Structural intensity measurement of cylindrical shell based on NAH technique and influences of a rib on the acoustic energy flow

Kenji Saijyou* and Shigeru Yoshikawa**

*Ship Systems Development Department, Technical R&D Institute, Japan Defense Agency,
1-2-24, Ikejiri, Setagaya-ku, Tokyo, 154-8511 Japan
**Department of Acoustic Design, Kyushu Institute of Design,
4-9-1, Shiobara, Minami-ku, Fukuoka, 815-8540 Japan
(Received 20 June 1998)

A measurement method of structural intensity was proposed by developing the signal processing in near-field acoustical holography (NAH) in our previous paper [Jpn. J. Appl. Phys., 35, 3167-3174 (1996)]. The present paper extends that method to the structural intensity measurement in cylindrical geometry. Two samples used in the experiment are SUS304 cylindrical shells. One of them has a rib and the other has no rib. The experimental data are obtained and reconstructed over the frequency range between about 300 and about 3,000 Hz. The data at 488 Hz and 1,877 Hz are selected for specific and detailed investigation of structural intensity distribution, where 488 Hz lies in a pass band and 1,877 Hz in a stop band of the ribbed shell. According to the discussion of experimental results, it may be understood that the attached rib acts as a “light beam” at a pass-band frequency 488 Hz and a part of the intensity flow is reflected at and the rest is transmitted through the rib. However, the rib almost completely reflects the intensity flow by acting as a “heavy beam” when the driving frequency is a stop-band frequency 1,877 Hz. The overall mode-cell structure is thus largely influenced by the action of a rib except for locally induced vortexlike structure. Also, the calculation of the power injected to the shell successfully leads to the identification and localization of the vibration source.

Keywords: Structural intensity, Injected power, Acoustic intensity, Generalized near-field acoustic holography, Ribbed shell, Pass band, Stop band

PACS number: 43. 40. Ey, 43. 40. Rj. 43. 60. Sx

1. INTRODUCTION

When we improve the performance of sonar system installed in underwater vehicles, it is very important to reduce self-radiation noise. Acoustic intensity measurements have been served our need for this noise reduction by localizing the radiation sources. However, the structure-borne energy flow, which is called structural intensity or vibration intensity, has recently attracted considerable interest of many engineers in the field of vibration control.1-14) Particularly, the importance of localizing the vibration source instead of the radiation source has been corrected recognized.

Structural intensity (SI) has usually been measured with an accelerometer10) or a strain gauge.11) These methods are suitable to measure local SI but unsuitable to measure the intensity distribution over the whole structure, because many attached sensors would considerably affect the structural vibration. Hayek et al.12) proposed a non-contacting measurement method of SI using a laser Doppler vibrometer (LDV). This method is able to give the accurate measurement of SI over the whole structure surface.
However, it is difficult to operate the LDV in the water, therefore, this LDV method has not been applied to the SI measurement of the submerged structures.

The structural intensity from the measurement of acoustic pressure (SIMAP) was proposed by Williams et al.\textsuperscript{13) in the application to submerged structures. Their SIMAP is based on the measurement of vibration on the structure using near-field acoustic holography (NAH).\textsuperscript{15) Since NAH is a non-contacting method, the surface vibration of a submerged structure is not affected by the measurement devices. Moreover, NAH enables us to examine the interrelationship between structural and acoustic intensities.

However, the finite difference method is used to obtain SI in SIMAP, so the acoustic pressure must be measured on a finely meshed contour near the vibrating structure to realize high accuracy. We proposed an improved method for measuring SI of the plate using NAH in the previous paper.\textsuperscript{14) It was our idea that the spatial derivatives for computing SI could be evaluated in wavenumber-frequency space (called K space) instead of real space. This K-space processing may reduce the spatial density of data sampling points while maintaining the measurement accuracy required. The effectiveness of the proposed method was confirmed by the experiment.\textsuperscript{14)"

The authors intend to extend this method to measure SI in cylindrical geometry. The effectiveness of our newly proposed method is shown by the measurement of radiation from two kinds of the submerged vibrating cylindrical shell; that is, a thin stainless steel shell and the same shell with a rib. Surface vibration of the cylindrical shell was measured with the generalized near-field acoustic holography (GENAH).\textsuperscript{10) The acoustic intensity and the power injected to the shell were also measured to investigate the interrelationship between acoustic source and vibration one.\textsuperscript{13,14) The injected power into the shell, which was studied in detail by Williams et al.,\textsuperscript{13) is used to localize the vibration source. The acoustic intensity from the shell is used to localize the acoustic source. The influence of the attached rib to the acoustic and structural intensities are visualized as shown in Fig. 9 to Fig. 16.

