}/a,$$

and

$$D_{mn} = r - R_n \sin \phi \cos(\Omega\tau + \Phi_m).$$

Equations (2.4) is an extension of the results of Lowson²⁾ and Morfey *et al.*,⁶⁾ which deal with the acoustic radiation from a moving point force. As will be shown, this relates the acoustic intensity spectral density function at the observation point with the power spectral density function of the turbulence experienced by rotating blades.

As shown in Eq. (2.4), the variation of the distance between the observer and the blade on a time axis causes a phase modulation in $l_{mn}(\tau)$ because $\tau \approx t - (r - R_n \sin \phi \cos(\Omega\tau + \Phi_m))/a$ where the numerator is the distance between the observer and the blade. Furthermore, the variation of the blade velocity Mach number in the direction of the observer, i.e., $(R_n\Omega/a) \sin \phi \sin(\Omega\tau + \Phi_m)$, behaves as an amplitude modulation signal of $l_{mn}(\tau)$ and $\partial l_{mn}(\tau)/\partial \tau$.

Utilizing the results shown in the Appendix, Eq. (2.4) can be rewritten as follows:

$$\mathcal{P}(r, \phi, t) = \frac{1}{4\pi ar} \sum_{m=1}^B \sum_{n=1}^N \sum_{p=-\infty}^{\infty} \left(e^{(p)}_{mn} \frac{\partial l_{mn}}{\partial \tau} - j\Omega f^{(p)}_{mn} l_{mn} \right) \times \exp \left[jp \left(\Omega t + \Phi_m + \frac{\pi}{2} \right) \right], \quad (2.5)$$

where

$$e^{(p)}_{mn} = -b_n^{(p)'} \sin \phi \sin \beta_{mn} - a_n^{(p)'} \cos \phi \cos \beta_{mn}, \quad (2.6)$$

$$f^{(p)}_{mn} = \sin \phi (\cos \phi \cos \beta_{mn} - \sin \beta_{mn}) c_n^{(p)'}, \quad (2.7)$$

$$c_n^{(p)'} = \sum_{p=-\infty}^{\infty} \frac{p J_p \left(p \frac{R_n\Omega}{a} \sin \phi \right)}{\frac{R_n\Omega}{a} \sin \phi} \quad (2.8)$$

$$a_n^{(p)'} = \sum_{s=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} J_s \left(s \frac{R_n\Omega}{a} \sin \phi \right) J_k \left(k \frac{R_n\Omega}{a} \sin \phi \right) \quad (2.9)$$

$p = s + k,$

and

$$b_n^{(p)'} = \sum_{s=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \frac{\delta_{os} - 1}{\frac{R_n\Omega}{a} \sin \phi}$$

$$\times J_s \left(s \frac{R_n\Omega}{a} \sin \phi \right) J_k \left(k \frac{R_n\Omega}{a} \sin \phi \right) \quad (2.10)$$

$p = s + k.$

The term, $e^{jp\Omega\tau}$, represents the explicit expression of the time variation of radiation path on a source time axis under the assumption that the typical time scale of $l_{mn}(\tau)$ is very large compared with R_n/a .

In Eq. (2.5), the second term dominates over the first term when $l_{mn}(\tau)$ is constant, i.e., the variation of the blade velocity Mach number in the direction of the observer gives rise to the blade-passing frequency and its multiples, and conversely the first term dominates over the second when $\phi = 0$, i.e., there is no variation of the blade velocity in the direction of the observer. Hence, the second term is strongly related to the blade acceleration. Accordingly, a model which does not take the dipole acceleration into consideration such as the rectilinear model, is applicable only where $\Omega \ll \omega$, since the first term dominates over the second term when $\Omega \ll \omega$.

Equation (2.5) reveals that even when $l_{mn}(\tau)$ is statistically stationary the resulted acoustical pressure is not so due to the term $e^{jp\Omega t}$. This is the reason why taking the time average over $2\pi/\Omega$ is done in the next section to get rid of the time-dependency of the intensity spectrum. Furthermore, the observer even at $\phi = 0$, i.e., $R_n\Omega \sin \phi/a \ll 1$, has a possibility to hear the blade-passing frequency and its multiples depending on the inflow condition, although the variation of the blade velocity in the direction of the observer can be neglected even when $R_n\Omega \sin \phi/a \ll 1$. This is to be shown in the numerical results of the later section.

