ACOUSTICAL LETTER

On the threshold pressure and the minimum sustaining pressure in the vocal fold oscillation

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1. Introduction

It is well known that a positive value of the lung air pressure is required to initiate the vocal fold oscillation in phonation, called the oscillation threshold pressure^{1-4,7)}. When the lung pressure is above this value, the energy transferred from the airflow to the vocal folds overcomes the energy dissipated in the tissues, and the oscillation can be generated. Typical threshold values for low oscillation frequencies are in the range of 200–400 Pa.^{1,2,4,5,7)} In experiments with excised canine larynges, it has been also observed that the threshold value decreases after the oscillation has started, *i.e.*, the minimum pressure to sustain the oscillation is lower than the pressure required to initiate it, with a difference of about 100 Pa.⁵⁾ However, no explanation for this difference between the threshold and the sustaining pressures has been offered yet, remaining unsolved.

In this letter, an analytical interpretation for this difference is presented, derived from the previous plank model of the vocal folds.⁶⁾ The analysis is performed studying the equivalent negative damping introduced by the airflow, previously indicated by Titze as the cause of the oscillation.⁸⁾

2. Equations of the plank model

A simplified version of the plank model⁶⁾ is adopted, under the following assumptions.

1) Small amplitude oscillations around the prephonatory position are considered. Under this condition, the glottis is relatively wide and its viscous aerodynamic resistance may be neglected as an approximation.

2) The pressure recovery at the exit of the glottis is small.⁹⁾

3) The oscillation frequency is assumed lower than about 300 Hz. At this range of frequencies, the effect of the reactance of the air mass (compliance and inertance) may be neglected.¹⁾

4) The supraglottal pressure is assumed equal to the atmospheric, *i.e.*, no vocal tract loading is included. This last condition corresponds to an excised larynx, where the experimental studies on the threshold pressure have been carried out.^{3,5)}

5) The aerodynamic resistance of the subglottal system is small, and hence the subglottal pressure is approximately equal to the lung pressure.^{1,3)}

The equations of motion are then reduced to

$$\frac{m}{3} \frac{d^2 x_1}{dt^2} + \frac{m}{6} \frac{d^2 x_2}{dt^2} + r_1 \frac{dx_1}{dt} + k_1 x_1 = -\frac{3}{4} F \quad (1a)$$

$$\frac{m}{6} \frac{d^2 x_1}{dt^2} + \frac{m}{3} \frac{d^2 x_2}{dt^2} + r_2 \frac{dx_2}{dt} + k_2 x_2 = -\frac{1}{4} F \quad (1b)$$

where m is the mass of the plank, x_1 and x_2 are the displacement of the lower and upper edges of the plank from the prephonatory position, respectively, r_i and k_i ($i=1, 2$) are their related viscous damping and stiffness, and F is the total force acting on the plank produced by the airflow, given by

$$F = \frac{BLP_L}{2} \frac{(x_1 - d)^2 - (x_2 - d)^2}{(x_1 - d)^2 + (\kappa - 1)(x_2 - d)^2} \quad (2)$$

where B and L are the width and length of the plank, P_L is the lung pressure, d is the prephonatory glottal half width, and κ is the pressure loss factor for the glottal inlet, with $\kappa=1.37$.⁷⁾

3. Analysis of the threshold and minimum sustaining pressures

The following approximate solution of the equations of motion is assumed, based on numerical computations using the original model.

$$x_1 = A_0 \sin \omega t \quad (3a)$$

$$x_2 = A_0 \sin (\omega t - \phi) \quad (3b)$$

In this solution, the upper and lower edges of the plank move with a phase delay between them, and approximately the same amplitude, as observed experimentally.⁵⁾

The linear approximation $\sin(\phi/2) = \phi/2$ is used to facilitate the calculations. For a phase delay $\phi=60^\circ$, typical for the oscillation, this approximation will introduce an error less than 5%. The above equations can then be written as

$$x_1 = A_0 \sin \theta + A_0 (\phi/2) \cos \theta \quad (4a)$$

$$x_2 = A_0 \sin \theta - A_0 (\phi/2) \cos \theta \quad (4b)$$

where $\theta = \omega t - \phi/2$. These equations are next introduced into the force equation (2), which is then approximated taking only the first harmonics terms of its expansion in Fourier series, according to the quasi-linearization method.⁸⁾ As a result, we obtain

$$F = -\frac{4LBP_L A_0 \phi}{\kappa d [1 + (1 - A_0^2/d^2)^{1/2}]} \cos \theta \quad (5)$$

Finally, using Eqs. (4a) and (4b),

$$F = -\frac{2LB\phi}{\kappa\omega d[1+(1-A_0^2/d^2)^{1/2}]} \frac{d}{dt} \left(\frac{x_1+x_2}{2} \right) \quad (6)$$

$$= -r_p \frac{d}{dt} \left(\frac{x_1+x_2}{2} \right) \quad (7)$$

In the above equations we can see that the force F acts as a negative damping with coefficient r_p , which is the cause of the oscillation. This effect was previously shown by Titze, assuming the presence of a traveling wave on the vocal folds.³⁾ It can be also noted that the absolute value of r_p increases with the oscillation amplitude A_0 .

