Scattering from edge corrugations of SAW $\Delta v/v$ waveguide on LiNbO_3

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The SAW energy is concentrated into the SAW waveguide and is propagating along it without radiation. However the energy can radiate from the waveguide if there are corrugations at waveguide edges. This paper described the radiation characteristics from the rectangular edge corrugations of SAW $\Delta v/v$ waveguide on YZ- LiNbO_3 . The theoretical characteristics is derived from the perturbation method and it is found that the SAW energy radiates toward only one side of the waveguide, if the corrugation size and position at the both edges of the waveguide are suitably designed. The experimental results are qualitatively in agreement with theoretical predication.

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1. INTRODUCTION

Surface acoustic waves (SAW) are widely used in many kinds of devices such as filters, oscillators, delay lines and convolvers. The SAW waveguides have been investigated for the purpose of long time delay lines or many kinds of circuit components. Since the SAW energy can be concentrated in a lateral direction by means of a guiding structure placed on the substrate surface, the nonlinear properties are great and the SAW waveguide is very effective for convolvers or correlators.

The SAW energy radiates away from the SAW waveguide which has rectangular corrugations at the edges, and the radiation characteristics were already reported theoretically for the thin gold ribbon waveguide on non-piezoelectric glass.^{1,2)} However, it is very hard to verify the theoretical results for such a non-piezoelectric waveguide, because the SAW can not be excited by a usual interdigital transducer.

The thin metal strip waveguide on a piezoelectric substrate which is so-called $\Delta v/v$ waveguide, is the best for experiments because of the easy excitation of the SAW. However the propagation characteristics in the waveguide are different from that of

normal isotropic waveguide, because anisotropic properties should be considered on a piezoelectric substrate.

In this paper, theoretical and experimental results of propagation and radiation of the $\Delta v/v$ waveguide on YZ- LiNbO_3 which has rectangular corrugations at both edges are described. The radiation characteristics were obtained by the equivalent transmission line method including the influence of anisotropic propagation characteristics. The experimental results are compared with theories, and it is found that these are qualitatively in agreement each other.

2. ANALYSIS

2.1 Theory for $\Delta v/v$ Waveguide

Since every piezoelectric crystal is anisotropic, the SAW velocity changes as the cut angle and direction of propagation on a substrate surface. Figure 1 shows velocity curves for surface waves on y -cut lithium niobate (LiNbO_3) as functions of direction of propagation. Note that the SAW velocities on both electrically open and shorted surface greatly change. The velocity propagating along the z direction on y -cut LiNbO_3 is the values for the angle θ is 90° in Fig. 1, and it is symmetrical and approximately

Fig. 1 Surface wave velocity *versus* direction of propagation on y-cut LiNbO₃.

parabolic curve against θ around 90° .

The thin conductive film $\Delta v/v$ waveguide was investigated theoretically by the parabolic approximation to include anisotropy and compared with experiments.³⁻⁵⁾ In this paper their theory is modified to obtain the propagation characteristics of $\Delta v/v$ waveguide on LiNbO₃ and the scattering properties from the edge corrugation of the waveguide.

It is well known that the equivalent transmission line⁶⁾ along the transverse direction of the waveguide is shown as in Fig. 2. In the figure K_{sx} and K_{ox} are the propagation constants for x direction in the waveguide and the outside respectively, and they are represented by the following equations.

$$K_{sx} = \sqrt{K_s(\theta)^2 - K_z^2} \quad (1)$$

$$K_{ox} = \sqrt{K_o(90^\circ)^2 - K_z^2} \quad (2)$$

where K_z is the propagation constant of the waveguide along z direction, and $K_s(\theta)$ designates the propagation constant of the SAW on the electrically shorted surface propagating along the direction of angle θ as shown in Fig. 2 (a). This value $K_s(\theta)$ can be easily obtained as the inverse value of the SAW velocity on y-cut LiNbO₃ as shown in Fig. 1. In Eq. (2) $K_o(90^\circ)$ designates the propagation constant of the SAW on the electrically open surface constant of the SAW along z direction.

Fig. 2 Structure of $\Delta v/v$ waveguide and the equivalent transmission line along the transverse (x) direction.