3. TIME-AVERAGED INTENSITY SPECTRAL DENSITY FUNCTION

Utilizing the following definition of the autocorrelation function of $\mathcal{P}(r, \phi, t)$,

$$\mathcal{R}_{\mathcal{P}\mathcal{P}}(r, \phi, t, \alpha) = E\{\mathcal{P}^*(r, \phi, t) \mathcal{P}(r, \phi, t + \alpha)\}, \quad (3.1)$$

and the following pair of Fourier integrals,

$$S_{\mathcal{P}\mathcal{P}}(r, \phi, t, \omega) = (1/2\rho a) \int_{-\infty}^{\infty} \mathcal{R}_{\mathcal{P}\mathcal{P}}(r, \phi, t, \alpha) e^{-j\omega\alpha} d\alpha \quad (3.2)$$

and

$$\mathcal{R}_{\mathcal{P}\mathcal{P}}(r, \phi, t, \alpha) = (\rho a/\pi) \int_{-\infty}^{\infty} S_{\mathcal{P}\mathcal{P}}(r, \phi, t, \omega) e^{j\omega\alpha} d\omega, \quad (3.3)$$

the expected value of the acoustic intensity at a large distance r is given by

$$E\{I(r, \phi, t)\} = (1/2\pi) \int_{-\infty}^{\infty} S_{\mathcal{D}\mathcal{D}}(r, \phi, t, \omega) d\omega \quad (3.4)$$

Thus, $S_{\mathcal{D}\mathcal{D}}(r, \phi, t, \omega)$ can be regarded as an instantaneous intensity spectral density function.

Assuming $S_{\mathcal{D}\mathcal{D}}(r, \phi, t, \omega)$ is continuous and finite with respect to t , the typical time scale of $l_{mn}(\tau)$ is very large compared with R_n/a (see Eq. (2.1)), $l_{mn}(\tau)$ is statistically stationary, replacing a blade with a dipole located at $R=R_T$, introducing a typical impulse response function $h(t)$ of a blade for all m such that

$$l_{mn}(\tau) = \int_{-\infty}^{\infty} h(\tau - \eta) u_m(\eta) d\eta \quad (3.5)$$

where $u_m(t)$ is the upwash at the m th blade, and time-averaging $S_{\mathcal{D}\mathcal{D}}(r, \phi, t, \omega)$ over $2\pi/\Omega$, the time-averaged intensity spectral density function can be obtained as follows

$$\begin{aligned} \langle S_{\mathcal{D}\mathcal{D}}(r, \phi, t, \omega) \rangle &= \{(\Delta R_T)^2 / (32\rho a^2 \pi^2 r^2)\} \sum_{p=-\infty}^{\infty} |H(\omega - p\Omega)|^2 \\ &\times \{(\omega - p\Omega)^2 (e_T^{(p)})^2 + \Omega^2 (f_T^{(p)})^2\} \\ &\times \left\{ \sum_{m=1}^B \sum_{k=1}^B \exp\{j(2\pi/B)p(k-m)\} S_{mk}^{uu}(\omega - p\Omega) \right\}. \end{aligned} \quad (3.6)$$

In Eq. (3.6), $S_{mk}^{uu}(\omega)$ is the Fourier transformed correlation function between $u_m(t)$ and $u_k(t)$ assuming the upwash is stationary, and $H(\omega)$ is the Fourier transformed impulse function, i.e., a transfer function. Equation (3.6) reveals the relation among the transfer function of a blade $H(\omega)$, the time variation of the propagation path p , the statistical property of the inflow $S_{mk}^{uu}(\omega)$, and the spatial relation among blades $e^{j[2\pi(k-m)]/B}$.