2. DERIVATION OF THE STRUCTURAL INTENSITY OF THIN CYLINDRICAL SHELL IN K SPACE

We have already proposed the measurement method of SI of the plate in K space.\textsuperscript{14) Let us extend it to measure SI of a thin cylindrical shell.

The cylindrical coordinate system $\vec{r}=(r, \theta, z)$ is adopted in this formulation (cf. Fig. 1). In the following expression, the variables expressed in a small letter mean the real-space variables, and the variables written in a capital letter mean the K-space variables (except the structural intensity $I$ and Young's Modulus $E$). Since we are now considering sinusoidal shell vibration, the shell surface velocity $v_r$, which is reconstructed by the GENAH signal processing, is described as

$$v_r = \frac{\partial w}{\partial t} = w \times j2\pi f,$$

where $w$ is the radial displacement of the shell, and $f$ is the driving frequency. The components of displacement vector $\vec{s}=(u, v, w)$ are expressed as follows,

$$u(\vec{r}) = \sum_{m=0}^{\infty} dk_z \sum_{l=0}^{\infty} U_l(k_z) \cos i\theta e^{jk_z r_0 s},$$

$$v(\vec{r}) = \sum_{m=0}^{\infty} dk_z \sum_{l=0}^{\infty} V_l(k_z) \sin i\theta e^{jk_z r_0 s},$$

$$w(\vec{r}) = \sum_{m=0}^{\infty} dk_z \sum_{l=0}^{\infty} W_l(k_z) \cos i\theta e^{jk_z r_0 s},$$

where $r_0$ is the shell radius, $k_z$ is the axial wavenumber, $\nu$ and $u$ are the circumferential and axial displacements of the shell. The K-space representation of the Flügge-Byrne-Lur'ye equation of motion\textsuperscript{17-19) is expressed as

$$L(U, V, W)^T = (0, 0, F_w)^T,$$

where $F_w$ is the axial force per unit length.

Fig. 1 Configuration and coordinate system for cylindrical radiation problems.
where, \( (0, 0, F_w) \) is the driving force vector, \( T \) means transposed matrix, \( c_p^2 = E / \rho(1 - \nu^2) \), \( \nu = 2\sqrt{f} \), \( \nu \) the Poisson's ratio, \( \rho \) the density of the shell, and \( h \) the thickness of the shell. The solution of the simultaneous linear equations Eq. (3) is given as,

\[
\begin{align*}
W & = \frac{L_0 L_3 - L_1 L_4}{L_1 L_4 - L_2 L_3} W, \\
U & = \frac{L_0 L_5 - L_1 L_6}{L_1 L_6 - L_2 L_5} W, \\
V & = \frac{L_0 L_7 - L_1 L_8}{L_1 L_8 - L_2 L_7} W.
\end{align*}
\]

\( W \) is obtained by Eqs. (1) and (2), therefore, \( U \) and \( V \) are computed by Eq. (6). Even if \( L_1 L_4 - L_2 L_3 = 0 \) in Eq. (6), the displacement vector satisfies \( (U, V, W) = (0, 0, 0) \) because \( W = 0 \).

According to Romano and Williams, SI of thin cylindrical shell is computed as,

\[
I_s = \left[ \frac{\partial W}{\partial t} q_x + \frac{\partial V}{\partial t} n_x + \frac{1}{r_0} \frac{\partial W}{\partial r} - \frac{1}{r_0} \frac{\partial V}{\partial \theta} \right] m_x + \frac{\partial U}{\partial t} m_x,
\]

\[
I_s = \left[ \frac{\partial W}{\partial t} q_x + \frac{\partial V}{\partial t} n_x + \frac{1}{r_0} \frac{\partial W}{\partial r} - \frac{1}{r_0} \frac{\partial V}{\partial \theta} \right] m_x + \frac{\partial U}{\partial t} m_x,
\]