Characteristic impedances Z_{sx} and Z_{ox} of the transmission line are shown as follows⁶⁾:

$$Z_{sx} = \omega \rho / K_{sx} \quad (3)$$

$$Z_{ox} = \omega \rho / K_{ox} \quad (4)$$

where ρ is the density of the substrate.

It is easy to obtain from the transmission line that the equivalent voltage amplitude $V(x)$ of the transmission line for the dominant guiding mode is given by as follows:

$$V(x) = \begin{cases} V_o \cos(K_{sx}x) & (\text{for } |x| \leq a) \\ V_o \cos(K_{sx}a/2) \cdot \exp\{K_{ox}(a/2 - |x|)\} & (\text{for } |x| > a) \end{cases} \quad (5)$$

Therefore the dispersion characteristics of the propagation constant K_z and the displacement amplitude can be obtained by the same method as isotropic waveguide, only if $K_s(\theta)$ is considered as function of propagation direction in the waveguide.⁴⁾

2.2 Equivalent Transmission Line for Leaky Wave

The analysis in this paper is concerned with the radiation from the SAW $\Delta v/v$ waveguide with edge corrugations on both sides as shown in Fig. 3 (a). It is well known that the presense of periodicity causes the SAW to be transformed into a leaky wave which radiates energy away from the structure and thus attenuates as it progresses along the guiding structure.

A realistic transmission-line model yields physically meaningful results for corrugations of arbitrary sizes. The equivalent circuit in the transverse direction of the waveguide, as shown in Fig. 3 (b), is derived from the same perturbation method as Ref-

erence 1) and 2). This equivalent transmission line network does not show the guided mode in the waveguide but the n -th space-harmonic field scattered away from the edge corrugations toward out-sides. Therefore the parameters of the transmission line are shown as follows, obtained by modifying the isotropic case²⁾ including anisotropy.

$$K_{s,zn} = \sqrt{K_s(\theta)^2 - K_{zn}^2} \quad (6)$$

$$K_{o,zn} = \sqrt{K_o(90^\circ)^2 - K_{zn}^2} \quad (7)$$

$$Z_{s,zn} = \omega \rho / K_{s,zn} \quad (8)$$

$$Z_{o,zn} = \omega \rho / K_{o,zn} \quad (9)$$

where K_{zn} is the perturbed propagation constant of n -th space-harmonic field when $n = \pm 1, \pm 2, \pm 3, \dots$ (with $n=0$ omitted) determined by the corrugation period l as follows:

$$K_{zn} = K_z - \frac{2\pi n}{l} \quad (10)$$

The distributed current sources $J_n(x)$ is physically equivalent to the energy sources of leaky wave in the corrugation region, which is given by

$$J_n(x) = \begin{cases} (K_s^2 - K_o^2) \frac{V(x)}{j\omega\rho} \frac{\sin(n\pi b/l)}{n\pi} & \text{(for } (a-h_1)/2 < x < (a+h_1)/2) \\ (K_s^2 - K_o^2) \frac{V(x)}{j\omega\rho} \frac{\sin(n\pi b/l)}{n\pi} e^{-j2n\pi P/l} & \text{(for } (a+h_2)/2 < x < -(a-h_2)/2) \\ 0 & \text{(elsewhere)} \end{cases} \quad (11)$$

where $V(x)$ denotes voltage amplitude of equivalent transmission network for the guided mode given by Eq. (5).

Furthermore, the effect of the distributed current density $J_n(x)$ may be approximated by replacing it with two current sources I_{n1} and I_{n2} located at $x = a/2$ and $x = -a/2$ respectively, as shown Fig. 3 (c), where

$$I_{n1} = \int_{(a-h_1)/2}^{(a+h_1)/2} J_n(x) dx \quad (12)$$

$$I_{n2} = \int_{(-a-h_2)/2}^{(-a+h_2)/2} J_n(x) dx \quad (13)$$

If the corrugation heights h_1 and h_2 are small enough, the current amplitude I_{n1} and I_{n2} are proportional to h_1 and h_2 , respectively. In connection with phase, since the corrugation of left side of waveguide is

Fig. 3 SAW $\Delta v/v$ waveguide with edge corrugations and two types of equivalent transmission lines.

shifted in length P from that of right side as shown in Fig. 3 (a), there is a phase difference ψ between current I_{n1} and I_{n2} , where

$$\begin{aligned} \psi &= \arg(I_{n2}) - \arg(I_{n1}) \\ &= \frac{2n\pi P}{l} \end{aligned} \quad (14)$$

As explained later, this phase difference is strongly related to the radiative direction of leaky energy away from the waveguide.

