Deformation of sound pressure pulse and that of particle velocity pulse with diffraction

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When ultrasonic pulse travels in homogeneous material, it shows deformation by diffraction. An important approach to analyze this effect is to compute retarded potential of ultrasonic field together with Fourier’s analysis. Deformation of sound pressure pulse with diffraction and that of axial component of particle velocity pulse with it are dealt with a numerical method by Fourier’s analysis. They are compared each other in the case that a pulse is transmitted from a circular flat transmitter into isotropic and non-absorptive medium. Analyzed transmitting pulses are assumed to have several envelopes of ramp followed by exponential decrease with sinusoidal carrier wave. Waveforms of receiving pulses through a few transmission systems with a circular flat transducer and a point one and those with a pair of coaxial circular flat ones are derived. Ratio of peak value of receiving pulse to that of transmitting one is obtained with a computing precision of 0.1%. It corresponds to apparent attenuation of pulsed ultrasonic wave by beam divergence when a pair of circular flat transducers are considered.

Keywords: Ultrasonic nearfield, Numerical analysis, Sound pressure, Particle velocity, Pulse

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1. INTRODUCTION

In the history of analysis of ultrasonic propagation, nearfield has been extensively investigated theoretically1-10) and experimentally.2,5,6) And, waveform of ultrasonic pulse has also been analyzed by many authors.11-17) But they investigated only the sound pressure11-13) or the waveform where a point receiver was set in the field.14-17) In this article are reported results of precise computation by Fourier’s analysis for deformation of axial component of particle velocity pulse with diffraction, together with sound pressure pulse, including the case where a circular flat receiver is set against the circular flat transmitter.

2. COORDINATE SYSTEM USED IN THE COMPUTATION AND WAVEFORM OF TRANSMITTING PULSE

Medium of ultrasonic propagation is assumed to be isotropic and to have no internal friction. Shape of transducer is circular flat, which vibrates longitudinally. Coordinate system used in the computation is shown in Fig. 1. Z-axis is the central axis of the circular flat transducers. Center of the transmitter is set to be the origin. R-axis is rectangular to Z-axis. Radius of the circular flat transducer is a. Point P(r', 0, 0) is on the transmitter (S'), and point Q(r, 0, z) on the receiver (S). The distance between P and Q is called d.

Transmitting waveform has envelope of ramp followed by exponential decrease with sinusoidal carrier wave. Figure 2 shows an example when both ramp rise time duration (t₁) and exponential
Fig. 1 Coordinate system used in the computation.

Fig. 2 Waveform of transmitting pulse \( t_1 = t_2 = t_0 \).

decrease time constant \( t_2 \) are equal to the period \( t_0 \) of the sinusoidal carrier wave. In the figure \( t/t_0 \) is the normalized time. The waveform can be explained as

\[
F(t) = \sum_{i=0}^{\infty} c_i \cos(\omega_d t - \varphi_i) \sin(\omega_0 t - \varphi_{\omega_0}), \tag{1}
\]

where

\[
\sum_{i=0}^{\infty} c_i \cos(\omega_d t - \varphi_i) \tag{2}
\]

expresses the envelope and

\[
\sin(\omega_0 t - \varphi_{\omega_0}) \tag{3}
\]

the carrier wave. In Eq. (1), \( \omega_0 T = 2\pi \) where \( T \) the repetition period of the envelope. \( F(t) \) can be developed as

\[
F(t) = \sum_{i=0}^{\infty} c_i \cos(\omega_d t - \varphi_i) \sin(\omega_0 t - \varphi_{\omega_0})
\]

\[
= \sum_{i=0}^{\infty} \frac{c_i}{2} \left[ \sin(\omega_d t - \varphi_i + \omega_0 t - \varphi_{\omega_0})
- \sin(\omega_d t - \varphi_i - \omega_0 t + \varphi_{\omega_0}) \right]
\]

\[
= \sum_{i=0}^{\infty} \frac{c_i}{2} \cos \left[ (\omega_d + \omega_0) t - \varphi_i - \varphi_{\omega_0} - \frac{\pi}{2} \right]
\]

The complex expression \( \hat{F}(t) \), in which the real part is shown in Eq. (4) as \( F(t) \), can be explained as

\[
\hat{F}(t) = \sum_{i=0}^{\infty} \frac{c_i}{2} \exp \left\{ j \left[ (\omega_d + \omega_0) t - \varphi_i - \varphi_{\omega_0} - \frac{\pi}{2} \right] \right\}
+ \sum_{i=0}^{\infty} \frac{c_i}{2} \exp \left\{ j \left[ (\omega_d - \omega_0) t + \varphi_i - \varphi_{\omega_0} + \frac{\pi}{2} \right] \right\}.
\]

In Eq. (5), \( \omega_d + \omega_0 \) is always plus, while \( \omega_d - \omega_0 \) becomes minus when \( i > \omega_0/\omega_d \). By dividing terms as in Eq. (5), we can find that \( \omega_d - \omega_0 \) is plus when \( i < \omega_0/\omega_d \), and that \( \omega_d - \omega_0 \) is plus when \( i > \omega_0/\omega_d \).