Letting the transfer function be the approximate form of Sears' function^{7,8)} such that

$$\begin{aligned} H(\omega) &\simeq \pi\rho\Omega c_T \\ &\times \frac{\exp\{-j(\omega c_T/2R_T\Omega)(1 - \pi^2/2(1 + \pi\omega c_T/R_T\Omega))\}}{(1 + \pi\omega c_T/R_T\Omega)^{1/2}}, \end{aligned} \quad (3.7)$$

utilizing the result of Lane⁹⁾ with respect to the correlation function, $\mathcal{R}_{mk}^{uu}(\alpha)$, and non-dimensionalizing Eq. (3.6), Eq. (3.6) becomes

$$\begin{aligned} \frac{\langle S_{\mathcal{D}\mathcal{D}}(r, \phi, t, \omega) \rangle}{\rho a^2 R_T} &= (1/32)(c_T/r)^2 (\Delta R_T/R_T)^2 (R_T\Omega/a)^2 ((\bar{u}^2)/a^2) \\ &\times \left\{ \sum_{m=1}^B \sum_{k=1}^B \exp\{j(2\pi/B)p(k-m)\} S_{mk}^{uu}(\omega - p\Omega) \right\}. \end{aligned} \quad (3.8)$$

where

$$\begin{aligned} S_B(U_z/A_1, R_T/A_1, \omega) &= \sum_{m=1}^B \sum_{k=1}^B \exp\{j(2\pi/B)p(k-m)\} \\ &\times S(U_z/A_1\Omega, R_T/A_1, \omega), \end{aligned} \quad (3.9)$$

$$S(U_z/A_1\Omega, R_T/A_1, \omega) = (\Omega/\bar{u}^2) S_{kk}^{uu}(\omega), \quad (3.10)$$

$$S_{mk}^{uu}(\omega) = \int_{-\infty}^{\infty} \mathcal{R}_{mk}^{uu}(\alpha) e^{-j\omega\alpha} d\alpha, \quad (3.11)$$

$$\mathcal{R}_{mk}^{uu}(\alpha) = \bar{u}^2 \{E(\alpha)(1 - G(\alpha)/2) + F(\alpha)/2G(\alpha)\} e^{-G(\alpha)}, \quad (3.12)$$

$$E(\alpha) = \cos^2 \beta + \cos(\Omega\alpha + 2\pi(k-m)/B) \sin^2 \beta, \quad (3.13)$$

$$\begin{aligned} G^2(\alpha) &= (U_z\alpha/A_1)^2 \\ &+ 2(R_T/A_1)^2 \{1 - \cos(\Omega\alpha + 2\pi(k-m)/B)\}, \end{aligned} \quad (3.14)$$

and

$$\begin{aligned} F(\alpha) &= (U_z\alpha/A_1)^2 \cos^2 \beta \\ &+ (R_T/A_1)^2 \sin^2 \beta \sin^2(\Omega\alpha + 2\pi(k-m)/B). \end{aligned} \quad (3.15)$$

The terms $E(\alpha)$ and $F(\alpha)$ in the above equation are introduced to make the equation shorten, and $G(\alpha)$ is the distance between two physical points in the converted turbulence with which the blades interact. Hence, $G(\alpha)$ is a function of time. In the derivation of $\mathcal{R}_{mk}^{uu}(\alpha)$, utilized are the assumptions: (1) an

Fig. 3 Rotor in cylindrical coordinates.

incompressible fluid medium, (2) frozen turbulence pattern convected with an axial flow velocity U , (3) homogeneous and isotropic turbulence, see Fig. 3, and (4) a longitudinal function, $\exp(-G/\Lambda_i)$, where Λ_i is the integral scale of the turbulence, and G is the distance between two physical points in the turbulence. As revealed in Eq. (3.14), the distance G is a function of time since the distance is the one between two physical points in the frozen turbulence pattern convected with an axial flow to be experienced by rotating blades. Due to homogeneous and isotropic turbulence assumption, the correlation function is stationary.

Utilization of Sears' function imposes that the effect of the flow fluctuation parallel to the chord line can be neglected as a second order effect. Hence, our concern is with the response of a thin airfoil with zero mean angle of attack to an upwash velocity fluctuation, where the airfoil is assumed to be lightly loaded so that the stagger angle β can be approximated by $U_z/R_T\Omega$. Another important consequence from utilizing Sears' function, under the assumption of the frozen turbulence pattern convected with an axial flow velocity U_z , is that a one-to-one correspondence between frequency and upwash wavenumber is attained so that the whole noise generation process can be treated on a time and frequency domain instead of a time and spatial domain. In addition, utilization of Sears' function is heavily relied on the results by Jackson *et al.*¹⁰⁾: the three-dimensional interaction between blades and flow fluctuations can be approximated by a two-dimensional strip theory using Sears' function. The two-dimensional strip approach imposes that the acoustic radiation due to parallel gusts (the spanwise wavenumber is zero) is dominant, as investigated by Amiet.¹¹⁾