where \( q_x \) and \( q_x \) are stress resultants which represent shear forces, \( m_x \) and \( m_x \) are bending moment resultants, \( m_x \) and \( m_x \) are twisting moment resultants, \( n_x \) and \( n_x \) are stress resultants which represent longitudinal forces, and \( n_x \) and \( n_x \) are stress resultants which represent in-shell shear forces. These variables include the 2nd and/or more higher spatial derivatives of \( u, v, \) and \( w \). Therefore, it is necessary for computation of SI to evaluate the spatial derivatives of \( u, v, \) and \( w \). In the present method, the spatial derivatives are evaluated in K space. It should be noted that the high wavenumber noise components of the higher order derivatives are remarkably amplified in K-space processing and give a terrible influence to the reconstructed result. However, it is necessary to measure high wavenumber components to secure higher resolution for better reconstruction. Therefore, a lighter K-space filtration should be applied to lower order spatial derivatives for assuring a higher resolution of SI, while a deeper K-space filtration is preferred for higher order spatial derivatives to eliminate the influence of noise. The filter function is defined as,

\[
f(k_w, k_l) = \text{circ}(k_w, k_l),
\]

\[
k_w = \sqrt{k_w^2 + \left( \frac{1}{r_0} \right)^2},
\]

where \( k_w \) denotes the highest wavenumber for the reconstruction of the \( l \)-th order spatial derivative. The highest wavenumber \( k_w \) is determined as follows:

**step. 1** K-space representation of \( W \times k_w f(k_w, k_l) \) is transformed to the real space.

**step. 2** The maximum value of the \( \partial^l w / \partial z^l \) (i.e. \( (\partial^l w / \partial z^l)_{\text{max}}(k_l) \)) is obtained.

**step. 3** The change rate \( \xi(k_l) \), which is defined as

\[
\xi(k_l) = \lim_{k_l \to 0} \left( \frac{\partial^l w / \partial z^l}{\partial k_l} \right)_{\text{max}}\left( k_l + \delta k_l \right) - \left( \frac{\partial^l w / \partial z^l}{\partial k_l} \right)_{\text{max}}(k_l)
\]

is computed, and the point of inflection of \( \xi(k_l) \) is obtained.

**step. 4** A suitable \( k_l \) is determined as the point of inflection of \( \xi(k_l) \).

The K-space representations of the variables in Eq. (7) are thus expressed as follow:

\[
N_w = \frac{E_0}{2(1 + \nu)} \left[ \frac{1}{r_0} \left( 1 + \frac{h^2}{12 r_0^2} \right) W f(k_w, k_l) \right] + \left( \frac{i}{r_0} V + j k_w U \right) f(k_w, k_l) - \frac{h^2}{12 r_0^2} W f(k_w, k_l),
\]

\[
N_w = \frac{E_0}{2(1 + \nu)} \left[ \frac{\nu}{r_0} W f(k_w, k_l) + \left( j k_w U + i \frac{\nu}{r_0} V \right) f(k_w, k_l) + \frac{h k_w^2}{12 r_0^2} W f(k_w, k_l) \right],
\]

\[
N_w = - \frac{E_0}{2(1 + \nu)} \left[ i \frac{r_0}{1 + \frac{h^2}{12 r_0^2}} U - j k_w V \right] f(k_w, k_l) + \frac{j k_w h k_w^2}{12 r_0^2} W f(k_w, k_l),
\]

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where \( D = \frac{Eh^3}{2(1+\nu)} \left[ \left( -\frac{i}{\rho_0} U + jk_0 \left( 1 + \frac{h^2}{12\rho_0^2} \right) V \right) f(k_\rho, k_z) 
+ \frac{jk_0 ih^2}{12\rho_0^2} W f(k_\rho, k_z) \right] \), 

\[ M_B = D \left[ -\frac{1}{\rho_0} U f(k_\rho, k_z) + \left( \frac{h^2}{\rho_0^2} + k_0^2 \nu \right) f(k_\rho, k_z) \right] W, \]

