Precise estimation of multiple transmission paths in an acoustic system

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A method is proposed for precisely identifying the multiple signal transmission paths of a multi-input single-output acoustic system by using correlated input signals. Such identification has not been previously achieved by a conventional straightforward generalization of the principle for identifying a single-input single-output acoustic system. To this end, pre-filters are applied to the system inputs so that the relation between inputs and output satisfies the condition of a unique solution set given by the framework of a diophantine equation

Keywords: A multi-input single-output acoustic system, Signal transmission path, Pre-filters, Diophantine equation, Unique solution set

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1. INTRODUCTION

Multi-input multi-output acoustic systems are being investigated for use in such applications as multi-point sound-pressure control,¹⁻³⁾ multi-point active noise control,^{4,5)} and stereo teleconferencing.⁶⁻⁸⁾ A common and important factor in advancing these applications is identification of the acoustic transmission paths between loudspeakers and microphones. For multi-point control schemes, on-line identification will help in tracking the changes in the acoustic transmission paths between secondary sources and controlled points and maintaining high sound control performance at the controlled points. With stereo teleconferencing, for example, such on-line identification is needed to prevent the "howling" that arises from the acoustic coupling between the loudspeakers and microphones; much effort has been made in the field of acoustic echo cancellation.

A straightforward generalization of the principle for identifying a single-channel acoustic path is insufficient for use in identification of multi-channel acoustic paths.^{7,8)} This is because the equation

relating the input signals with the output signal of a multi-input single-output (MISO) acoustic system has many possible solution sets when the input signals are identical or correlated. As a result, multiple acoustic paths cannot uniquely be determined. Consider the two-input single-output acoustic system shown in Fig. 1.

The non-uniqueness problem can be clarified by considering the relation between two identical input signals, $x_1(k)$ and $x_2(k)$. This relation is given by

$$Y(z) = H_1(z)X_1(z) + H_2(z)X_1(z) \quad (1)$$

where $H_1(z)$ and $H_2(z)$ are the transfer functions of the two acoustic paths and $X_1(z)$, $X_2(z)$, and $Y(z)$ are the z -transforms of input and output signals, $x_1(k)$, $x_2(k)$ and $y(k)$.

Taking arbitrary polynomial $E(z)$, the z polynomials $H_1(z) + E(z)$ and $H_2(z) - E(z)$ also satisfy this equation. Thus, Eq. (1) has infinite solution sets. This means that the convergence of the adaptive filters shown in Fig. 1 to the actual acoustic paths is not always guaranteed.

Several methods for identifying the acoustic paths in a two-input single-output acoustic system have

Fig. 1 Configuration for identification of acoustic paths $H_1(z)$ and $H_2(z)$; S_1 and S_2 are loudspeakers, M_1 is a microphone, and AP_1 and AP_2 are adaptive filters for estimating acoustic transmission paths and $H_1(z)$ and $H_2(z)$.

Fig. 2 Proposed configuration for identification of acoustic paths $H_1(z)$ and $H_2(z)$ by using pre-filters $G_1(z)$ and $G_2(z)$; S_1 and S_2 are loudspeakers, M_1 is a microphone, and AP_1 and AP_2 are adaptive filters for estimating acoustic transmission paths.

been proposed⁶⁻⁸⁾ to reduce the inter-channel coherence. These include independent modulation of each input signal with random noise⁶⁾ and nonlinear transformation of each signal itself.⁷⁾ Both of these methods circumvent the non-uniqueness problem by introducing a distortion that is tuned to be barely audible by means of psycho-acoustic masking effects. However, the two methods are specific to two-channel input signals and are not easily extensible for MISO systems. Also, the referenced discussions on the convergence of the solutions were incomplete.

We have developed a method for precisely identifying multiple acoustic transmission paths. Application of prime pre-filters to each input signal guarantees a unique solution. The properties required for the pre-filters were determined through analysis using a diophantine equation. The capabilities of this method were evaluated by numerical simulation.