From these results, both amplitude and phase of current sources I_{n1} and I_{n2} can arbitrary chosen by changing the corrugation parameters such as h_1 , h_2 , P and l in Fig. 3 (a).

2.3 Radiation from Edge Corrugations

The leakage angle θ_r of n -th space-harmonics from the waveguide is derived from the phase-matching condition as,

$$\sin \theta_r = \frac{K_{zn}}{K_o(\theta_r)} \quad (15)$$

The calculated values θ_r of phase velocities for several space-harmonics are displayed as the solid lines in Fig. 4, where the dashed line shows the leakage angle of group velocity for the first space-harmonic field. The difference between the angle of phase and

Fig. 4 Leakage angle from the waveguide with edge corrugations.

group velocities can be calculated by the power flow angle caused by piezoelectric anisotropy.

It is clear from Fig. 4 that there is no radiation below the cut off values of $K_{sz}a$, which is proportional to frequency, and that leakage angle changes from backward ($\theta_r = -90^\circ$) direction to forward ($\theta_r = 90^\circ$) of guided wave as frequency increases.

The leakage energy radiates away from the waveguide along x direction on y -cut LiNbO_3 when $\theta_r = 0^\circ$, and the direction of group velocity is same as that of phase velocity *i.e.* power flow angle is equal to zero in this case.

Therefore these curves are important to determine the transducer's position to receive the leaky wave energy radiated from the corrugated waveguide.

If energy radiates from the waveguide, guided SAW propagates as a leaky mode and non-zero attenuation constant α should be obtained. This attenuation constant α is divided into two terms, which are represented by α_1 and α_2 from radiation power ΔP_1 and ΔP_2 toward the right and left sides, respectively.

$$\alpha = \alpha_1 + \alpha_2 = \frac{\Delta P_1}{2P_z} + \frac{\Delta P_2}{2P_z} \quad (16)$$

where P_z is propagation power of guide wave along z direction across $-\infty < x < \infty$.

According to the basic circuit theory, the radiation power per unit length (along z) that leaked away from the waveguide is given by as follows:

$$\Delta P_1 = \text{Re} \left[\frac{|V_n(a/2)|^2}{Z_{o2n}} \right] \quad (17)$$

$$\Delta P_2 = \text{Re} \left[\frac{|V_n(-a/2)|^2}{Z_{o2n}} \right] \quad (18)$$

where $V_n(a/2)$ and $V_n(-a/2)$ are the voltages at $x = a/2$ and $-a/2$ in the equivalent circuit respectively in Fig. 3 (c). As all $|V_n(a/2)|^2$ and $|V_n(-a/2)|^2$ are proportional to P_z , Eq. (16), (17), and (18) yield α in terms of the corrugation parameters only. Therefore the leakage characteristics can be obtained from these equations.

The leakage energy usually radiates to both sides away from the waveguide. However, corrugation parameters can be determined so that the leaky wave energy to the left side vanishes and the SAW energy radiates to only the right side, if $|V_n(-a/2)|$ is equal to zero in Eq. (18). This condition is very important to design SAW devices by using this type of waveguide with edge corrugation, because the achievable power for leaky wave received by one IDT increases 3 dB.

2.4 Numerical Results of Leakage Power

Numerical calculation was performed for the $\Delta v/v$ waveguide propagating along z direction on y -cut LiNbO_3 substrate. Figure 5 shows the ratio of radiation power ΔP_1 toward the right side to total radiation power $\Delta P_1 + \Delta P_2$ for both sides against phase difference ψ which is represented by $360 P/l$ degrees from Eq. (14) for the first space-harmonics when $n=1$. In this case, the solid and dashed lines are shown for wavenumber-width product $K_{sz}a = 10$ and 12 respectively, when $l/a = 0.5$ and $h_1/a = h_2/a = 0.1$.