\[ M_x = D \left[ \left( \frac{jk_0}{\rho_0} U + \frac{j\nu}{\rho_0} V \right) f(k_\rho, k_z) 
+ \left( k_0^2 + \frac{\nu^2}{\rho_0^2} \right) W f(k_\rho, k_z) \right], \]

\[ M_m = \frac{Eh^3}{24(1+\nu)\rho_0^3} \left[ \left( \frac{i}{\rho_0} U + jk_0 V \right) f(k_\rho, k_z) 
+ 2jk_0 i W f(k_\rho, k_z) \right], \]

\[ M_m = \frac{Eh^3}{12(1+\nu)\rho_0^3} \left[ V f(k_\rho, k_z) + iW f(k_\rho, k_z) \right], \]

\[ Q_u = -\frac{D}{a} \left[ \frac{i}{\rho_0} \frac{1}{a} V f(k_\rho, k_z) 
+ \left( k_0^2 - \frac{(i)^2}{a^2} \right) W f(k_\rho, k_z) \right], \]

\[ Q_v = jk_0 D \left[ \frac{i}{\rho_0} V f(k_\rho, k_z) + (k_0^2 + i^2) W f(k_\rho, k_z) \right], \tag{10} \]

where \( D = \frac{Eh^3}{12(1-\nu^2)} \). It should be noted here that each variable in real space as seen in Eq. (7) was already obtained by Romano,\(^{20}\) the K-space filter function of \( f(k_\rho, k_z) \) in Eq. (10) was originally introduced by the present authors.

Hence, SI of the thin shell is computed according to the following step:

**step. 1** K-space representation of the radial displacement \( W \) is reconstructed by GENAH back propagation.

**step. 2** K-space representation of axial and circumferential displacement \( U \) and \( V \) are computed by Eq. (6).

**step. 3** The component of structural intensity \((N_\theta, N_z, etc.)\) are computed by Eq. (10).

**step. 4** All variables in Eq. (10) are transformed to real-space variables.

**step. 5** Structural intensity is computed by Eq. (7).

The direction of the vibration power flow may be determined by the following line integral:

\[ \Pi = \int_C \vec{T} \cdot \vec{n} dl, \tag{11} \]

where the SI vector \( \vec{T} \) is defined as

\[ \vec{T} = \vec{u}I_\rho + \vec{v}I_\theta. \tag{12} \]

The \( \Pi \) is the power created in the shell, \( C \) a contour shown in Fig. 2, \( \vec{u} \) the outer vector normal to the contour \( C \), and \( \vec{v} \) and \( \vec{\nu} \) are the unit vector of the \( u \) and \( v \) direction, respectively. The \( \Pi \) is regard as an estimate of the power delivered to the shell from the vibrational source, and called the "injected power," hereafter.\(^{13}\)

### 3. EXPERIMENTAL APPARATUS AND RECONSTRUCTED SURFACE VIBRATION

#### 3.1 Data Acquisition System

The experiment was carried out in the test basin of 5th Research Center of Japan Defense Agency, which has 15 m in length, 9 m in breadth and 8 m in depth. An automated scanning system was developed for the experiment of GENAH at this facility (cf. Fig. 3).

A simply-supported shell is immersed at about 3 m depth. A WILCOXON F9/Z9/F3 shaker, which drives the sample shell, is attached inside a shell. A Bruel \& Kjaer Type 8103 hydrophone is placed at the prescribed position with automatic \( X- \), \( Y- \), and \( Z- \)axis scanners. A two-dimensional measurement contour is thus scanned by stepwise rotational and vertical movement of the shell. Measured data of the measurement contour are taken in 6-degree steps along the rotational scanning and in 25-mm steps along the vertical scanning. The mesh size is sufficiently smaller than the size of the vibration mode cell. Thus, 60×64 points data make one hologram. The received and amplified signals from the hydrophone are digitized and transferred to the HP-9000 computer through the interface bus (GP-IB:IEEE-488) line, and input into a floppy diskette. Sampling inception time is accurately controlled by the computer, which acts as the reference source for holographic interference. The
Fig. 3 A block diagram of the measurement system for the GENAH experiment.

Fig. 4 The experimental models.

analysis computation is executed with the HP-9000 model 755 computer.