3. FREQUENCY CHARACTERISTICS OF ULTRASONIC TRANSMISSION SYSTEM

When transmitter vibrates coherently as

\[
\dot{v}_s \exp(j\omega t), \tag{6}
\]

velocity potential \( \Phi(r, 0, z, t) \) at a point \( Q(r, 0, z) \) is expressed as

\[
\Phi(r, 0, z, t) = \dot{v}_s \exp(j\omega t)
\times \frac{1}{2\pi} \int \frac{\exp(-jkd)}{d} dS', \tag{7}
\]

in which \( k \) denotes angular wave number, \( S' \) the transmitting area. From axial symmetry of ultrasonic field, integration of Eq. (7) can be accomplished by a single numerical one.\(^9\)

Sound pressure at this point is expressed as

\[
\dot{p}(r, 0, z, t) = \dot{v}_s \exp(j\omega t)
\times \rho c \left( \frac{j}{\lambda} \right) \int \frac{\exp(-jkd)}{d} dS', \tag{8}
\]

\[
\equiv \dot{v}_s \exp(j\omega t) \times r \exp(-\theta), \tag{8'}
\]

where \( \rho \) the density of the medium, \( c \) the velocity of the ultrasonic wave and \( \lambda \) the wavelength of it. Equation (8) can be expressed as Eq. (8'), where \( r \) and \( \theta \) depend only on space.\(^9\) Axial component
of particle velocity at the point is expressed as
\[ \dot{v}(r, 0, z, t) = \dot{v}_0 \exp(j\omega t) \times \frac{1}{2\pi} \int_S \frac{z(1+jkd) \exp(-jkd)}{d^3} dS' \text{ (9)} \]

Axial component of acoustic impedance density at the point is expressed as
\[ \dot{\rho}(r, 0, z) = \rho c \left(2\pi f / \lambda \right) \times \frac{1}{2\pi} \int_S \frac{\exp(-jkd)}{d^3} z(1+jkd) \exp(-jkd) dS' \text{ (10)} \]

The central wavelength (\(\lambda_0\)) is connected with the period (\(T_0\)) of the sinusoidal carrier wave as \(\lambda_0 = cT_0\). And the central frequency (\(f_0\)) and the period (\(T_0\)) have the relation \(f_0T_0 = 1.0\). In the following computation, \(a/\lambda_0 = 8.55\). This condition is fulfilled when the radius (\(a\)) of the transducer is 10 mm, and ultrasonic longitudinal wave of 5 MHz is propagated in steel (\(c = 5.848 \times 10^3 \text{ m/s}\)). Using Eqs. (8) and (8)', amplitude (\(r\)) and phase delay (\(\theta\)) of the sound pressure is computed. Figure 3 shows frequency characteristics of normalized sound pressure at \(r=0\) and \(z=a\) where \(a/\lambda_0 = 8.55\). The abscissa indicates the normalized frequency, and the ordinates indicate the amplitude (\(r\)) and the phase delay (\(\theta\)), respectively. Amplitude has regular repetitive variation, while phase delay has irregular leaps where amplitude is zero. This is because the phase leaps \(\pi\) where the amplitude is zero, which can be explained reasonably by particle velocity and acoustic impedance density. Using Eqs. (9) and (9)', amplitude (\(r_0\)) and phase delay (\(\theta_0\)) of the axial component of particle velocity is computed. Figure 4 shows frequency characteristics of axial component of particle velocity at \(r=0\) and \(z=a\) where \(a/\lambda_0 = 8.55\). Phase delay has no leap, but has irregular feature when amplitude is minimum. This compensates the phase delay of the sound pressure to reach a regular leap on the phase delay of the axial component of acoustic impedance density. Figure 5 shows frequency characteristics of normalized axial component of acoustic impedance density at \(r=0\) and \(z=a\) where \(a/\lambda_0 = 8.55\). Phase delay leaps regularly from \(\pi/2\) to \(-\pi/2\) when amplitude is zero.