2. DIOPHANTINE EQUATION AND UNIQUE SOLUTION SET

2.1 Basic Ideas for Identifying Two Acoustic Paths

The problem mentioned in the previous section was overcome by applying pre-filters $G_1(z)$ and $G_2(z)$ to the input signals prior to their radiation via the loudspeakers. When input signals $x_1(k)$ and $x_2(k)$ are identical, the input-output relation can be formulated as

$$Y(Z) = X_1(z)(G_1(z)H_1(z) + G_2(z)H_2(z)) \quad (2)$$

$$\triangleq X_1(z)H_0(z).$$

This is equivalent to the following diophantine equation⁹⁻¹²⁾ when the input signals are sufficiently broadband :

$$G_1(z)H_1(z) + G_2(z)H_2(z) = H_0(z) \quad (3)$$

This diophantine equation has a unique solution set,^{9,12)} $H_1(z)$ and $H_2(z)$, under the conditions :

- The orders of $H_1(z)$ and $H_2(z)$ are less than those of $G_2(z)$ and $G_1(z)$, respectively, and
- $G_1(z)$ and $G_2(z)$ have no common zero in the z plane; in other words, $G_1(z)$ and $G_2(z)$ are mutually prime.

When pre-filters $G_1(z)$ and $G_2(z)$ satisfy these conditions, $H_1(z)$ and $H_2(z)$ can be determined uniquely even when the input signals, $x_1(k)$ and $x_2(k)$, are identical. That is, the adaptive filters shown in Fig. 2 converge to $H_1(z)$ and $H_2(z)$, respectively (see Appendix A).

2.2 Extension to n Loudspeakers

The idea of introducing mutually prime pre-filters can easily be extended to the case of n loudspeakers. The set of $H_1(z) \cdots H_n(z)$ satisfying Eq. (4) should be unique in order to precisely identify n acoustic paths between the loudspeakers and a microphone.

$$G_1(z)H_1(z) + \cdots + G_n(z)H_n(z) = H_0(z) \quad (4)$$

Assume that the order of the acoustic paths are $m-1$. The requirements for a unique solution set given by Appendix B are

- (a) The order of pre-filters $G_i(z)$ is $(n-1)m$,
- (b) The coefficient of the order $(n-1)m$ of $G_n(z)$ is not 0, and
- (c) $[G_1(z), \dots, G_{n-1}(z)]^T$ and $G_n(z)$ are mutually prime.

Therefore, to identify n acoustic paths from n identical input signals, we simply need to design pre-filters $G_i(z)$ to satisfy requirements (a), (b), and (c).

2.3 Extension to Correlated Signals

Using single-input multi-output (SIMO) FIR filters that simulate room transfer functions, correlated multi-channel signals can be modeled as :

- (a) multi-channel signals produced by such an FIR filter and a single sound source,
- (b) the sum of multi-channel signals produced by each sound source and each SIMO FIR filter.

Consider the case where correlated two-channel signals are produced from a single sound source, $x_1(k)$, and a pair of FIR filters, $F_1(z)$ and $F_2(z)$, as shown in Fig. 3.

The relation between signal $x_1(k)$ and $y(k)$ in the z -transform is given by

$$Y(z) = X_1(z)(G_1(z)F_1(z)H_1(z) + G_2(z)F_2(z)H_2(z)). \quad (5)$$

Let the greatest common divisor of z polynomials $F_1(z)$ and $F_2(z)$ be denoted by $GCD(F_1(z), F_2(z))$. Let the degree of z polynomials $F_1(z)$ be denoted by $\deg F_1(z)$. Again, a set of $H_1(z)$ and $H_2(z)$ satisfying the diophantine equation Eq. (5) is unique when $G_1(z)F_1(z)$ and $G_2(z)F_2(z)$ satisfy the following relations.^{9,12)}

$$\begin{aligned} \deg G_1(z)F_1(z) &> \deg H_2(z) \\ \deg G_2(z)F_2(z) &> \deg H_1(z) \\ GCD(G_1(z)F_1(z), G_2(z)F_2(z)) &= 1 \end{aligned} \quad (6)$$

This means that the convergence of adaptive filters $\hat{H}_1(z)$ and $\hat{H}_2(z)$ to actual acoustic transmission paths $H_1(z)$ and $H_2(z)$ is guaranteed if $F_1(z)$, $F_2(z)$, $G_1(z)$ and $G_2(z)$ satisfy Eq. (6).

