Multiple reflections between rigid plane panels

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The early reflections in the sound field of an auditorium have important roles in the impulse response and transfer function. The practical calculations are already shown for the first reflections from the boundaries and compared with measured reflections in good agreement. In this paper, the estimation of the multiple reflections between rigid plane panels is treated, based on these. When a panel is small, or the distance between a point source or a receiving point and the panel is large, the multiple reflection of a boundary wave cannot be neglected compared with that of a geometrical wave. Because the first reflection of a rigid plane panel has separate geometrical wave and boundary waves, the multiple reflections between them are obtained by those of each wave. Since a boundary wave is explained as the contribution of lined point sources with directivity on an edge, its multiple reflection is calculated as their geometrical reflection and their boundary waves of a reflecting panel. This calculation method is applied to two rigid plane panels which have a broad angle in between, and two which are in parallel. The calculated results are compared, in the time and frequency domains, with measured values in good agreement.

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1. CALCULATION OF THE MULTIPLE REFLECTION BETWEEN TWO RIGID PLANE PANELS

A reflected wave $h(P, t)$ from a rigid plane panel in Fig. 1 is given¹⁾ by,

$$h(P, t) = -\frac{\delta\left(t - \frac{d}{C}\right)}{d} + \epsilon \cdot \frac{\delta\left(t - \frac{d'}{C}\right)}{d'} + \frac{1}{4\pi} \oint_r \frac{\delta\left(t - \frac{l+m}{C}\right)}{lm} \cdot \left\{ \frac{l \times m'}{lm' + l \cdot m'} + \frac{l \times m}{lm + l \cdot m} \right\} \cdot dg \quad (1)$$

where $\delta(t)$ is a Delta function, d is a distance between a point source and a receiving point, d' is a direct distance from the image source to a receiving point, ϵ takes 1, 1/2 and zero, when a receiving point is inside, on and outside the cone formed by

the image source P_s' and F , respectively; m, m' and l are vectors from a point source, an image source and a receiving point to the edge, respectively, dg is an element of a vector tangential to the edge, l is the edge of the panel, and C is a sound velocity.

The line integral in Eq. (1) $h_l(P, t)$ is also written in the form,²⁾

$$h_l(P, t) = \frac{1}{4\pi} \oint_r \frac{\delta\left(t - \frac{l+m}{C}\right)}{lm} \cdot \left\{ \frac{\cos(n', l) \sin(m', dg)}{1 + \cos(l, m')} + \frac{\cos(n, l) \sin(m, dg)}{1 + \cos(l, m)} \right\} dg \quad (2)$$

where n and n' are the inward normals on the cones formed by a real source P_s and an edge l , and an image source P_s' and the edge l , respectively. (a, b) shows an angle between two vectors in the brackets.

Equation (1) shows that when a specular reflection

Fig. 1 Sound reflection of a rigid plane panel by the line integral along Γ . A point source is at P_s and a receiving point is at P . P_s' is the image source of P_s .

Fig. 2 Second reflection of a geometrical wave having its boundary waves at the second panel.

point is on the panel, there is no reflection after a geometrical wave till a wave front reaches an edge to cause a boundary wave.

As regards the line element dq in the line integral, it has a Delta function, whose phase is already delayed by m/C , and a magnitude $1/m \cdot dq$ with the directivity expressed by the inner product of dq and the vector in the brackets, and it decays by $1/l$ with the phase shift of l/C . It means that each line element is a point source with the directivity which could be replaced by some point sources.

This interpretation of the first reflection of a rigid plane panel produces the following procedure of calculation^{3,4)} of the multiple reflections between two rigid plane panels.

1.1 Multiple Reflection of a Geometrical Wave

a) When the projection of a panel F_1 onto a panel F_2 from an image source P_s' covers F_2 as in Fig. 2, the second reflection by F_2 of a geometrical wave is obtained by the first reflection of F_2 from the image source P_s' . It is calculated by the second and third terms in the right hand side of Eq. (1) with,

$$\left. \begin{aligned} m &= m_2, & m' &= m_2', & l &= l_2 \\ d' &= d'' = \overline{P_s'P'}, & \text{and } \Gamma &= \Gamma_2. \end{aligned} \right\} \quad (3)$$

ϵ takes 1, 1/2 and zero, when the image of a receiving point P' is inside, on and outside the cone formed by the source P_s' and F_2 , respectively.

b) When it cuts F_2 by F_2' as in Fig. 3, the second reflection of a geometrical wave is the reflection from the area F_2' and is calculated by the second and

Fig. 3 Second reflection of a geometrical wave having its boundary waves on the projection of the first panel to the second.

third terms in Eq. (1) with,

$$\left. \begin{aligned} m &= m_2, & m' &= m_2', & l &= l_2 \\ d' &= d'', & \text{and } \Gamma &= \Gamma_2' = \overline{ABCD}. \end{aligned} \right\} \quad (4)$$

ϵ takes 1, 1/2 and zero, when the image of a receiving point P' is inside, on and outside the cone formed by the image source P_s' and F_2' , respectively.