Equation (3.8) shows the relation between $\langle S_{\mathcal{GP}}(r, \phi, t, \omega) \rangle$ and the terms: (1) amplitude modulation terms due to rotation, i.e., $e_T^{(p)}$ and $f_T^{(p)}$, (2) the blade transfer function $H(\omega)$, and (3) the upwash-related spectrum $S_B(U_z/\Lambda_i\Omega, R_T/\Lambda_i, \omega/\Omega)$. The non-dimensional parameters, $U_z/\Lambda_i\Omega$ and R_T/Λ_i , represent the relations between the turbulence and the rotor, dynamic and kinematic relation, respectively. The dynamic relation implies how many times the blade chops the statistically related turbulence on the course of time passing. On the other hand, the kinematic relation implies how many blades can chop the statistically related turbulence

at an instance time. The next section shows how these two non-dimensional parameters govern the profile of $\langle S_{\mathcal{GP}}(r, \phi, t, \omega) \rangle$.

4. NUMERICAL RESULTS

Computations are done for the results in Sections 2 and 3 by changing the number of blades, the observation angle, and non-dimensional parameters such as $U_z/\Lambda_i\Omega$ and R_T/Λ_i . In particular, the variation of these non-dimensional parameters is made around an axial flow velocity of $U_z = 30$ m/s, a rotor, radius of $R_T = 0.30$ m/s, a rotor rotational speed of $\Omega = 220$ rad/s, an inflow turbulence length scale $\Lambda_i = 0.15$ m.

Figures 4, 5, and 6 show the non-dimensionalized $\langle S_{\mathcal{GP}}(r, \phi, t, \omega) \rangle$ dependent on the non-dimensional parameters, $U_z/\Lambda_i\Omega$ and R_T/Λ_i . These figures show that as $U_z/\Lambda_i\Omega$ decreases, the level of the blade-passing frequency and its multiples increases, and as R_T/Λ_i increases those levels also increase. Furthermore, as R_T/Λ_i increases, the overall level increases.

Hence, it is clear that the kinematic relation R_T/Λ_i cannot explain fully the spectral profile without the dynamic relation $U_z/\Lambda_i\Omega$. The effect of R_T/Λ_i on the spectrum for constant $U_z/\Lambda_i\Omega$ is the opposite of that claimed by Mani.¹²⁾ This is probably due to the fact that Mani did not consider the variation of $U_z/\Lambda_i\Omega$ when he varied R_T/Λ_i as a parameter. Further investigation of this matter is necessary.

Furthermore, as the rotational speed increases, the bandwidth and the level at the blade-passing frequency and its multiples are increased, and as the axial flow velocity decreases, the blade-passing frequency and its multiples appear clearly. All these variations, under the constant turbulence length scale constraint, make rotating blades interact with the statistically correlated turbulence more. Hence, the above phenomenon can be observed through the variation of $U_z/\Lambda_i\Omega$ in Figs. 4, 5, and 6.

Figure 7 shows that as B increases, the bandwidth of the peak at the blade-passing frequency and its multiples increases. In this example, as B is increased from 3 to 9, the width at the blade-passing frequency has approximately triples. This tendency is mentioned also by Homicz.¹³⁾ This is probably due to the fact that as B increases the correlation among the noise sources (rotating dipoles) is increased so that the spectral profile is flatten.

Figure 8 shows the observer angle dependence

Fig. 4 Non-dimensionalized $\langle S_{FF}(r, \phi, t, \omega) \rangle$ for $B=3$ and $U_z/\Lambda_1\Omega=0.868$.

Fig. 5 Non-dimensionalized $\langle S_{FF}(r, \phi, t, \omega) \rangle$ for $B=3$ and $U_z/\Lambda_1\Omega=0.217$.

 Fig. 6 Non-dimensionalized $\langle S_{\mathcal{P}\mathcal{P}}(r, \phi, t, \omega) \rangle$ for $B=3$ and $U_z/\Lambda_1\Omega=0.0542$.