Fig. 5 Ratio of radiation power $\Delta P_1/(\Delta P_1 + \Delta P_2)$ versus phase difference ψ between two series of corrugations at both edges.

Fig. 6 Frequency characteristics of attenuation α per one period of corrugation l .

Note that leaky wave energy radiates toward only right side of the waveguide when $\psi = 326^\circ$ for $K_{sz}a = 10$. It is easily understood from physical consideration that leakage energy radiate equally toward both sides of the waveguide when $\psi = 0^\circ, 180^\circ$, and 360° .

Figure 6 shows the frequency characteristics for the attenuation constant αl which means the attenuation per period l , when two corrugations are shifted so that the phase difference ψ is 326 degrees i.e. $P/l = 0.906$ or $P/l = -0.094$. Note that there is no radiation below cut off frequency at $K_{sz}a = 6.4$ and that radiation power to right or left side changes steeply as function of frequency.

As explained above, the leakage angle changes by period l and frequency, but the corrugation period can be determined so that the leakage energy propagates along x direction on y -cut LiNbO_3 , which is obtained from the condition of $\theta_r = 0^\circ$ in Eq. (15). The corrugation parameters are especially determined from both this condition and the condition that the energy radiates only one side of the waveguide. The results obtained are expressed as follows:

$$\begin{cases} K_{sz}a = 15.8, & l/a = 0.4 \\ h_1/a = h_2/a = 0.1, & \psi = 278^\circ \end{cases} \quad (19)$$

Fig. 7 Configuration of $\Delta v/v$ waveguide with edge corrugations and interdigital transducers for experiments.

3. EXPERIMENTS

3.1 Experimental Waveguide and Results

Experiments were performed to verify the theoretical prediction. The exciting and receiving interdigital transducers and $\Delta v/v$ waveguide with edge corrugations were made by lithographic technique on y -cut LiNbO_3 as shown in Fig. 7. The propagation direction of waveguide is chosen to be z -axis and the leaky wave radiated from corrugations propagating along x -axis can be received by two IDTs on both sides.

The waveguide width a is designed to be 0.25 mm and the center frequency is determined as 34.2 MHz, $l = 0.1$ mm, $h_1 = h_2 = 0.025$ mm and $P = 0.0772$ mm from Eq. (19).

The theoretical attenuation constants α_1 and α_2 , which are caused by leaky waves toward right side and left side of the waveguide, are displayed in Fig. 8 for designed corrugations of the waveguide. Note that the α_2 is very small at the center frequency 34.2 MHz, though α_1 is almost constant in the frequency range in Fig. 8.

The parameters of exciting and receiving IDTs made for experiments are shown in Table 1. The electrode periods of exciting and receiving transducers are different from each other, because they depend on the SAW velocities along z and x direction on y -cut LiNbO_3 respectively.

Figure 9 shows the experimental frequency response received by two transducers of leaky wave radiated away from the waveguide with edge corrugations. The solid and dashed lines show the receiving power of leaky waves radiated away from the waveguide by right and left sides respectively. Note that

Fig. 8 Theoretical attenuation constants α_1 and α_2 for designed waveguide with corrugations.

Fig. 9 Experimental frequency response of leaky wave from the waveguide received by two transducers.

the radiation toward right side is greater than left side at the center frequency as theoretically expected. This difference is only 8 dB but this might be in the cause of size errors of corrugations and waveguide from the theoretical design, which will be discussed later.

Table 1 Parameters of exciting and receiving interdigital transducers.

	Exciting	Receiving
Center frequency (MHz)	34.18	34.18
Electrode period (mm)	0.1008	0.11
Electrode width (mm)	0.0252	0.0275
Aperture (mm)	0.5	15
Finger pairs	30	30

Fig. 10 Model for analysis to obtain the receiving power of oblique incidence.

3.2 Receiving Power of Leaky Wave

It is interesting to note in Fig. 9 that experimental frequency response is very narrow compared with the frequency characteristics of attenuation constants shown in Fig. 8. And this is much narrower than frequency response of receiving interdigital transducer itself with 30 pairs of fingers. This is because that the leakage angle from the periodical corrugations changes as function of frequency, and then the leaky wave reaches to receiving transducers at oblique incidence. Therefore this influence should be taken account into theory to obtain the receiving power of leaky wave.