For a sustained oscillation, the negative damping must equal the positive damping of the tissues, i.e.

$$r_p = r_1 + r_2 \quad (9)$$

This condition can be verified introducing Eqs. (4a), (4b) and (7) into the equations of motion (1a) and (1b) and solving. Finally, using Eq. (6), we obtain the required value of the lung pressure

$$P_L = \frac{\kappa\omega d(r_1+r_2)[1+(1-A_0^2/d^2)^{1/2}]}{2LB\phi} \quad (10)$$

Letting $A_0=0$, which corresponds to the start of the oscillation, we obtain the threshold pressure P_{L0}

$$P_{L0} = \frac{\kappa\omega d(r_1+r_2)}{LB\phi} \quad (11)$$

which agrees with the equation obtained by Titze.³⁾

We can also see in Eq. (10) that the sustaining value of the lung pressure decreases with the oscillation amplitude A_0 . According to this equation, its minimum value occurs at $A_0=d$, i.e., at the glottal closure, with $P_L = P_{L0}/2$. This last result is only valid as a reference, since it is beyond the small-amplitude restriction adopted for the analysis.

Fig. 1 Value of the lung pressure P_L required for the oscillation versus the oscillation amplitude A_0 , using Eq. (10) with $d=0.2$ mm, $B=14$ mm, $L=3$ mm, $f=150$ Hz, $\phi=60^\circ$, and $r_1+r_2=50$ dyn s/cm. The threshold value is $P_{L0}=296$ Pa for $A_0=0$. The minimum value occurs at $A_0=d$, with $P_L=P_{L0}/2$.

A numerical example using Eq. (10) is given in Fig. 1, for the values $B=14$ mm, $L=3$ mm, $r_1+r_2=50$ dyn s/cm,⁶⁾ and $d=0.2$ mm, $f=150$ Hz, $\phi=60^\circ$, which correspond to a normal phonation.^{1,3,5,7)} In this case the threshold pressure is $P_{L0}=296$ Pa, within the range of experimental values, and a minimum sustaining pressure of 148 Pa is predicted as a reference. The difference between both values is close to the result obtained on excised canine larynges, which gives experimental support to the analytical results.

4. Conclusion

The difference between the oscillation threshold pressure and the minimum sustaining pressure observed experimentally in the vocal fold oscillation can be explained considering the equivalent negative damping introduced by the airflow. The absolute value of this negative damping increases with the oscillation amplitude, thus lowering the minimum value of the pressure to sustain the oscillation. An analytical expression including this effect was derived, which was confirmed by previous experimental values.

It must be noted that this analytical explanation was derived from the simple plank model, with linear mechanical components and within the several simplifying assumptions adopted such as small-amplitude oscillations.

We believe that these results may be of interest for speech studies, providing further insight into the underlying mechanism of phonation.

References

- 1) J. L. Flanagan, *Speech Analysis, Synthesis and Perception* (Springer-Verlag, New York, 1972), pp. 11, 43-50.
- 2) W. J. Gould, "The pulmonary-laryngeal system," in *Vocal Fold Physiology*, K. N. Stevens and M. Hirano, Eds. (University of Tokyo Press, Tokyo, 1981), Chap. 4, pp. 33-44.
- 3) I. R. Titze, "The physics of small-amplitude oscillation of the vocal folds," *J. Acoust. Soc. Am.* **83**, 1536-1552 (1988).
- 4) I. R. Titze, "Phonation threshold pressure: A missing link in glottal aerodynamics," *J. Acoust. Soc. Am.* **91**, 2926-2935 (1992).
- 5) T. Baer, "Observation of vocal fold vibration: Measurement of excised larynges," in *Vocal Fold Physiology*, K. N. Stevens and M. Hirano, Eds. (University of Tokyo Press, Tokyo, 1981), Chap. 10, pp. 119-133.
- 6) J. C. Lucero and T. Gotoh, "Equilibrium positions in the plank model of the vocal cords and stability analysis," *Proc. Autumn Meet. Acoust. Soc. Jpn.*, 343-344 (1992).
- 7) K. Ishizaka and J. L. Flanagan, "Synthesis of voiced sounds from a two-mass model of the vocal cords," *Bell Syst. Tech. J.* **51**, 1233-1268 (1972).

- 8) A. Blaquiere, *Nonlinear System Analysis* (Academic Press, New York, 1966), pp. 56–57, 153–165.

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