Energy dissipations in underwater and aerial organ pipes

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(Received 21 March 1984)

A fundamental theory on the overall $Q$ of resonance in underwater and aerial organ pipes is proposed. It contains the following five mechanisms of energy dissipation or conversion governing the $Q$: wall vibrations, wall-borne hysteresis, wall radiation, wall boundary effects, and open end radiation. Our theory is encouraged by a relatively good agreement between theoretical and experimental $Q$-values. Numerical calculation of the $Q$ due to each dissipation mechanism except the wall-borne hysteresis is carried out on two column mediums, five wall materials, and four pipe geometries. Underwater measurement of wall vibrations is moreover done, and its result makes surer the validity of our theory. An essential difference between underwater and aerial organ pipes clearly manifests itself as a decisive difference in their own governing dissipation mechanisms due to the differences between water and air.

PACS number: 43.20.Hq, 43.20.Rz, 43.30.Bp, 43.30.Jx, 43.30.Yj, 43.40.At, 43.75.Np

1. INTRODUCTION

The main purpose of this paper is to develop the theory on the resonant $Q$-value in underwater organ pipes. In the previous paper,\textsuperscript{1} the author reported that the magnitude of the measured $Q$-value and its dependence on the mode number of resonance were probably governed by the material of pipe wall: (1) The $Q$ of acryl pipes is much smaller than that of aluminium pipes. (2) The $Q$ decreases with ascending mode number in aluminium pipes, while it almost keep constant in acryl pipes. The effects of wall material are thus significant in underwater organ pipes.

The wall of a pipe immersed in water becomes compliant and vibrates in the presence of sound pressure. Therefore, wall vibrations always accompany the resonance of the column confined by the pipe wall when an organ pipe is self-excited. A part of the mechanical energy of these wall vibrations is then dissipated by the hysteresis between stress and strain in the wall interior and by the sound radiation from the outer wall surface. “Wall effects” on energy dissipations in underwater organ pipes thus mean (1) the conversion of input acoustic energy into the mechanical one by the wall vibration associated with the column resonance, (2) the loss of such mechanical energy by the internal hysteresis, and (3) the loss of the mechanical energy by the wall radiation.

The reason why the mechanical energy of wall vibrations should be considered as the energy dissipated is as follows: Strictly speaking, a closed or isolated system does not exist in our environment. An organ pipe as a system always couples with some external systems. For example, an underwater organ pipe is supported on a V-shaped stand, and through a metallic rod this stand couples with the electrical elevator to move the organ pipe into water. Further, this elevator is installed on the bridge built over the water tank. Owing to such successive couplings, the vibratory energy of pipe wall can not be stored in the wall but transmitted to external systems. The con-
version of acoustic energy into vibratory energy thus operates as one of dissipation mechanisms if the efficient transmission of vibrations to external systems is possible. In this paper, for the simplicity, it is assumed that all of the converted vibratory energy is lost from the wall at the established steady-state of self-excitation. Such assumption then defines the theoretical minimum of the overall $Q$.

Another secondary purpose of this paper is to theoretically examine the degree of the above wall effects in aerial organ pipes too. The predominant energy dissipations in aerial organ pipes are caused by the so-called wall boundary effects and sound radiation from open ends. Wall effects are thus only slight in air, but the question whether tone colour in wind instruments is detectably affected by changing their wall material or not may forever continue in controversy because it is a subtle and anxious matter for professional players. Experimental investigation on this problem has been done by many researchers and by various methods, while theoretical consideration on it is not full.

We can thus generally formulate the overall loss factor determining the $Q$ of resonance in organ pipes by including the wall effects. Theoretical expression of each term involved in it is then derived except that of hysteresis loss. Numerical calculation on these loss factors or $Q$s is carried out on two kinds of column medium (i.e., water and air), on five kinds of wall material, and on four kinds of pipe geometry. Theoretical $Q$-value of resonance in underwater organ pipes is compared with experimental one measured in the previous study on four models. Moreover, the author carried out the underwater measurement of wall vibrations on these models to know their amplitudes and to make sure our theory on the loss factor due to wall vibrations.