3.2 Reconstructed Results of Surface Vibration

Two samples which used in the experiment were SUS304 stainless steel shells of the same size (800 mm in length, 216.3 mm in outer diameter and 3 mm in thickness). The shaker is attached at the distance of 500 mm from the bottom of the shell. One shell has no rib and the other has a rib, and these shell is filled with air (Fig. 4). The rib is welded at 400 mm from the bottom of the shell. The rib is circular, 5 mm wide, 20 mm high, and made of SUS304 stainless steel. The ends of the shells are sealed with rubber packing to approximate the simply-supported boundary condition. The shaker is driven by a broadband (300-3,000 Hz) linear frequency modulation (LFM) signal. See Ref. 18) and 19) for more information about the shell and its experimental setup.

Figure 5 shows the surface velocity of an unribbed shell, where the real part of the velocity is normalized by the maximum value and drawn with color linear intensity (10 shades of red for positive and 10 shade of green for negative values). In this figure, the horizontal axis indicates the circumferential angle $\theta$ and the vertical axis the shell axis $z$, and the upper and lower horizontal lines indicate the shell ends. The distribution of the surface velocity varies according to the ascending frequency, and the surface velocity is large over the whole shell at every frequency. The vibration mode is shown in $(m, n)$, where $m$ and $n$ mean the $z$-axis mode number and circumferential mode number, respectively. The vibration mode changes from $(2, 3)$ to $(6, 7)$ at 488 Hz to 2,808 Hz.

The surface velocity of the ribbed shell is shown in Fig. 6. The surface velocity of the shell is large over the whole shell when the frequencies are selected as 488 Hz, 732 Hz, 1,099 Hz and 1,160 Hz in the experiment. At these frequencies, the vibration mode is clearly shown in the figure $(2, 3)$ at 488 Hz, $(2, 4)$ at 732 Hz, etc.), and these mode numbers are the same as that of the unribbed shell. At the other frequencies, the upper part of the shell is strongly vibrated, but the lower part of the shell is weakly vibrated. These two types of characteristic vibration (the former: pass-band vibration; the latter: stop-band vibration) may depend on the existence of a rib. Although the terminologies of “stop band” and “pass band” are exactly defined when wave propagation in periodic structures is considered,22) we apply them to our finite ribbed structure, where the cylinder ends work as a strong reflector.

Figures 7 and 8 show the driving mobility of the shells without and with a rib, respectively. The mobility is normalized at the maximum point. In these figures, vertical arrows indicate the vibration mode. The comparison between surface velocity and driving mobility of the unribbed shell (cf. Figs. 5 and 7) shows that the modal frequencies of the unribbed shell almost correspond to the local peaks of the driving-point mobility.

On the contrary, modal frequencies of the ribbed shell correspond to those only at 488 Hz, 732 Hz, 1,099 Hz, and 1,160 Hz in the measured frequencies (cf. Fig. 6). But we can find many local peaks of
Fig. 5 The reconstructed surface velocity of an unribbed shell. Note that two horizontal lines in a frame correspond to the shell ends.

Fig. 6 The reconstructed surface velocity of a ribbed shell. Note that a middle horizontal line in a frame corresponds to an attached rib on the shell.

the driving-point mobility in Fig. 8.

The ribbed shell shows vibrational behavior similar to that of the unribbed shell at the pass bands, therefore, the modal frequencies of the ribbed shell correspond to those of the unribbed shell. On the contrary, the attached rib can act like the edge of the shell at the stop bands. This is why we can find many local peaks of the driving-point mobility over the stop bands, which suggest modal vibrations occurring on the upper half of the ribbed shell.
4. EXPERIMENTAL RESULTS OF STRUCTURAL AND ACOUSTIC INTENSITIES

We intend to investigate the surface vibration at 488 Hz and 1,877 Hz, because 488 Hz and 1,877 Hz exist in the pass band and stop band of the ribbed shell, respectively. Also, it should be noticed that the wavelength of the bending wave at 1,877 Hz is half the wavelength at 488 Hz. The measured results of the structural and acoustic intensities are shown in Fig. 9 to Fig. 12. The upper and lower lines indicate the shell ends, and the driving point is shown by a small triangle symbol, hereafter.