More ordered reflection of a geometrical wave can be estimated by the first reflection of the last image source from a panel or the part on the last panel projected through the preceding panel as in a) or b).

1.2 Second Reflection of a Boundary Wave

1.2.1 Geometrical reflection

Since a geometrical wave in the second term in the right hand side of Eq. (1) is calculated as the contribu-

Fig. 4 Geometrical reflection of a boundary wave.

Fig. 5 Second reflected boundary wave of first reflected one.

tion from a singular point on a panel¹⁵⁾ which yields a specular reflection, the geometrical reflection of each line element dq is also obtained by finding the singular point on a panel F_2 corresponding to it. Boundary waves reflected at the edge of F_1 have such singular points of the lines $E'B'$, $B'C'$ and $C'G'$ on F_2 , where the straight line between each line element of F_1 and the image P' of a receiving point crosses F_2 as shown in Fig. 4. Geometrical reflection as the second reflection of boundary waves is then estimated by the line integral in Eq. (1) with,

$$\left. \begin{aligned} m &= m, \quad m' = m', \quad l = l_B, \quad \text{and} \\ \Gamma &= \vec{EB} + \vec{BC} + \vec{CG}. \end{aligned} \right\} \quad (5)$$

1.2.2 Second reflected boundary wave

The second reflected boundary wave of the first reflected one is considered very small compared with other reflections, but it can not be neglected in some cases. Such a secondary reflection can be also calculated by the interpretation discussed above.

Treating a line element dq_1 as a point source in

Fig. 5, its boundary wave $h_B(P, t)$ reflected by the panel F_2 is obtained by the following double line integral,

$$h_B(P, t) = - \left(\frac{1}{4\pi} \right)^2 \iint_{\Gamma_1} \iint_{\Gamma_2} \frac{\delta \left(t - \frac{m + l_1 + l}{C} \right)}{m l_1 l} \cdot \left\{ \frac{l_1 \times m'}{l_1 m' - l_1 \cdot m'} + \frac{l_1 \times m}{l_1 m - l_1 \cdot m} \right\} \cdot dq_1 \cdot \left\{ \frac{l \times l_1'}{l l_1' - l \cdot l_1'} + \frac{l \times l_1}{l l_1 - l \cdot l_1} \right\} \cdot dq_2 \quad (6)$$

where Γ_1 and Γ_2 are the edges of the panels F_1 and F_2 , whose line elements are dq_1 and dq_2 , respectively, and l_1' is the image of l_1 as for F_2 .

Numerical integrations in Eqs. (1) and (6) are estimated by dividing an edge into the segments.

2. COMPARISON OF MEASURED AND CALCULATED RESULTS

2.1 In the Case of a Broad Angle between Two Rigid Plane Panels

A bakelite panel of dimension $22.8 \text{ cm} \times 34.0 \text{ cm}$ and a plywood panel covered by hard plastic sheets of dimension $32.0 \text{ cm} \times 31.8 \text{ cm} \times 1.1 \text{ cm}$, with tapered edges, are used for two rigid plane panels. The arrangement of them in Fig. 6 is the case where they have the condition in Fig. 2 and that in Fig. 7 is the case in Fig. 3.

A spherical wave is generated by an electric dis-

Fig. 6 Multiple reflection between two rigid plane panels having the case in Fig. 2.

Fig. 7 Multiple reflection between two rigid plane panels having the case in Fig. 3.

Fig. 8 Filter's transfer function (a) and its inverse Fourier transform (b).

Fig. 9 (a) Impulse response of multiple reflections between the panels in Fig. 6; the upper curve shows a measured impulse response and the lower shows a calculated one.

Fig. 9 (b) Comparison of a measured and a calculated transfer functions for the panels in Fig. 6.

charger. Direct and reflected waves are received by a 1/4 inch condenser microphone which is omnidirectional till 15 kHz and is directed to measure them with the same directivity. They are recorded on digital memory and separated each other by processing. Each of them is Fourier transformed and a transfer function is obtained by their ratio. A practical impulse response is then found as the inverse Fourier transform of the high-cut filtered transfer function. The transfer function of the filter and its inverse Fourier transform are shown in Fig. 8.

Measured impulse response in the case of Fig. 6 is shown by a solid line in Fig. 9(a) and is compared with a calculated one below. The kind of reflections is shown at its position by a number along the calculated impulse response; a second reflection of a geometrical wave is shown by 1, a geometrical

reflection of a boundary wave by 2, a geometrical reflection of the boundary wave caused at the second reflection of a geometrical wave by 3, and a second reflected boundary wave by 4. Preceding waves are the first reflections by the panels F_1 and F_2 . Measured and calculated transfer functions are shown by solid and dotted line in Fig. 9(b), respectively.

Measured and calculated results in the case of Fig. 7 are shown in Figs. 10(a) and (b) in the same way.

They show the validity of this method with good agreements in the time and frequency domains. Especially, the introduction of the geometrical reflection of a boundary wave explains the measured results much better.