The estimation of the $Q$-value is one of important problems in acoustics. We must know all of mechanisms of energy dissipation or conversion involved in an acoustical system to accurately determine the $Q$-value. Although our present theory is not complete in this sense, appreciable dissipation mechanisms may be introduced into it. Particularly our consideration on the wall effects will serve to estimate the $Q$-value of an underwater sound projector with a resonant system, for example a Helmholtz resonator or a tubular resonator.

\section{2. GENERAL APPROACH}

Along the above explanation we can generally define the overall loss factor $\eta$ of the resonant column:

$$\eta = \sum I_i / 2\pi I_p ,$$

(1)

$$\sum I_i = I_w + I_{wh} + I_{wr} + I_b + I_r ,$$

(2)

where $\sum I_i$ expresses the total power lost from the column and $I_p$ the acoustic power finally stored in the column. The total acoustic power introduced into the column by the jet corresponds to the sum $I_p + \sum I_i$.

The vibratory power $I_w$ of the pipe wall involved in Eq. (2) participates in the energy dissipation because the wall vibration accompanied with the column resonance is transmitted to external systems. The powers $I_{wh}$ and $I_{wr}$ define the powers lost from the wall due to the internal hysteresis and sound radiation from the wall surface respectively. The power $I_b$ shows the viscous and thermal losses due to the wall boundary effects, and $I_r$ the radiation loss at the open ends.

From Eqs. (1) and (2) we can factorize the overall loss factor as follows:

$$\eta = \eta_w [1 + 2\pi (\eta_{wh} + \eta_{wr})] + \eta_b + \eta_r ,$$

(3)

where

$$\eta_w = I_w / 2\pi I_p ,$$

(4)

$$\eta_{wh} = I_{wh} / 2\pi I_p ,$$

(5)

$$\eta_{wr} = I_{wr} / 2\pi I_p ,$$

(6)

$$\eta_b = I_b / 2\pi I_p ,$$

(7)

$$\eta_r = I_r / 2\pi I_p .$$

(8)

Let us define $\eta_w$, $\eta_{wh}$, $\eta_{wr}$, $\eta_b$, and $\eta_r$ as loss factors due to the wall vibration, wall-borne hysteresis, wall radiation, wall boundary effects, and pipe end radiation respectively.

The $Q$ of column resonance is defined as

$$Q = \eta^{-1} .$$

(9)

Hence, from Eq. (3) we get

$$Q^{-1} = Q_w^{-1} [1 + 2\pi (\eta_{wh}^{-1} + \eta_{wr}^{-1})] + Q_b^{-1} + Q_r^{-1} ,$$

(10)

where $Q_w^{-1}$, $Q_{wh}^{-1}$, $Q_{wr}^{-1}$, $Q_b^{-1}$, and $Q_r^{-1}$ equal $\eta_w$, $\eta_{wh}$, $\eta_{wr}$, $\eta_b$, and $\eta_r$ respectively. Note that loss
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factors or Qs generally depend on the frequency and resonant mode number. In latter sections we quantitatively consider the above each loss factor or Q concerning the Nth \((N=1, 2, 3, \ldots)\) mode resonance of the column. Also, note that Eq. (10) defines the minimum value of the Q because we have assumed that all of the vibratory energy of the wall is dissipated through the transmission mechanisms of vibrations.

3. WALL VIBRATION

Acoustic driving power of the jet induces the vibrations of column medium and pipe wall simultaneously. In the steady state of column resonance, the pressure acting on the wall is identified with the acoustic pressure of the standing wave in the column. Rigorously speaking, we must consider the acoustic pressure acting on the external wall surface, but it is much smaller than the above internal one. In this section we attempt to estimate the loss factor \(\eta_w\) of Eq. (4) due to the induced wall vibration under such a condition.