 Fig. 7 Blade number dependence of non-dimensionalized $\langle S_{\mathcal{P}\mathcal{P}}(r, \phi, t, \omega) \rangle$ for $U_z/\Lambda_1\Omega = 0.217$ and $R_T/\Lambda_1 = 0.5$.

of the non-dimensionalized $\langle S_{\mathcal{P}\mathcal{P}}(r, \phi, t, \omega) \rangle$. As shown in this figure, the amplitude modulation terms due to propagation path variation, i.e., $e_r^{(p)}$ and $f_T^{(p)}$ in Eq. (3.8), gradually influence the spectral

profile as the observer angle increases. Thus, the domination of the blade-passing frequency is reduced as the observer angle increases.

Fig. 8 ϕ -Dependence of non-dimensionalized $\langle S_{\phi\phi}(r, \phi, t, \omega) \rangle$ for $B=3$, $U_z/A_1\Omega=0.0542$, and $R_T/A_1=1.0$.

5. CONCLUSION

The time-averaged intensity spectral density function of the acoustic radiation from rotating blades was derived by replacing blades with rotating dipoles. The following conclusions can be drawn from the derivation and the numerical calculation:

1) The rectilinear model of a rotating blade row is applicable only for the frequency range, $\omega \gg \Omega$ for which the 2nd term in Eq. (2.5) can be neglected compared with other terms.

2) The effect of the variation of the distance between the observer and rotating dipoles can be neglected when $R_T\Omega \sin \phi/a \ll 1$. However, even when $\phi=0$, the blade-passing frequency and its multiples appear according to the inflow condition; see Fig. 8 and Eq. (3.8).

3) Two non-dimensional parameters, $U_z/A_1\Omega$ and R_T/A_1 , govern the profile of the noise spectrum. As $U_z/A_1\Omega$ decreases, the blade-passing frequency and its multiples appear clearly, and as R_T/A_1 increases they do also; see Figs. 4, 5, and 6.

4) The non-dimensionalized time average acoustic intensity density function is proportional to $(\bar{u}^2/a^2)(R_T/a)^3$ under the condition that other non-dimensional parameters are fixed; Eq. (3.8).

5) The acoustic intensity at the off-angle from the rotor axis is not statistically stationary even if the inflow turbulence is frozen and homogeneous; see Eq. (2.5).

6) Increasing B , and/or increasing Ω under the condition that $U_z/A_1\Omega$ is fixed increases the bandwidth of the peak of the blade-passing frequency and its multiples; see Fig. 7.

ACKNOWLEDGEMENTS

The author greatly acknowledges Dr. D. E. Thompson, who suggested the topic. His guidance will always be remembered.

The author acknowledges the support of Applied Research Laboratory, The Pennsylvania State University, under contract with the U. S. Naval Sea Systems Command.

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NOMENCLATURE

- a : speed of sound [m/s]
 B : number of blades
 c_T : chord length of typical segment of blade [m]
 D_{mn} : distance between observer between n th segment of m th blade
 $e_{mn}^{(p)}$: defined in Eq. (2.6)
 $f_{mn}^{(p)}$: defined in Eq. (2.7)
 $\vec{f}$: body force defined in Eq. (2.3) [N/m³]
 $H(\omega)$: transfer function of typical segment of blade
 k : integer
 $\vec{l}_{mn}(\tau)$: lift per unit span on n th segment of m th blade [N/m]
 $\bar{l}_{mn}(\tau)$: modular of $\vec{l}_{mn}(\tau)$ [N/m]
 m : integer
 N : number of segments
 p : harmonic number
 $\mathcal{P}(r, \phi, t)$: pressure at observer [N/m²]
 q : harmonic number
 r, θ, ϕ : spherical coordinates for observer
 R_T : radial position of typical segment of