Figure 9(a) represents the surface velocity of the unribbed shell at 488 Hz. The vibration mode appears to be (2, 3). The acoustic intensity (AI) radiated from the shell is shown in Fig. 9(b), where AI is normalized by the maximum value and drawn with the linear scale defined as 10 steps for positive values. Figure 9(c) exhibits the reconstructed SI. The highest wavenumbers for computing SI at 488 Hz are $k_0=94$ (rad/m), $k_1=63$ (rad/m), $k_2=46$ (rad/m), and $k_3=31$ (rad/m). The vector length is normalized by the maximum value. The injected power computed from SI is shown in Fig. 9(d). It should be noted that this scale is defined as 20 steps for positive values in Fig. 10(d) and that the scale of Fig. 9(d) is renormalized by the maximum value of Fig. 10(d) for the convenience of the mutual comparison. The solid contour shows the positive value, and the dotted contour the negative value.

Comparison of Figs. 9(a) with 9(b) shows that the radiation of the acoustic energy at the antinode of the vibration mode near the driving point. It becomes weak as going far away from the driving point along the circumferential direction. Figure 9(c) shows that strong SI near the driving point. It seems to be possible to infer the vibration source from Fig. 9(b), but it is difficult to distinguish the real vibration energy source from many acoustic radiation sources. On the other hand, the injected power shows a large positive value near the driving point (Fig. 9(d)), therefore, we are able to identify the vibration energy source. In Fig. 9(c), SI does not diminish at the edge because the particle velocity of the surrounding water is induced by cylinder vibration. A similar phenomenon is found in SI shown below.

Surface velocity of a ribbed shell is shown in Fig. 10(a). At the driving frequency of 488 Hz the ribbed shell has a vibration mode similar to that of the unribbed shell. In Fig. 10(a) the driving point exists on the node of the vibration mode. This phenomenon seems to be happened by the local property of the welded rib. Figure 10(b) indicates that the acoustic energy is strongly radiated at the antinode of the vibration mode but the external radiation is weak at the driving point. Figure 10(c) shows that SI diverges from the driving point to the shell edges. The injected power also shows a large positive value at the driving point (Fig. 10(d)).

Figure 11(a) shows the surface velocity of the
Fig. 9 The experimental results of an unribbed shell. The driving frequency is 488 Hz. (a) The amplitude of the reconstructed surface velocity. (b) The acoustic intensity radiated from the shell. (c) The structural intensity of the shell. (d) The power injected into the shell. The solid and broken contours represent positive and negative values of the injected power.

Fig. 10 The experimental results of a ribbed shell. The driving frequency is 488 Hz. (a) The amplitude of the reconstructed surface velocity. (b) The acoustic intensity radiated from the shell. (c) The structural intensity of the shell. (d) The power injected into the shell.

Fig. 11 The experimental results of an unribbed shell. The driving frequency is 1,877 Hz. (a) The amplitude of the reconstructed surface velocity. (b) The acoustic intensity radiated from the shell. (c) The structural intensity of the shell. (d) The power injected into the shell.

Fig. 12 The experimental results of a ribbed shell. The driving frequency is 1,877 Hz. (a) The amplitude of the reconstructed surface velocity. (b) The acoustic intensity radiated from the shell. (c) The structural intensity of the shell. (d) The power injected into the shell.
unribbed shell at 1,877 Hz. The vibration mode is (4, 6), and the driving point corresponds to the antinode of the vibration mode. The AI is strongly radiated from the antinodes of the vibration mode as shown in Fig. 11(b). The SI diverges from the antinodes of the vibration mode to the surroundings in Fig. 11(c). The highest wavenumbers for computing SI at 1,877 Hz are \( k_0 = 120 \) (rad/m), \( k_1 = 80 \) (rad/m), \( k_2 = 58 \) (rad/m), and \( k_3 = 40 \) (rad/m). The local maxima of \( \Pi \) appear to correspond to each antinode of vibration modes in Fig. 11(d). The \( \Pi \) at the driving point does not show a maximum value. Therefore, the localization of the vibration energy source by using \( \Pi \) is not effective in this case.