By the way, the resonant frequency \(f_r\) of radially breathing vibration is approximately given by\(^6\)

\[
f_r = (1/2\pi) \sqrt{E'/\rho_w a_0^5}, \tag{11}\]

where \(\rho_w\) is the density of wall material, \(a_0\) the inner pipe radius, and \(E'\) the effective Young's modulus defined as

\[
E' = E(1 - \mu^2), \tag{12}\]

where \(E\) and \(\mu\) are Young's modulus and Poisson's ratio of wall material respectively.

This so-called "ring frequency" \(f_r\) is of the order of 10 kHz in usual pipes. On the other hand, the resonant frequency of longitudinal column oscillation is much lower than \(f_r\) (cf. Tables 4 and 6 in Sec. 8). Hence, we can treat our low frequency wall vibration due to the resonant pressure in the column by neglecting the inertial term, in other words, according to the theory of elasticity.

The solution on the elastic deformation of the pipe wall caused by the internal uniform pressure \(p\) is given as follows\(^7\):

\[
\tau_{rr} = \frac{a_0^5}{a_1^3 - a_0^3} \left(1 - \frac{a_1^2}{r^2}\right) p, \quad a_0 \leq r \leq a_1, \tag{13}\]

\[
\tau_{\varphi\varphi} = \frac{a_0^5}{a_1^3 - a_0^3} \left(1 + \frac{a_1^2}{r^2}\right) p, \tag{14}\]

\[
\varepsilon_{rr} = \frac{a_0^5}{a_1^3 - a_0^3} \left[1 + \frac{a_1^2}{r^2}\right] \frac{1 - a_1^2}{E} \left[1 - \frac{1 + a_1^2}{r^2} - 2\mu\right] p, \tag{15}\]

\[
\varepsilon_{\varphi\varphi} = \frac{a_0^5}{a_1^3 - a_0^3} \left[1 + \frac{a_1^2}{r^2}\right] \frac{1 - a_1^2}{E} \left[1 - \frac{1 + a_1^2}{r^2} - 2\mu\right] p, \tag{16}\]

where

\(r\): radial argument in the cylindrical coordinates,

\(\varphi\): circumferential argument in the above one,

\(z\): axial argument in the above one,

\(\tau_{rr}\): radial component of stress tensor,

\(\tau_{\varphi\varphi}\): circumferencial component of stress tensor,

\(\varepsilon_{rr}\): axial component of stress tensor,

\(\varepsilon_{\varphi\varphi}\): circumferencial component of strain tensor,

\(a_1\): outer radius of the pipe.

All other components of stress and strain tensors equal zero. Note that in deriving the above solution we have supposed that the pipe does not be deformed longitudinally and keeps its length constant.

The elastic energy per unit length \(W\) of the wall is thus derived from the above solution as follows:

\[
W = \frac{1}{2} \int_{a_0}^{a_1} \left(\tau_{rr} \varepsilon_{rr} + \tau_{\varphi\varphi} \varepsilon_{\varphi\varphi}\right) 2\pi r \, dr
= \pi \left(\frac{a_0^5}{a_0 + a_1}\right) \left(\frac{a_0^5 + a_1^5}{a_0^5 - 2\mu}\right) \left(\frac{1 + \mu}{t_w E}\right) p^2, \tag{18}\]

where \(t_w (= a_1 - a_0)\) is the wall thickness. If the pipe wall is thin or \(a_1 \approx a_0\), Eq. (18) takes the following simple form:

\[
W \approx \pi (a_0^5/t_w E) p^2. \tag{19}\]

Hence, it is effective to rewrite Eq. (18) as

\[
W = \pi \alpha (a_0^5/t_w E) p^2, \tag{20}\]

where the correction factor

\[
\alpha = [2 + (t_w/a_0)]^{-1} [2 + 2(t_w/a_0) + (t_w/a_0)^2 - 2\mu] \times (1 - \mu)^{-1}. \tag{21}\]

Note that we must take time-averaging of Eq. (20) in our vibrational problem where \(p\) is an alternating quantity.

Now let us imagine the Nth mode resonance of the column whose geometrical length is \(l\). For the simplicity, we assume that the acoustic pressure at the mouth \((z = 0)\) and the open end \((z = l)\) are both null.