Since not all two-channel signals are produced from a single sound source, we will consider the case of two-channel correlated input signals produced from mutually independent broadband signals, $p(k)$ and $q(k)$, as shown in Fig. 4.

The relation between signals $p(k)$, $q(k)$, and $y(k)$

Fig. 3 Proposed estimator of acoustic paths $H_1(z)$ and $H_2(z)$ and configuration synthesizing two-channel correlated input signals from single sound source $x_1(k)$ and FIR filters $F_1(z)$ and $F_2(z)$.

Fig. 4 Signal flow for synthesizing correlated two-channel signals from two sound sources.

is given by the following equation in the z -transform :

$$Y(z) = H_p(z)P(z) + H_q(z)Q(z), \quad (7)$$

where

$$\begin{aligned} H_p(z) &\triangleq F_{p1}(z)G_1(z)H_1(z) + F_{p2}(z)G_2(z)H_2(z), \\ H_q(z) &\triangleq F_{q1}(z)G_1(z)H_1(z) + F_{q2}(z)G_2(z)H_2(z). \end{aligned} \quad (8)$$

$H_p(z)$ and $H_q(z)$ given by Eq. (7) are unique because $p(k)$ and $q(k)$ are independent. The set $H_1(z)$ and $H_2(z)$ satisfying Eq. (8) can be determined uniquely when two sets of simulation filters, $(F_{p1}(z), F_{p2}(z))$ and $(F_{q1}(z), F_{q2}(z))$, and pre-filters $G_1(z)$ and $G_2(z)$ satisfy Eq. (9), because Eq. (8) has the same structure as diophantine equation Eq. (5).

$$\begin{aligned} \deg G_1(z)F_{p1}(z) &> \deg H_2(z) \\ \deg G_2(z)F_{p2}(z) &> \deg H_1(z) \\ GCD(G_1(z)F_{p1}(z), G_2(z)F_{p2}(z)) &= 1 \end{aligned} \quad (9)$$

$$\begin{aligned} \deg G_1(z)F_{q1}(z) &> \deg H_2(z) \\ \deg G_2(z)F_{q2}(z) &> \deg H_1(z) \\ GCD(G_1(z)F_{q1}(z), G_2(z)F_{q2}(z)) &= 1 \end{aligned}$$

This means that the relation between the four input signals, $p_1(k)$, $p_2(k)$, $q_1(k)$, and $q_2(k)$, uniquely determines the set $H_1(z)$ and $H_2(z)$. This also means that correlation matrix R_A defined by Eq. (10) is symmetric and positive definite, where $L-1$ is the degree of $H_1(z)$ and $H_2(z)$.

$$R_A \triangleq \begin{bmatrix} p_1(k) \\ p_2(k) \\ q_1(k) \\ q_2(k) \end{bmatrix} [p_1^T(k) \ p_2^T(k) \ q_1^T(k) \ q_2^T(k)], \quad (10)$$

$$\begin{aligned} p_1(k) &\triangleq [p_1(k-L+1) \cdots p_1(k)] \\ p_2(k) &\triangleq [p_2(k-L+1) \cdots p_2(k)] \\ q_1(k) &\triangleq [q_1(k-L+1) \cdots q_1(k)] \\ q_2(k) &\triangleq [q_2(k-L+1) \cdots q_2(k)] \end{aligned}$$

$$\begin{aligned}
p_1(k) &\triangleq G_1(z)F_{p_1}(z)p(k) \\
p_2(k) &\triangleq G_2(z)F_{p_2}(z)p(k) \\
q_1(k) &\triangleq G_1(z)F_{q_1}(z)q(k) \\
q_2(k) &\triangleq G_2(z)F_{q_2}(z)q(k) \\
\left[\begin{array}{c} \dots \\ \dots \end{array} \right] [\dots] &: \text{time average}
\end{aligned}$$