The surface vibration pattern of the ribbed shell is different from that of the unribbed shell (Fig. 12(a)). Because of the influence of an attached rib, the vibration mode pattern is distorted and the surface vibration becomes weak at the lower part of the shell. The AI of the ribbed shell also shows that the acoustic radiation becomes weak at the lower part of the shell (Fig. 12(b)). The vibration energy flow is restricted at the upper part of the shell because the frequency of 1,877 Hz exists in the stop band of the ribbed shell (Fig. 12(c)). The injected power shows the maximum value near the driving point (Fig. 12(d)), from which we can detect the vibration source.

5. THE INFLUENCE OF AN ATTACHED RIB TO THE SHELL VIBRATION

5.1 The Influence of an Attached Rib at the Pass Band

Let us discuss influences caused by the attached rib to the shell vibration in the pass band in more detail. A comparison between Figs. 9(a) and 10(a) shows that there is a similar vibration mode of (2, 3). However, we can see an appreciable difference in the acoustic energy radiation pattern as shown in Figs. 9(b) and 10(b).

This difference should be seen in the pattern of structural intensity flow. The bold lines in Fig. 13 are the typical flow of SI of the unribbed shell at 488 Hz. The flows in the circumferential direction are mainly from the driving point in Fig. 13. These flows collide with the counter-direction flows, which diverge from the right and/or left antinodes, and in turn the flow direction changes to the vertical (axial) direction. This axial flow becomes small at the both edges and around the center of the shell. The cell in the vibration mode (cf. Fig. 9(a)) may be formed by such structural intensity flow. As the result, the mode cell which is distant from the vibration source becomes weak and the structural intensity combined at the periphery of the cell becomes indistinct. Corresponding to a very distinct mode cell near the driving point, the acoustic energy is strongly radiated from its vicinity. The SI at the both edges of the shell seems to be the same as that at the center of the shell. In the vibration problem, a simply-supported shell is closely approximated as an infinite shell whose finite part corresponding to the shell is enclosed with the rigid baffle. Since the distribution of vibration is spatially periodic, the wavenumber is restricted to

\[
k_m = \frac{m\pi}{L},
\]

where the axial mode number \( m \) takes integer and \( L \) denotes the shell length. Since the vibration mode shown in Fig. 9(a) is (2, 3), the edges and center of the shell are the nodes of the vibration mode.

We can also find that a part of the vibration energy flow is rotated around the area which is indicated by a circle in Fig. 13. The generation mechanism of such a rotating flow is explained by
Tanaka et al. like that the vortex flow in the plate is produced by the interference of two modes. The applicability of this explanation to the shell vibration will be confirmed in the near future.

Figure 14 shows SI of the ribbed shell at 488 Hz. First we examine the flow at the upper part of the ribbed shell. The driving point of the ribbed shell is just located near the boundary between adjacent two cells as shown in Fig. 10(a). Therefore, the axial flow should be originated from the driving point and may be directed toward the rib. The energy flow which reflects from the rib is absorbed at an antinode located nearest from the driving point. The acoustic energy is thus strongly radiating at this antinode (cf. Fig. 10(b)). Therefore, we may consider that the vibration energy is converted into the acoustic energy at this antinode. Of course, energy propagation toward the circumferential direction is not prevented according to our rib configuration.

Next let us consider the vibration energy flow at the lower part of the shell. The lower and upper SI patterns between $|\theta| \leq \pi/5$ (rad) show high symmetry with respect to the rib. Moreover, the lower SI pattern shows good symmetry in circumferential direction with respect to $\theta = 0$ except for that near the rear side of the driving point ($\theta = \pi$). The flows which diverge from the rib and the bottom end joins with each other around the point of $(-200$ mm, $\pm 0.56\pi$ (rad)), and the resultant flow is directed to the circumferential direction, and finally returned to the rib and the shell bottom. Such axial and circumferential flow formation, which depends on the rib configuration, is responsible for clear symmetry of mode cell pattern.

The influence of the attached rib is summarized as follows:

No. 1 The structural intensity is spread out over the whole shell by the attached rib.

No. 2 Major vibration energy from the driving point is turned to the circumferential direction by a combination of axial reflections from the rib and the bottom end, and then returned to the axial direction according to the interaction between adjacent mode cells.