- blade
 R, Φ, Z : cylindrical coordinates for rotor
 ΔR_T : length of typical segment of blade
 $\mathcal{R}_{mk}^{uu}(\alpha)$: correlation function of upwash between m th blade and k th blade
 $S_{mk}^{uu}(\omega)$: Fourier transform of $\mathcal{R}_{mk}^{uu}(\alpha)$
 t : time associated with observer [s]
 $\overline{u^2}$: mean square value of velocity fluctuation of inflow turbulence
 U_z : axial flow velocity [m/s]
 α : time difference
 β_{mn} : stagger angle at n th segment of m th blade
 β_T : stagger angle at typical segment of blade
 $\delta(x)$: Dirac delta function
 δ_{s0} : Kronecker delta where s is zero
 ω : angular frequency associated with sound
 Ω : rotational speed of blade [rad/s]
 ρ : density
 A_i : integral length scale [m]
 τ : time associated with noise source
 Φ_m : $2\pi m/B$
 $\langle \rangle$: time average over $2\pi/\Omega$ [s]

Superscripts

- (p): harmonic number
(q): harmonic number
 u : upwash of blade
 $*$: complex conjugate

Subscripts

- B : B rotor blades
 k : k th blade
 m : m th blade
 n : n th blade
 $\mathcal{P}$: acoustic pressure
 T : typical segment

APPENDIX

Lowson²⁾ derived the following Fourier coefficients of the amplitude modulation for 3rd and 4th terms in Eq. (2.4), under the condition $\Phi_m = 0$,

$$\frac{\cos \Omega(t - D_{mn}/a)}{\left(1 + \frac{R_n \Omega}{a} \sin \phi \sin \Omega(t - D_{mn}/a)\right)^3} = \sum_{p=-\infty}^{\infty} \frac{-jpJ_p\left(p \frac{R_n \Omega}{a} \sin \phi\right)}{\frac{R_n \Omega}{a} \sin \phi} e^{jp(\Omega t + \pi/2)}. \quad (\text{A.1})$$

Taking the phase Φ_m , of the amplitude modulation in Eq. (2.4) into account yields

$$\begin{aligned} & \frac{\cos\{\Omega(t - D_{mn}/a) + \Phi_m\}}{\left(1 + \frac{R_n\Omega}{a} \sin\phi \sin\{\Omega(t - D_{mn}/a) + \Phi_m\}\right)^3} \\ &= \sum_{p=-\infty}^{\infty} \frac{-jpJ_p\left(p\frac{R_n\Omega}{a} \sin\phi\right)}{\frac{R_n\Omega}{a} \sin\phi} e^{jp(\Omega t + \pi/2 + \Phi_m)}. \end{aligned} \quad (\text{A.2})$$

Now, consider the Fourier coefficient of the other amplitude modulation in Eq. (2.4), for instance, the Fourier coefficient of $(1 + (R_n\Omega/a) \sin\phi \sin\{\Omega(t - D_{mn}/a) + \Phi_m\})^{-2}$. Taking the time derivative of $(1 + (R_n\Omega/a) \sin\phi \sin\{\Omega(t - D_{mn}/a) + \Phi_m\})^{-1}$ yields

$$\begin{aligned} & \frac{d}{dt} \{1 + (R_n\Omega/a) \sin\phi \sin\gamma\}^{-1} \\ &= -\frac{\Omega^2 R_n \sin\phi \sin\gamma}{a \left(1 + \frac{R_n\Omega}{a} \sin\phi \sin\gamma\right)^3}, \end{aligned} \quad (\text{A.3})$$

where the following relations are used:

$$\frac{d}{dt} = \frac{d\gamma}{dT} \frac{dT}{dt} \frac{d}{d\gamma}, \quad (\text{A.4})$$

$$\gamma = \Omega(t - D_{mn}/a) + \Phi_m, \quad (\text{A.5})$$

and

$$T = \Omega(t - r/a) + \Phi_m = \gamma - \frac{R_n\Omega}{a} \sin\phi \cos\gamma. \quad (\text{A.6})$$

By comparing Eq. (A.2) with Eq. (A.3), one obtains the following expansion:

$$\begin{aligned} & \left(1 + \frac{R_n\Omega}{a} \sin\phi \sin\{\Omega(t - D_{mn}/a) + \Phi_m\}\right)^{-1} \\ &= \sum_{p=-\infty}^{\infty} J_p\left(p\frac{R_n\Omega}{a} \sin\phi\right) e^{jp(\Omega t + \pi/2 + \Phi_m)}, \end{aligned} \quad (\text{A.7})$$

where the steady state component can be computed by

$$\frac{\Omega}{2\pi} \int_0^{2\pi/\Omega} \left(1 + \frac{R_n\Omega}{a} \sin\phi \sin\gamma\right)^{-1} dt = 1, \quad (\text{A.8})$$

since $d\gamma = \left(1 + \frac{R_n\Omega}{a} \sin\phi \sin\gamma\right)^{-1} dt$.