The receiving voltage of oblique incident wave to the transducer, as shown in Fig. 10, are proportional to the following value calculated by summing up all receiving signals at each finger.

$$V_r = A \sum_{n=1}^N \left[\int_{d \tan \theta_r}^w \exp \{ -jK_0(\theta)(z \cos \theta + nlp \sin \theta) \} dz \right] \quad (20)$$

where A is a constant related to the exciting power, N is the number of fingers and $N=30$ in this paper. When $\theta_r=0^\circ$ at the center frequency, the leaky wave reaches normally to the receiving transducer. There-

Fig. 11 Theoretical insertion loss between exciting and receiving transducers of right and left sides of waveguide with edge corrugations.

fore the insertion loss of receiving power can be calculated by Smith's equivalent network, and the constant value A of Eq. (20) can be determined in comparison with equivalent network when $\theta_i = 0^\circ$.

Figure 11 shows the theoretical insertion loss between exciting and two receiving transducers. This results are obtained by summing up the insertion losses of exciting and receiving transducer including influence of oblique incidence expressed by Eq. (20), and radiation power ΔP_1 and ΔP_2 from corrugation given by Eq. (17) and (18). The insertion loss of interdigital transducer with 30 pairs of fingers is also plotted in the same figure, and it is found that the frequency response does not explain by the transducer characteristics.

As compared experimental results in Fig. 9 with theoretical prediction in Fig. 11, the experimental and theoretical frequency responses of L_1 for the leaky wave toward right side are in good agreement each other from a view-point of frequency bandwidth. This means that the propagation direction of leaky wave changes as function of frequency as predicted by theory. However, experimental insertion loss L_2 does not agree with theory, and this will be discussed in the next.

3.3 Discussion on the Experimental Errors

The corrugation parameters of waveguide were determined theoretically and made by lithographic technique for experiments. However actual size of

Table 2 Size errors of corrugations.

Waveguide width	a (mm)	0.250	0.259
Corrugation period	l (mm)	0.100	0.1004
Corrugation shift	P (mm)	0.0772	0.0760

Fig. 12 Modified theoretical insertion loss including corrugation errors.

corrugations are a little bit different from theoretical design, as shown in Table 2. The size errors are very small but the influence on frequency response was theoretically checked.

The theoretical frequency responses including corrugation errors are shown in Fig. 12. Note that the insertion loss L_2 toward left side greatly changes from ideal response shown in Fig. 11, even though size errors are very small. Frequency response of insertion loss L_1 is almost same as the ideal design, but the minimum loss is larger as compared with Fig. 11.

The theoretical curves as shown in Fig. 12 are in good agreement with experimental results of Fig. 9. This agreement suggests us that the corrugations must be made without error from the theoretical design. However the actual deviation of size is very small and only micron order as listed in Table 2, and it is very hard to make more precisely waveguide with edge corrugations in our laboratory.

The investigation on this paper was first motivated to obtain narrow band filters by using the SAW scattering from corrugations of waveguide as shown in Fig. 7. Because, the narrower frequency band width of the filter must be easily obtain, if the receiving IDT-1 is placed farther away from the waveguide. However, the more precise manufactur-

ing technique should be required in order to obtain the practical devices by the principle in this paper as explained in this section.

4. CONCLUSIONS

The radiation characteristics from the edge corrugations of SAW $\Delta v/v$ waveguide on y -cut LiNbO_3 was investigated theoretically and experimentally. First, the theoretical analysis was shown for scattering away from the periodic corrugations of waveguide by perturbation approach using equivalent transmission network including anisotropy of LiNbO_3 crystal. Second, the corrugations were especially designed in order to radiate leaky wave toward only one side of waveguide.

Then the experiments were performed to verify the theoretical prediction, and the frequency responses are qualitatively in agreement with theoretical results. However, the null radiation toward another side could not be obtained by experiments. The reason of this disagreement was proven by theory to be because that the actual size of corrugations was different from designed values, even though

the deviative error was very small. Therefore the better agreement between theory and experiment must be expected if the waveguide corrugations could be made more precisely.

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