The cause of No. 1 is explained using the “mass control theory” of the attached rib. According to Heckl, a “light beam,” whose width is negligible compared with the bending wavelength, acts according to its “mass” only. Therefore, a part of the vibration energy from the driving point is reflected by the rib, and the rest is propagated across the rib. The reflected energy is ordinarily dominant in the oblique direction. Therefore, the propagation to the circumferential direction may occur. On the other hand, a more physical and specific explanation relevant to No. 2 should be required to make clear the interrelation between the mode-cell formulation and the associated structural intensity flow pattern in a ribbed shell.

5.2 The Influence of an Attached Rib at the Stop Band

We would like to discuss an example of what influence the attached rib gives to shell vibration at a stop band. The vibration mode of Fig. 12(a) is obviously different from that of Fig. 11(a). Let us examine SI in more detail to address this difference.

Three typical patterns of SI flow exist on the unribbed shell. The first of them appears at the driving point (100 mm, $0$ (rad)) and at another point ($-100$ mm, $0$ (rad)), from which vibration energy diverges strongly. Oblique and circumferential energy flows make the flow pattern as shown in Fig. 15. The second flow pattern appears at $(\pm 100$ mm, $\pm \pi/6$ (rad)). The circumferential flow from $(\pm 100$ mm, $0$ (rad)) changes to the axial direction at
(±100 mm, ±π/6 (rad)), and the energy is divided upward and downward. As the result, the energy propagation is almost restricted to the area surrounded by these points, that is, the very vicinity of the driving point.

The third type of the flow pattern appears at the antinodes of the vibration mode except for those near the points above. The oblique flows are dominant at the antinodes, and intersect each other. The oblique intersection of SI at the antinode does not interfere with energy propagation to the circumferential direction. The structural intensities are comparatively strong at the antinodes, even if they are far from the driving point.

The energy flow on the ribbed shell is complicated (Fig. 16). At the driving point and the neighboring antinode of the vibration mode, SI diverges to all direction. Strong flows collide with each other, and induce strong radiation of the acoustic energy (Fig. 12(b)). At the antinode which is far from the driving point, the oblique SI flow is dominant. This oblique pattern is the same as that shown in Fig. 15, although the pattern itself is considerably disordered. The distortion of this pattern forms the oblique pattern of vibration mode cell (Fig. 12(a)).

The vibration energy diverges to vertical direction in the areas surrounded by ellipses indicated in Fig. 16. Apparently SI is strong over the upper part of the shell, and weak over the lower part of it.

The attached rib intercepts the structural intensity flow, and changes the direction of the strong energy flow to the upper part of the shell. This disturbance by the attached rib against the vibration energy propagation is explained by Heckl. When the beam width compared with the bending wavelength is not negligible, the beam is called “heavy beam”. Since the mode of our unribbed shell turns into (4,6) of 1,877 Hz (Fig. 11(a)) from (2,3) of 488 Hz (Fig. 9(a)), the bending wavelength at 1,877 Hz is almost half of that at 488 Hz. The attached rib, hence, may act as a “heavy beam” at 1,877 Hz. Because 1,877 Hz exists in the stop band of the ribbed shell, the propagation to the lower part of the shell is intercepted. Also, many reflections of the energy flow appear at the peripheries of mode cells which contact with the shell edges and/or the attached rib.

6. CONCLUSION

The structural intensity was computed from the reconstruction of surface vibration based on GENAH to localize the vibration source of a sub-
merged cylindrical shell. To investigate the interrelationship between the acoustic and vibrational sources, we measured the acoustic intensity as well as the power injected to the shell. A thin stainless steel shell and a ribbed shell were investigated. The experimental results are summarized as follows:

- The acoustic energy is radiated at the antinode of the vibration mode.
- The driving point does not always coincide with the antinode of the vibration mode. Therefore, it is difficult to estimate the vibration source from the acoustic intensity.
- The local maxima of the injected power always coincide with the driving point.

Moreover, the experiment at 488 Hz showed that an attached rib acted as a “light beam.” On the other hand, the experiment at 1,877 Hz showed that an attached rib acted as a “heavy beam.” These “light” and “heavy” beams are responsible for the characteristic mode-cell formation in a pass band and in a stop band, respectively.

A rotating pattern of structural intensity was also found in this paper. The explanation of this particular phenomenon will be tried in the near future.

REFERENCES