Mean sound pressure is expressed as
\[ \dot{p}_m(z, t) = \frac{1}{S} \int_S \dot{p}(r, 0, z, t) dS \text{ (11)} \]

\[ \equiv \dot{v}_0 \exp(j\omega t) \times r_m \exp(-j\theta_m) \text{, (11)'} \]

where \(r_m\) and \(\theta_m\) depend only on space. In Eq. (11), \(S\) denotes the receiving area. Mean axial component of particle velocity is expressed as
\[ \dot{v}_m(z, t) = \frac{1}{S} \int_S \dot{v}(r, 0, z, t) dS \text{ (12)} \]

\[ \equiv \dot{v}_0 \exp(j\omega t) \times r_0m \exp(-j\theta_0m) \text{, (12)'} \]
where \( r_{bm} \) and \( \theta_{bm} \) depend also only on space. Mean axial component of acoustic impedance density is expressed as

\[
\frac{\hat{\rho}_m(z)}{\hat{\rho}_{bm}} = \frac{r_m}{r_{bm}} \exp[-j(\theta_m - \theta_{bm})].
\] (13)

Figure 6 shows frequency characteristics of normalized mean sound pressure over the coaxial circular flat receiver at \( z=a \) where \( a/\lambda_0 = 8.55 \). Amplitude is zero when \( f=0 \), and rises sharply to a small undulation which reaches a stable level. This is because, when the frequency is small, the amplitudes of complex functions are always small, and when it becomes large they approach to 1.0 in the large area of the receiver. In this area, phase delay is similar to that of plane wave. Comparing with Fig. 3, mean sound pressure resembles more the plane wave. Figure 7 shows frequency characteristics of mean axial component of particle velocity over the coaxial circular flat receiver at \( z=a \) where \( a/\lambda_0 = 8.55 \). Amplitude starts from minimum at \( f=0 \), and reaches a stable level with a shoulder. As is the mean sound pressure, mean particle velocity resembles the plane wave. Figure 8 shows frequency characteristics of normalized mean axial component of acoustic impedance density over the coaxial circular flat receiver at \( z=a \) where \( a/\lambda_0 = 8.55 \). Amplitude rises sharply from zero, when \( f=0 \), to maximum, and undulates and reaches a stable level of 1.0. Phase delay rises from minus to stable 0.0. These results confirm that the wave resembles plane wave when the frequency is large.
4. DEFORMATION OF SOUND PRESSURE PULSE AND THAT OF PARTICLE VELOCITY PULSE WITH DIFFRACTION

Each frequency of \( F(t) \) in Eq. (5) has respective \( c_i \) and \( \varphi_i \) values. Frequency characteristics of ultrasonic field can be computed from Eqs. (8), (9), (11) and (12). Each response of a particular frequency on the receiver can be obtained by multiplying each complex frequency characteristic to each term in Eq. (5). And, by Fourier’s synthesis, receiving waveform is derived. Receiving waveform of \( \hat{F}(t) \) in Eq. (5) is expressed as

\[
\hat{G}(t) = \sum_{i=0}^{\infty} \frac{c_i r_i^+}{2} \times \exp \left\{ j \left[ (\omega_0 + \omega_d) t - \varphi_i - \omega_d \right] \right\} + \sum_{i=0}^{\infty} \frac{c_i r_i^-}{2} \times \exp \left\{ j \left[ (\omega_0 - \omega_d) t - \varphi_i + \omega_d \right] \right\}
\]

where \( r_i^+ \) and \( r_i^- \) denote amplitude of each frequency, which are \( r, r_D, r_m \) and \( r_Dm \), respectively. And \( \theta_i^+ \) and \( \theta_i^- \) denote phase delay of it, which are \( \theta, \theta_D, \theta_m \) and \( \theta_Dm \), respectively. In Eq. (14), \( r_i^+ \) and \( r_i^- \) correspond to \( \omega_0 + \omega_d \), while \( r_i^- \) and \( \theta_i^- \) correspond to \( |\omega_0 - \omega_d| \). Hereafter, changing \( t_1 \) and \( t_2 \) in Fig. 2, deformation of ultrasonic pulse is investigated, when \( a/\lambda_s = 8.55 \). Integer \( i \) is increased from zero to 2,000, where \( \omega_0 \) equals to 0.001\( \omega_s \). In the time domain, time interval to be computed is 1/200 \( \times t_0 \).

