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The effect of incident pulse width on transient waveform from a circular plate

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1. Introduction

In our previous papers,¹⁻³⁾ the mechanism of the transient sound radiation from a baffled circular plate was discussed in the time domain in terms of the waveform radiated when the plate is excited by a plane single triangular sound pulse at normal incidence. It was noted that one can obtain an arbitrary waveform in the far-field when the mechanical constants of the plate are selected.

The waveform radiated is changed not only by the mechanical properties of the plate itself but also by the characteristics of the incident pulse. It appears, however, that this change due to the incident pulse has not been explained in the time domain.

In the present paper, the effect of the pulse width of the incident triangular pulse on the waveform radiated is examined in the time domain by the use of the results obtained in the previous papers. The amplitude of the incident triangular pulse and the mechanical constants of the plate are kept constant. This procedure means that the sharpness of the incident pulse is changed in terms of the pulse width.

2. Effect of Pulse Width on Transient Waveform

Before considering the waveform due to the change of the pulse width, the brief résumé of the transient waveform radiated is as follows. A plane triangular sound pulse with the amplitude P and the half width T , as shown in Fig. 1 and formulated by Eq. (1), impinges normally on a thin elastic clamped circular plate of radius a set in a plane rigid baffle.

$$p_i(t) = \begin{cases} Pt/T & (T \geq t \geq 0), \\ -Pt/T + 2P & (2T \geq t \geq T), \\ 0 & (t \geq 2T, t \leq 0). \end{cases} \quad (1)$$

The resultant far-field axial pressure waveform $p(t)$ is given by Eqs. (2) to (4) in the semi-infinite space, that is,

Fig. 1 Pressure-time history of a triangular sound pulse.

$$p(t) = \frac{\rho a^2}{mz} \frac{P}{T} \sum_{n=1}^{\infty} \{S_n^2 \cdot y_n(t)\}, \quad (2)$$

$$y_n(t) = \begin{cases} f_n(t) & (T \geq t \geq 0), \end{cases} \quad (3.1)$$

$$y_n(t) = \begin{cases} f_n(t) - 2f_n(t-T) & (2T \geq t \geq T), \end{cases} \quad (3.2)$$

$$y_n(t) = \begin{cases} f_n(t) - 2f_n(t-T) + f_n(t-2T) & (t \geq 2T), \end{cases} \quad (3.3)$$

and

$$f_n(t) = \sin \omega_n t / \omega_n, \quad (4)$$

where m is the surface density of the plate, ρ the density of the air, z the distance from the center of the circular plate to the observation point, ω_n and S_n the n th resonant angular frequency and the amplitude of each vibration mode (see, Table 1 of Ref. 2)), respectively. When the plate moves with the piston-like vibration, the waveform is similar to the incident one, that is, triangular pulse.

Let's consider three kinds of T as the pulse width, namely $T=T'/2$, T' and $2T'$. Firstly, the waveform is estimated in the time interval between 0 and T of the first shock. If the waveform for $T=2T'$ is obtained, another ones for $T=T'/2$ and T' are given automatically. This is resulted from the fact that the amplitude is inversely proportional to T , as mentioned in Eq. (2), whereas the quantity in the bracket of Eq. (2) is not affected by T , as given by Eq. (3.1). Therefore, as shown in Fig. 2 as a model, the amplitude for $T=T'$ within the time interval between 0 and T' is doubled compared with that for $T=2T'$ within the same time interval, i.e., $MM'=M'M''$. The same relationship as before is also obtained in the case for $T=T'/2$ and $T=T'$, i.e., $NN'=N'N''$, when $T'/2 \geq t \geq 0$. Since with the lapse of time the waveform for $T=2T'$ deviates from the straight line resulted from the piston-

Fig. 2 Effect of pulse width on the first shock of waveform as a model, with T varied.

Fig. 3 Frequency characteristics of incident triangular sound pulse, as a function of ka , with T varied.

Quantity 0 dB refers to $|F_A(\omega)|=1$.

like vibration of the plate, the initial peak level for $T=T'$ becomes higher than for $T=2T'$ and that for $T=T'/2$ becomes higher than for $T=T'$. This means that the initial peak level increases with decreasing the pulse width, i.e., with increasing the sharpness of the incident pulse. It appears that the explanation for this

Fig. 4 Waveform radiated from circular plate of duralumin of thickness of 0.1 cm, with T varied. ---, First mode only.

has been given uncertainly in the frequency domain, that is, the sharp pulse contains the higher frequency components in comparison with the broader pulse.

If the plate moves with the piston-like vibration, the initial peak level is not affected by T , as mentioned in résumé.

Secondly, in the successive part ($t \geq 2T$) of the waveform following the first shock, it's convenient to estimate the waveform by means of the frequency characteristics of the incident pulse $F_{\Delta}(\omega)$, as given by Eq. (5) and illustrated in Fig. 3, with T varied.

$$F_{\Delta}(\omega) = -\frac{P}{T} \frac{(e^{-j\omega T} - 1)^2}{\omega^2}. \quad (5)$$

Since the frequency characteristics of the radiated wave is given by multiplying $F_{\Delta}(\omega)$ by the transfer function²⁾ of the pressure field, with increasing T only the resonant mode of the small number of n remains, whereas the higher mode terms are reduced, and then the simple oscillating waveform with the relatively large amplitude of the lower mode terms is obtained. On the other hand, with decreasing T the complicated oscillating waveform with the small amplitude of the higher mode terms is obtained.

After all, in the case of the large width, the waveform with the low initial peak level and the high amplitude of the successive part is obtained, and vice versa in the case of the small width.

3. Numerical Results

Numerical examples of the waveform radiated from the circular plate of duralumin of the thickness of 0.1 cm are shown in Fig. 4, with T varied. The constants used in the calculation are as follows;

a : 5 cm, m : 0.267 g/cm², Young's modulus: 5.98×10^{11} dyn/cm², Poisson's ratio: 0.34, z : 100 cm, ρ : 0.001215 g/cm³, P : 1 mbar, n : 7.

A small circle above the initial peak shown in Fig. 4 indicates the initial peak level resulted from the piston motion of the plate. One can recognize that the tendency of the change of the waveform as mentioned in the previous section is obtained. In the case of $T = 100 \mu\text{s}$, the successive part is determined only by a first few vibration modes. Figure 5 shows the initial peak level for the plate of duralumin as a function of T . Its 0 dB refers to the peak level caused by the piston motion of the plate.

Fig. 5 Effect to T on peak level of the first shock. Quantity 0 dB refers to the first peak level caused by the piston motion of the plate.

4. Conclusion

The effect of the pulse width on the transient waveform radiated from the clamped circular plate has been examined in the time domain when the plane triangular sound pulse impinges normally on the plate. It has been noted that with increasing the pulse width the initial peak level becomes small and the successive part becomes to be a simple oscillating waveform with the large amplitude. It may be valid that in the impulsive sound insulation problem one should direct ones attention to the properties of the incident wave.

References

- 1) I. Nakayama, A. Nakamura, and R. Takeuchi, "Transient radiation field from an elastic circular plate excited by a sound pulse," *Acustica* **46**, 276-282 (1980).
- 2) I. Nakayama and A. Nakamura, "Mechanism of transient sound radiation from a circular plate for impulsive sound wave," *J. Acoust. Soc. Jpn.(E)* **2**, 151-159 (1981).
- 3) I. Nakayama and A. Nakamura, "Transient waveform from a baffled circular plate," *J. Sound Vib.* **86**, 129-141 (1983).