Therefore, one obtains

$$\begin{aligned} & \left(1 + \frac{R_n\Omega}{a} \sin\phi \sin\{\Omega(t - D_{mn}/a) + \Phi_m\}\right)^{-2} \\ &= \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} J_p\left(p\frac{R_n\Omega}{a} \sin\phi\right) J_q\left(q\frac{R_n\Omega}{a} \sin\phi\right) \\ & \quad \times e^{j(p+q)(\Omega t + \pi/2 + \Phi_m)}. \end{aligned} \quad (\text{A.9})$$

Now, consider the Fourier coefficient of $\sin\{\Omega(t - D_{mn}/a) + \Phi_m\}/(1 + (R_n\Omega/a) \sin\phi \sin\{\Omega(t - D_{mn}/a) + \Phi_m\})^2$. Since it can be expressed by

$$\begin{aligned} & \frac{\sin\{\Omega(t - D_{mn}/a) + \Phi_m\}}{1 + \frac{R_n\Omega}{a} \sin\phi \sin\{\Omega(t - D_{mn}/a) + \Phi_m\}} \\ &= \frac{1}{1 + \frac{R_n\Omega}{a} \sin\phi \sin\{\Omega(t - D_{mn}/a) + \Phi_m\}}, \end{aligned}$$

the Fourier coefficient can be obtained through coefficients for each term in the above.

Taking the time derivative of $\sin\{\Omega(t - D_{mn}/a) + \Phi_m\}/(1 + (R_n\Omega/a) \sin\phi \sin\{\Omega(t - D_{mn}/a) + \Phi_m\})$ yields

$$\begin{aligned} & \frac{d}{dt} \left(\frac{\sin\gamma}{1 + \frac{R_n\Omega}{a} \sin\phi \sin\gamma} \right) \\ &= \frac{\Omega \cos\gamma}{\left(1 + \frac{R_n\Omega}{a} \sin\phi \sin\gamma\right)^3}, \end{aligned} \quad (\text{A.10})$$

where Eq. (A.4), Eq. (A.5), and Eq. (A.6) are utilized.

By comparing Eq. (A.10) with Eq. (A.2), the following expansion can be obtained:

$$\begin{aligned} & \frac{\sin\gamma}{1 + \frac{R_n\Omega}{a} \sin\phi \sin\gamma} \\ &= \sum_{p=-\infty}^{\infty} \frac{J_p\left(p\frac{R_n\Omega}{a} \sin\phi\right)}{\frac{R_n\Omega}{a} \sin\phi} e^{jp(\Omega t + \pi/2 + \Phi_m)} (\delta_{0p} - 1), \end{aligned} \quad (\text{A.11})$$

where the steady state component is obtained by

$$\begin{aligned} & \frac{\Omega}{2\pi} \int_0^{2\pi/\Omega} \frac{\sin\gamma}{1 + \frac{R_n\Omega}{a} \sin\phi \sin\gamma} dt = \frac{1}{2\pi} \int_0^{2\pi} \sin\gamma d\gamma \\ &= 0. \end{aligned} \quad (\text{A.12})$$

Therefore, utilizing Eq. (A.9) and Eq. (A.12), the following expansion is obtained:

$$\begin{aligned} & \frac{\sin\left\{\Omega\left(t - \frac{D_{mn}}{a}\right) + \Phi_m\right\}}{\left(1 + \frac{R_n\Omega}{a} \sin\phi \sin\left\{\Omega\left(t - \frac{D_{mn}}{a}\right) + \Phi_m\right\}\right)^2} \\ &= \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{\delta_{0p} - 1}{\frac{R_n\Omega}{a} \sin\phi} J_p\left(p\frac{R_n\Omega}{a} \sin\phi\right) \\ & \quad \times J_q\left(q\frac{R_n\Omega}{a} \sin\phi\right) e^{j(p+q)(\Omega t + \pi/2 + \Phi_m)}. \end{aligned} \quad (\text{A.13})$$