Using Eq. (14), Figs. 9 and 10, and Tables 1—4 are computed. Figure 9 shows receiving waveform of sound pressure at \( r=0 \) and \( z=a \) where \( a/\lambda_s = 8.55 \), of which the transmitting waveform is in Fig. 2 \( (t_1=t_2=t_0) \). It has a transmit resembling pulse followed by a polarity inverted pulse, which corresponds to the plane wave and to the edge wave, respectively.\(^{17} \) As a typical example, the peak value of the transmitting pulse in Fig. 2 is 0.7981, while the peak value of the receiving sound pressure pulse in Fig. 9 is 0.8212. So, the ratio of the peak value of receiving sound pressure to the transmitting peak value \((R/T)\) becomes 1.029. Figure 10 shows receiving waveform of axial component of particle velocity at \( r=0 \) and \( z=a \) where \( a/\lambda_s = 8.55 \), of which the transmitting waveform is in Fig. 2 \( (t_1=t_2=t_0) \). It has a similar feature as sound pressure, but the following polarity inverted pulse has smaller peak compared with the first transmit resembling pulse, which corresponds to the plane wave. So, the ratio of the peak value of receiving axial component of particle velocity to the transmitting peak value \((R/T)\) becomes 1.0. Receiving waveform of mean sound pressure of the system of a pair of coaxial circular flat transducers and that of mean axial component of particle velocity of it have a very little difference from the transmitting one.

Peak value of each pulse is obtained with a computing precision of better than 0.1%, because the frequency characteristics of the ultrasonic transmission system is computed to the precision of 0.0001. Table 1 shows ratio of peak value of receiving sound pressure along \( r=0.0 \) to transmitting peak value \((R/T)\) where \( a/\lambda_s = 8.55 \). It varies most when the wave is sinusoidal, and approaches to 1.0 when the pulse becomes sharp. Table 2 shows ratio of peak value of receiving axial component of particle velocity along \( r=0.0 \) to transmitting peak value
Table 1 Ratio of peak value of receiving sound pressure along \( r=0.0 \) to transmitting peak value \((R/T)\) where \(a/\lambda_0=8.55\).

<table>
<thead>
<tr>
<th>(z)</th>
<th>(0.5a)</th>
<th>(a)</th>
<th>(2a)</th>
<th>(5a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sinusoidal</td>
<td>1.558</td>
<td>1.983</td>
<td>0.115</td>
<td>0.927</td>
</tr>
<tr>
<td>(t_1=t_2=10\tau_0)</td>
<td>1.285</td>
<td>1.721</td>
<td>0.230</td>
<td>0.908</td>
</tr>
<tr>
<td>(t_1=t_2=5\tau_0)</td>
<td>1.151</td>
<td>1.526</td>
<td>0.433</td>
<td>0.891</td>
</tr>
<tr>
<td>(t_1=t_2=2.5\tau_0)</td>
<td>1.047</td>
<td>1.275</td>
<td>0.893</td>
<td>0.877</td>
</tr>
<tr>
<td>(t_1=t_2=\tau_0)</td>
<td>1.000</td>
<td>1.029</td>
<td>1.000</td>
<td>0.973</td>
</tr>
<tr>
<td>(t_1=t_2=0.5\tau_0)</td>
<td>1.000</td>
<td>1.001</td>
<td>1.001</td>
<td>1.007</td>
</tr>
</tbody>
</table>

Table 2 Ratio of peak value of receiving axial component of particle velocity along \( r=0.0 \) to transmitting peak value \((R/T)\) where \(a/\lambda_0=8.55\).

<table>
<thead>
<tr>
<th>(z)</th>
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<th>(a)</th>
<th>(2a)</th>
<th>(5a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sinusoidal</td>
<td>1.179</td>
<td>1.693</td>
<td>0.151</td>
<td>0.918</td>
</tr>
<tr>
<td>(t_1=t_2=10\tau_0)</td>
<td>1.073</td>
<td>1.472</td>
<td>0.306</td>
<td>0.899</td>
</tr>
<tr>
<td>(t_1=t_2=5\tau_0)</td>
<td>1.000</td>
<td>1.244</td>
<td>0.491</td>
<td>0.881</td>
</tr>
<tr>
<td>(t_1=t_2=2.5\tau_0)</td>
<td>1.000</td>
<td>1.000</td>
<td>0.903</td>
<td>0.863</td>
</tr>
<tr>
<td>(t_1=t_2=\tau_0)</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.973</td>
</tr>
<tr>
<td>(t_1=t_2=0.5\tau_0)</td>
<td>1.000</td>
<td>0.999</td>
<td>1.001</td>
<td>1.007</td>
</tr>
</tbody>
</table>

Table 3 Ratio of mean peak value of receiving coaxial sound pressure over the area of receiver to transmitting peak value \((R/T)\) where \(a/\lambda_0=8.55\).