However, as shown in Fig. 4, the actual input signals are not the four input signals, $p_1(k)$, $p_2(k)$, $q_1(k)$, and $q_2(k)$, but are instead the two input signals, $p_1(k) + q_1(k)$ and $p_1(k) + q_2(k)$. Therefore, it must be proved that correlation matrix R_B , defined from these two input signals, is always positive definite and invertible.

$$R_B \triangleq \frac{\begin{bmatrix} p_1(k) + q_1(k) \\ p_2(k) + q_2(k) \end{bmatrix} \begin{bmatrix} p_1^T(k) + q_1^T(k) & p_2^T(k) + q_2^T(k) \end{bmatrix}}{L} \quad (11)$$

Let $L-1$ be the degree of the acoustic paths. Equation (12) shows how the constant matrix T relates the correlation matrix of the four input signals, R_A , to that of two input signals, R_B .

$$\begin{aligned}
R_B &\triangleq T^T \begin{bmatrix} p_1(k) \\ p_2(k) \\ q_1(k) \\ q_2(k) \end{bmatrix} \begin{bmatrix} p_1^T(k) & p_2^T(k) & q_1^T(k) & q_2^T(k) \end{bmatrix} T \\
&= T^T R_A T, \\
T &\triangleq \begin{bmatrix} I & O & I & O \\ O & I & O & I \end{bmatrix}^T \quad (12) \\
I &: L \times L \text{ unit matrix} \\
O &: L \times L \text{ zero matrix}
\end{aligned}$$

Since R_A is a positive definite symmetric matrix, Eq. (13) holds for R_B :

$$\forall v (Tv \neq 0), v^T R_B v = (Tv)^T R_A (Tv) > 0. \quad (13)$$

Thus, two acoustic paths can be identified from the two input signals, $p_1(k) + q_1(k)$ and $p_2(k) + q_2(k)$, and a single output signal because all eigenvalues of R_B are positive and R_B is invertible. This also holds for cases with more than two independent sound sources. Therefore, multiple acoustic paths can be identified with correlated input signals produced from multiple sound sources.

3. SIMULATION

3.1 Settings

We evaluated our proposed identification method using numerical simulation. The acoustic paths,

$H_1(z)$ and $H_2(z)$, to be estimated were 512-point room-transfer functions measured in an actual room at a sampling frequency of 8 kHz. This room had a reverberation time of 200 ms. We used the NLMS algorithm with a step-size matrix of exponentially weighted diagonal elements¹³⁾ as the adaptive algorithm.

Maximum phase filter $G_1(z)$ and delayed minimum phase filter $G_2(z)$ were used as the pre-filters. Their transfer functions were as follows.

$$\begin{aligned}
G_1(z^{-1}) &= 0.2 + 1.0z^{-L} \\
G_2(z^{-1}) &= 1.0z^{-1} + 0.2z^{-2L} \\
L &= 512
\end{aligned} \quad (14)$$

This pair of pre-filters has the following properties¹⁴⁾:

- Pre-filters $G_1(z)$ and $G_2(z)$ are mutually prime,
- Zeros of these filters are symmetrical with respect to the unit circle in the z plane,
- Pre-filters $G_1(z)$ and $G_2(z)$ have the same magnitudes.

3.2 Simulation A

In simulation A, we tested our proposed method for three kinds of input signal pairs: (A1) a pair of uncorrelated white noise signals, (A2) a pair of identical white noise signals, and (A3) a pair of correlated noise signals. The signals were synthesized using white noise and two 512-tapped FIR filters representing the actual room-transfer functions. The corresponding results are shown in Fig. 5. Curve a represents the coefficient error energy of the adaptive filters defined in Eq. (15), without using the pre-filters and curve b represents that with the pre-filters.

$$|e|^2 \triangleq \frac{|\mathbf{h}_1 - \hat{\mathbf{h}}_1|^2 + |\mathbf{h}_2 - \hat{\mathbf{h}}_2|^2}{|\mathbf{h}_1|^2 + |\mathbf{h}_2|^2}, \quad (15)$$

where

$\mathbf{h}_1, \mathbf{h}_2$: original acoustic transmission paths given by the 512-tapped FIR filters,

$\hat{\mathbf{h}}_1, \hat{\mathbf{h}}_2$: acoustic transmission paths adaptively estimated.

