Recently, new approaches for sound-image localization\(^1,2\) have been developed based on a model of hearing. However, they are applicable only to two-loudspeaker one-listener case. The method proposed here is their extension to multi-loudspeaker multi-listener case. Although our method cannot offer the optimal configuration of loudspeakers and listeners, it provides the least-squares solution for the transfer characteristics of the channels to the loudspeakers for given configuration of loudspeakers and listeners. This method is one of the applications of the generalized inverse of matrices to acoustic signal processing.\(^3\)

1. Theory of Sound-Image Localization for Multi-loudspeaker Multi-listener Case

The signals at the ears of listeners are characterized by acoustic transfer functions associated with the propagation paths from the sound source to the ears as illustrated in Fig. 1, where \(S\) denotes the source characteristics.

The sound pressures \(P_L\) and \(P_R\) produced acoustically at the left and right ears are expressed as

\[
\begin{align*}
P_L &= H_L S \\
P_R &= H_R S.
\end{align*}
\]

The acoustic transfer functions between the loudspeakers \(S_j\) \((j=1,\ldots,n)\) and both ears of the listeners \(L_i\) \((i=1,\ldots,m)\) are shown in Fig. 2, where \(T_{ijL}\) and \(T_{ijR}\) denote the transfer functions of the sound propagation paths from \(S_j\) to the left and right ears of \(L_i\), respectively.

Multi-loudspeaker multi-listener sound-image localization can be designed by inserting channel compensation network \(C_j\) into each signal channel to the loudspeaker \(S_j\). The sound pressures \(P'_{L}\) and \(P'_{R}\) produced at the left and right ears are the summation of the products of the voltages supplied to the loudspeakers and the transfer functions of the sound propagation paths as follows:

\[
\begin{align*}
P'_{L} &= \sum_{j=1}^{n} T_{ijL} C_j \left( P_j - H_j S \right) \\
P'_{R} &= \sum_{j=1}^{n} T_{ijR} C_j \left( P_j - H_j S \right)
\end{align*}
\]
where \( S \) denotes the source characteristics.

In order to obtain exact sound-image localization, the following relations should hold:

\[
\begin{align*}
P_{L}' &= \sum_{j=1}^{m} T_{ij}C_jS \quad \text{for } i=1, \ldots, m \ \ (2) \\
P_{R}' &= \sum_{j=1}^{m} T_{ij}C_jS \quad \text{for } i=1, \ldots, m
\end{align*}
\]

or

\[
\begin{align*}
\sum_{j=1}^{m} T_{ijL}C_j &= H_L \quad \text{for } i=1, \ldots, m \\
\sum_{j=1}^{m} T_{ijR}C_j &= H_R \quad \text{for } i=1, \ldots, m
\end{align*}
\]

With matrix notation, Eq. (4) can be rewritten as

\[
T \ C = H
\]

where

\[
T = \begin{bmatrix}
T_{11L} & T_{12L} & \cdots & T_{1mL} \\
T_{w11L} & T_{w12L} & \cdots & T_{w1mL} \\
T_{11R} & T_{12R} & \cdots & T_{1mR} \\
T_{w11R} & T_{w12R} & \cdots & T_{w1mR}
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
C_1 \\
\vdots \\
C_m
\end{bmatrix}, \quad H = \begin{bmatrix}
H_L \\
H_R
\end{bmatrix}
\]

(5)

The exact solution for \( C \) in Eq. (5) does not always exist for given \( T \) and \( H \). However, we can get the optimal solution for \( C \) in the sense of least-square error as follows:

\[
\hat{C} = T^+ H
\]

where \( T^+ \) denotes the generalized inverse of \( T \), and \( \hat{C} \) means the least-squares estimate for \( C \).

If different sound-image localization is required among listeners, the definition of \( H \) should be modified as

\[
H = \begin{bmatrix}
H_{L1} \\
\vdots \\
H_{Lm} \\
H_{R1} \\
\vdots \\
H_{Rm}
\end{bmatrix}
\]

(6)

where \( H_{L1} \) and \( H_{R1} \) correspond to \( H_L \) and \( H_R \) for listener \( L_1 \).

2. Exact Solutions for Some Special Cases

The proposed method gives only the least-squares solution for the characteristics of the channel compensation network \( C_j \) to be inserted into the channel to loudspeaker \( S_j \). That means the proposed method cannot realize the desired sound field exactly. The sound field generated by the proposed method is merely the approximated one under the least-squares criterion. There, however, can be some cases in which the exact solution exists. Those are the following two cases:

2.1 Two-loudspeaker one-listener case

For this case, Eq. (7) yields the exact solution

\[
C = \begin{bmatrix}
T_{11L} & T_{12L} \\
T_{11R} & T_{12R}
\end{bmatrix}^{-1} \begin{bmatrix}
H_L \\
H_R
\end{bmatrix}
\]

(9)

provided that the inverse matrix exists for any frequency point under consideration.

By locating the two loudspeakers at the symmetrical positions concerning the median plane of the listener, the following approximation holds:

\[
T_{11L} \approx T_{12R} (= T_S) \\
T_{11R} \approx T_{12L} (= T_C)
\]

where \( T_S \) and \( T_C \) denote the transfer characteristics of the channels from a loudspeaker to the ear of the same side and that to the opposite side, respectively. Under this approximation, Eq. (9) can be simplified to

\[
C = \begin{bmatrix}
T_S \\
T_C
\end{bmatrix}^{-1} \begin{bmatrix}
H_L \\
H_R
\end{bmatrix}
\]

(10)

2.2 Multi-listener case with head-sets

For this case, we can set \( n=2 \) and \( m=1 \) even if the actual number of listeners is more than one provided that the transfer characteristics or the coupling characteristics between each head-set and the corresponding ear are same and that the crosstalk between the left and the right channel is negligible. Under these assumptions, Eq. (5) yields

\[
\begin{bmatrix}
T_{11L} & 0 \\
0 & T_{12R}
\end{bmatrix} C = \begin{bmatrix}
H_L \\
H_R
\end{bmatrix}
\]

(12)

and we get

\[
C = \begin{bmatrix}
H_L \\
H_R
\end{bmatrix} / T_{11L} T_{12R}
\]

(13)

Further, \( T_{11L} \) and \( T_{12R} \) can be assumed to be identical for most of the listeners, and Eq. (13) might be simplified to

\[
C = \frac{1}{T_H} \begin{bmatrix}
H_L \\
H_R
\end{bmatrix}
\]

(14)

where

\[
T_H = T_{11L} T_{12R}
\]

(15)

3. Discussions

In practical applications, the most wanted is the method to determine the number and the configura-
tion of loudspeakers for a given number and configuration of listeners. The proposed method gives the least-squares solution for a given configuration of loudspeakers and listeners, but it does not give any information about the desirable configuration at all. It, however, is possible to calculate the least-squares error for the given numbers and the configuration of loudspeakers and listeners, therefore we can evaluate the degree of goodness of the numbers and the configuration of them. Consequently, some conventional search techniques in a multi-dimensional space can be employed to find the optimal solution for the numbers and the configuration of loudspeakers and listeners.

At present, any practical experiment has not been conducted yet, but the proposed method gives the exact solution for a two-loudspeaker one-listener case as a special case of the multi-loudspeaker multi-listener case. Therefore, the method seems to be the most relevant way for realizing multi-loudspeaker multi-listener reproduction scheme as far as the least-squares criterion has a sense in sound perception.

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References