<table>
<thead>
<tr>
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<th>(5a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sinusoidal</td>
<td>0.946</td>
<td>0.926</td>
<td>0.898</td>
<td>0.838</td>
</tr>
<tr>
<td>(t_1=t_2=10\tau_0)</td>
<td>0.949</td>
<td>0.928</td>
<td>0.900</td>
<td>0.843</td>
</tr>
<tr>
<td>(t_1=t_2=5\tau_0)</td>
<td>0.950</td>
<td>0.930</td>
<td>0.902</td>
<td>0.848</td>
</tr>
<tr>
<td>(t_1=t_2=2.5\tau_0)</td>
<td>0.953</td>
<td>0.933</td>
<td>0.907</td>
<td>0.858</td>
</tr>
<tr>
<td>(t_1=t_2=\tau_0)</td>
<td>0.960</td>
<td>0.944</td>
<td>0.922</td>
<td>0.881</td>
</tr>
<tr>
<td>(t_1=t_2=0.5\tau_0)</td>
<td>0.971</td>
<td>0.958</td>
<td>0.940</td>
<td>0.907</td>
</tr>
</tbody>
</table>

Table 4 Ratio of mean peak value of axial component of receiving coaxial particle velocity over the area of receiver to transmitting peak value \((R/T)\) where \(a/\lambda_0=8.55\).

<table>
<thead>
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<td>0.838</td>
</tr>
<tr>
<td>(t_1=t_2=10\tau_0)</td>
<td>0.948</td>
<td>0.927</td>
<td>0.899</td>
<td>0.843</td>
</tr>
<tr>
<td>(t_1=t_2=5\tau_0)</td>
<td>0.949</td>
<td>0.929</td>
<td>0.902</td>
<td>0.848</td>
</tr>
<tr>
<td>(t_1=t_2=2.5\tau_0)</td>
<td>0.951</td>
<td>0.932</td>
<td>0.906</td>
<td>0.857</td>
</tr>
<tr>
<td>(t_1=t_2=\tau_0)</td>
<td>0.959</td>
<td>0.943</td>
<td>0.921</td>
<td>0.881</td>
</tr>
<tr>
<td>(t_1=t_2=0.5\tau_0)</td>
<td>0.968</td>
<td>0.955</td>
<td>0.939</td>
<td>0.906</td>
</tr>
</tbody>
</table>

It has a similar tendency as sound pressure, but the ratio reaches 1.0 even when the pulse is less sharp with narrower frequency characteristics. These effects are typical when the point receiver is near the circular flat transmitter. Table 3 shows ratio of mean peak value of receiving coaxial sound pressure over the area of receiver to transmitting peak value \((R/T)\) where \(a/\lambda_0=8.55\). It is smaller when the distance between transducers becomes longer. It is more large when the pulse becomes more sharp, because sharp pulse has much higher frequency components which approach to 1.0. Table 4 shows ratio of mean peak value of axial component of receiving coaxial particle velocity over the area of the circular flat receiver to transmitting peak value \((R/T)\) where \(a/\lambda_0=8.55\). The feature of peak ratio tendency is similar to mean sound pressure in Table 3, but the data are a little smaller than those in it. These correspond to apparent attenuation of pulsed ultrasonic waves by beam divergence, because the mean over the whole circular flat receiver is obtained.

5. CONCLUSION

A computational example for deformation of ultrasonic pulse with diffraction is reported. Receiving waveform for sound pressure of an ultrasonic system, that has a circular flat transducer and a point one on the axis of it, has remarkable difference from transmitting waveform, which corresponds to plane wave and to edge wave. This is also the case for axial component of particle velocity of the same system. Receiving waveform of mean sound pressure of the system of a pair of coaxial circular flat transducers and that of mean axial component of particle velocity of it have a very little difference from the transmitting one. Ratios of peak values of receiving pulses to those of transmitting ones are computed to a precision of 0.1% and are tabulated. They correspond to apparent attenuation of pulsed ultrasonic waves by beam divergence when the system of a pair of circular flat transducers is considered, because the mean over the whole circular flat receiver is obtained.

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T. IMAMURA: SOUND PRESSURE PULSE AND PARTICLE VELOCITY PULSE


Tohru Imamura was born in Nagoya, Japan on September 29, 1945. He received the B. Sci. from the University of Tokyo in 1969. Since then, he has been with the National Research Laboratory of Metrology, AIST, MITI. His interests are in numerical analysis of ultrasonic propagation, ultrasonic measurement of materials, and history and role of standards.