In case A1, using a pair of uncorrelated white noise signals, the coefficient error energy dropped to -30 dB in only 1 s, whether the pre-filters were used or not. In case A2, using a pair of identical white noise signals, the coefficient error energy when using the pre-filters (curve b) was much lower than that when the pre-filters were not used (curve a). Curve a became saturated at around -4 dB, while

Fig. 5 Results of simulation A. Input signals of (A1), (A2), and (A3) were, respectively, a pair of uncorrelated white noise signals, a pair of identical white noise signals, and a pair of correlated noise signals. The coefficient error of the adaptive filters without pre-filters is shown by (a) and that with pre-filters is shown by (b).

curve b continued to decrease even after passing -20 dB in about 5 s. This tendency was also found in case A3, using a pair of correlated white noise signals. Curve a became saturated at around -9 dB, while curve b continued to decrease

even after passing -20 dB in about 8 s. These results demonstrate the effectiveness of our fundamental strategy of using mutually prime pre-filters.

3.3 Simulation B

In simulation B, we tested our proposed method for three kinds of correlated two-channel signals synthesized using two independent white noise signals and four 512-tapped FIR filters representing the actual room transfer functions. The changes in the coefficient error energy defined by Eq. (15) are shown in Fig. 6. Curve a shows the case in which the amplitude ratio of one input signal to the other was 1 to 10; curve b shows the case for 3 to 10; and curve c shows the case for 10 to 10. The advantage of using the pre-filters was not clear when the ratio was 10 to 10, but became more obvious as the ratio became smaller.

The continuous decrease of the estimation error in the case without pre-filters (see the graph in Fig. 6) may be explained using the framework of the pre-filters. When the correlation between two input

Fig. 6 Results of simulation B. Amplitude ratios: (a) 1 : 10 (b) 3 : 10 (c) 10 : 10.

signals, $x_1(k)$ and $x_2(k)$, is high enough, Eq. (16) can be a good approximation, where $J_1(z, r)$ and $J_2(z, r)$ are finite-order z polynomials and their coefficients are function of r , the ratio of the amplitudes of the two sound sources.

$$X_2(z) \cong \frac{J_2(z, r)}{J_1(z, r)} X_1(z) \quad (16)$$

Using this approximation allows the relation between the input signals and the output signal in the z -transform to be given by

$$(H_1(z)J_1(z, r) + H_2(z)J_2(z, r))X_1(z) = J_1(z, r)Y(z), \quad (17)$$

The continuous decrease suggests that $J_1(z, r)$ and $J_2(z, r)$ defined in Eq. (16) work similarly to the pre-filters. The tendency for the convergence speed to improve as r approaches 1 suggests that an r closer to 1 enlarges the spaces of the zeros of $J_1(z, r)$ and that of $J_2(z, r)$ in the z plane.

4. SUMMARY

We have developed a method for precisely identifying multiple transmission paths of an acoustic system. By applying prime pre-filters to each input signal, a unique solution is guaranteed. The properties required for the pre-filters were determined by using a diophantine equation. The effectiveness of the proposed method was verified by numerical simulation. The design of pre-filters, which introduce almost inaudible distortion, is still a problem to be examined.

Stereo echo cancellation in a room having a short reverberation time seems not to be problematic when the room is connected to a room having a long reverberation time.⁷⁾ This may be interpreted to mean that the long reverberation works like the pre-filters proposed here.

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Appendix A

The mean-square error performance surface for the adaptive filter¹⁵⁾ is a quadratic function of its weights $\hat{h}$ and the actual transmission path h as shown in Eq. (A-1) when the input signals and the desired response are statistically stationary¹⁵⁾:

$$\text{Mean Square Error} \triangleq \hat{\xi} = \xi_{\min} + (\hat{\mathbf{h}} - \mathbf{h})^T \mathbf{R} (\hat{\mathbf{h}} - \mathbf{h}) \quad (\text{A-1})$$

$$\mathbf{R} \triangleq E[\mathbf{X}_k \mathbf{X}_k^T],$$

where $\mathbf{X}_k$ is a column vector of input signal.