A sound source omnidirectional from 0.6 to 6.5 kHz, which is generated from the end of a cylindrical pipe of 16 mm diameter attached to a driver unit,

Fig. 10 (a) Impulse response of multiple reflections between the panels in Fig. 7; the upper curve shows a measured impulse response and the lower shows a calculated one.

Fig. 10 (b) Comparison of a measured and calculated transfer functions for the panels in Fig. 7.

Fig. 11 Frequency characteristics by the pure tone sweep and those by the measured and calculated transfer functions of the multiple reflections in Fig. 9(b).

Fig. 12 Frequency characteristics by the pure tone sweep and those by the measured and calculated transfer functions of the multiple reflections in Fig. 10 (b).

replaces the pulse generator and pure tone is swept between the panels in both cases. Frequency characteristics thus measured in the steady state is shown in Fig. 11 for the case of Fig. 6 and in Fig. 12 for the case of Fig. 7 by solid lines. A thin solid line is a direct sound pressure level from the source in both figures. Measured and calculated transfer functions in Figs. 9(b) and 10(b) are added to the direct sound pressure level and they are plotted by $\circ$ and $\times$, respectively.

The frequency characteristics from such a transfer

Fig. 13 Locations of a point source and a receiving point between two rigid plane panels in parallel.

function explain well those in the steady state with standard deviation of about 1.2 dB for both cases. It can be said that almost all the main reflections among the panels are included by the orders of reflections estimated above.

2.2 In the Case of Two Rigid Plane Panels in Parallel⁽⁴⁾

The two rigid plane panels which were used in 2.1 are arranged in parallel and a point source and a microphone are set on the axis which passes through their centers as in Fig. 13. The microphone faces the panel F_1 , in order to get a higher omnidirectional frequency range of the reflections from it and only the multiple reflections from F_1 are compared with calculated values. Sampling time is $5 \mu\text{s}$ with 4096 sampling points.

For example, on the calculation of the third reflections the following reflections are summed; reflections of the last panel of second reflected geometrical waves, geometrical reflections of boundary waves caused on the second panel by first reflected geometrical waves, second geometrical reflections of boundary waves on the first panel, and geometrical reflections of second reflected boundary waves. All of them except the last ones can be estimated by the single line integral Eq. (1). Second reflected boundary waves are calculated by the double line integral Eq. (6).

Measured and calculated impulse responses are compared in Fig. 14. Because of the direction of the microphone, the parts A_1 , A_2 and A_3 must be compared with each other. The positive boundary waves just after negative ones are composed by second reflected boundary waves and their geometrical reflections. Since the point source and the receiving

Fig. 14 Impulse responses measured and calculated with the second reflected boundary waves; they must be compared with each other at the parts A_1 , A_2 and A_3 .

(a)

(b)

Fig. 15 Comparison of the transfer functions of the second and third reflections; calculated [I] is with the second reflected boundary waves and calculated [II] without them.

(a) Second reflection A_2 and (b) third reflection A_3 .

Fig. 16 Transfer functions by the superpositions of the parts A_1 , A_2 and A_3 in Fig. 14; a solid line shows measured values, a dashed one shows calculated values with the second reflected boundary waves and a dotted one shows those without them.

point are on the central axis as in Fig. 13, a higher ordered amount of multiple reflections of boundary waves reaches in phase and becomes not negligible. Measured and calculated transfer functions of the parts A_2 and A_3 are compared in Figs. 15(a) and (b), respectively. Calculated values with second reflected boundary waves and their geometrical reflections are shown by a chained line and those without them are shown by a dotted line. The former calculation explains the measured results better than the latter in low frequency. A second reflected boundary

wave seems also to have more components in low frequency.

Transfer functions from the superposition of the parts A_1 , A_2 and A_3 are compared in Fig. 16. A solid line shows a transfer function from the measured reflections. A dashed line shows the calculated transfer function with second reflected boundary waves and a dotted line shows the calculated one without them. Second reflected boundary waves seem not to be neglected from 0.5 to 2 kHz even in this case. Difference less than 0.5 kHz is thought to be caused by the approximation of the zero velocity potential on the backs of the panels and the shortage of the signal to noise ratio in this frequency range.

3. CONCLUSIONS

Boundary waves from a geometrical wave multiple reflected between two rigid plane panels are obtained by the first reflection from the last panel or its part projected from the last image source through the preceding panel. This calculation, however, includes an excessive estimation of the geometrical reflection of a geometrical wave.

The first reflected boundary wave is treated as lined point sources on an edge. A geometrical reflection of them is one of its second reflections and shows the role of an iris to the excessive estimation. When there are more ordered multiple reflections of a boundary wave, multiple geometrical reflections of its second geometrical reflection should be estimated, which can be also calculated by a single line integral.

The second reflected boundary waves of the first reflected one are comprised of a smaller amount of reflections, but they can not be neglected when they are in phase as in between parallel rigid plane panels.

Such second reflected boundary waves can also be estimated by treating the first reflected boundary wave as lined point sources expressed by the double line integral.

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