When the solution of the input-output relation is unique and the input signal's bandwidth is sufficient, matrix $\mathbf{R}$ in Eq. (A-1) is positive-definite and the performance surface has a single global optimum. In the process of adaptive filtering, the weight vector seeks the minimum of the performance surface by the gradient method, and reaches the single global optimum in the mean.

However, when multiple solutions exist, $\mathbf{R}$ is no longer positive-definite and the minimum of performance surface is not a single point but sub-space.^{6,7)} Hence, the solution given by the minimum mean-square-error does not always coincide with the true solution, and gradient search does not guarantee that the weight will converge to the actual solution.^{6,7)}

Hence the adaptive filter converges to the actual solution when the solution is unique and the performance surface has a single global optimum.

Appendix B

Consider the identification of n acoustic paths between n loudspeakers and a single microphone. Assume that the order of the acoustic paths is given by $m-1$, that n input signals are identical, and that n pre-filters are inserted prior to radiation via such n loudspeakers. Let $y(k)$ be the signal received by the microphone. Let $H_0(z)$ be the transfer function from signal $x(k)$ to signal $y(k)$. The relation among the transfer functions of the acoustic paths and pre-filters is thus given by Eq. (B-1). $H_i(z)$ which satisfies Eq. (B-1), must be unique for the acoustic paths to be identified from n signals filtered by pre-filters and a single output signal.

$$G_1(z)H_1(z) + \cdots + G_n(z)H_n(z) = H_0(z), \quad (\text{B-1})$$

We prove that there exist pre-filters that allow unique estimation of the acoustic paths from n

identical input signals and a single output signal, and that the order of the pre-filters is given by $(n-1)m$.

Let the order of the n pre-filters be $L-1$ and define $nm \times (L+m-1)$ matrix $S(m)$ according to Eq. (B-2).

$$\begin{aligned} G_i(z) &= g_{iL-1}z^{L-1} + \cdots + g_{i0}z^0 \\ G_i(m) &\equiv \begin{bmatrix} \overbrace{g_{iL-1} \cdots \cdots g_{i0} \ 0 \ 0}^{L+m-1} \\ 0 \ g_{iL-1} \cdots \cdots g_{i0} \\ \vdots \\ g_{iL-1} \cdots \cdots g_{i0} \end{bmatrix}_m \\ S(m) &\equiv \begin{bmatrix} G_1(m) \\ \vdots \\ G_n(m) \end{bmatrix} \\ H_i(z) &= h_{im-1}z^{m-1} + \cdots + h_{i0} \\ \mathbf{h}_i^T &= [h_{im-1} \cdots h_{i0}] \\ H_0(z) &= h_{0L+m-1}z^{L+m-1} + \cdots + h_{00} \\ \mathbf{h}_0^T &= [h_{0L+m-1} \cdots h_{00}] \end{aligned} \quad (\text{B-2})$$

Equation (B-3) gives the relation between the coefficients of the transfer functions of the acoustic paths and pre-filters.

$$[\mathbf{h}_1^T \cdots \mathbf{h}_n^T] S(m) = \mathbf{h}_0^T \quad (\text{B-3})$$

If $S(m)$ is a square matrix and invertible, $H_1(z) \cdots H_n(z)$ can be uniquely determined from $H_0(z)$.

If a square matrix $S(m)$ satisfies the following relations:

- (a) The order of $G_n(z)$ is $L-1$,
- (b) $[G_1(z), \dots, G_{n-1}(z)]^T$ and $G_n(z)$ are mutually prime,

the rank of $S(m)$ is proved to satisfy Eq. (B-4) based on the discussion of the generalized resultant matrix¹⁶⁾ obtained by row exchanges of $S(m)$.

$$\text{rank } S(m) = m + \deg G_n(z) = m + L - 1 \quad (\text{B-4})$$

Therefore, when $S(m)$ is a square matrix ($nm = L+m-1$ or $(n-1)m = L-1$) and $G_1(z) \cdots G_n(z)$ satisfy the two above relations, $S(m)$ is invertible by Eq. (B-4). Hence, a set of $H_1(z) \cdots H_n(z)$ can be uniquely determined from the relation between n input signals and a single output signal.