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We can then express the pressure \( p \) involved in the righthand side of Eq. (20) as
\[
\rho s = \hat{p}_N \sin (k_N z) \cos (\omega_N t)
\] (22)
by adding the suffix \( N \), where \( \hat{p}_N \) is the magnitude of resonant pressure \( p_N \) of \( N \)th mode, \( k_N \) the \( N \)th mode wave number, \( \omega_N \) the \( N \)th mode angular frequency, and \( t \) the time variable.

Hence, from Eqs. (20) and (22) we get the vibratory power \( \Pi_w \) of the wall when the \( N \)th mode resonance of the column takes place:
\[
\Pi_w = \int_0^l \dot{\hat{W}} \, dz
= N \alpha (\pi c/8) (a_N^5 t_s E') \hat{p}_N^5
\] (23)
where the bar on \( W \) denotes the time-averaging, \( f_N \) the \( N \)th mode resonant frequency, and \( c \) the sound velocity in the column. Note that we have applied the resonance condition
\[
k_N l = N \pi
\] (24)
to Eq. (23).

On the other hand, the acoustic power \( \Pi_p \) of \( N \)th mode resonance stored in the column is given by
\[
\Pi_p = f_N \int_0^l (\rho s^2 / \rho c^2) \pi a_N^5 \, dz
= N (\pi a_N^5 / 8) (\hat{p}_N^5 / \rho_0 c)
\] (25)
where Eqs. (22) and (24) are used, and \( \rho_0 \) is the density of column medium.

Therefore, from Eqs. (4), (23), and (25) we get the following expression of loss factor \( \eta_w \):
\[
\eta_w = (\alpha/2\pi) (a_N^5 t_s E') (\rho_0 c^2)
\] (26)
And from the relation of Eq. (9) we get the \( Q \) corresponding to this \( \eta_w \) as
\[
Q_w = (2\pi/\alpha) (t_s E' / a_0) (1/\rho_0 c^2)
\] (27)
Note that \( Q_w \) does not depend on the mode number \( N \) as shown in Eq. (27).

The sound velocity \( c \) in the column is given by
\[
c^2 = c_0^2 [1 + (D/K)]
\] (28)
where \( c_0 \) is the sound velocity in the free space, \( K \) the compressibility of column medium, and \( D \) the distensibility of wall material. Fluid compressibility \( K \) is defined as
\[
K = (\rho_0 c_0^2)^{-1}
\] (29)
Wall distensibility \( D \) is defined as
\[
D = S^{-1} (dS/dp)_{p=0}
= 2 - \frac{1}{E} \left[ \left[ 1 + (t_s/ao) \right]^2 + 1 - 2\mu \right],
\] (30)
where \( S \) is the cross-sectional area of the pipe and regarded as a function of the internal pressure. In deriving Eq. (30), we have used Eq. (17) and the relation that the radial displacement of inner wall surface is given by \( a_0 \pi r (r = a_0) \).

If we apply the quantities \( D \) and \( K \) to Eq. (27), we can roughly rewrite it as follows:
\[
Q_w = 4\pi (K/D)(c_0/c)^2
\] (31)
where we have used the approximation \( D \approx 2a_0/\rho_0 c E' \approx 2a_0/\rho_0 t_s E \) which is valid in the case of \( \mu \approx 0 \) and \( a_1 \approx a_0 \). Hence, we may state that the ratio \( D/K \) governs the \( Q \)-value as well as the sound velocity in the column confined by the elastic pipe wall.

4. WALL-BORNE HYSTERESIS

The theory developed in the previous section bases on the major premise, Hooke's law. That is, we have assumed the proportionality between stress and strain in a solid body. However, Hooke's law is exact only for ideal situations, and some phase difference between them always exists in practical situations. This phase difference causes the energy dissipation into heat or the so-called internal hysteresis loss.

For this hysteresis loss it is convenient to introduce the following complex Young's modulus \( E^* \) instead of real one \( E \):
\[
E^* = E (1 + j\eta_{wb})
\] (32)
From the following explanation we can understand that \( \eta_{wb} \) in Eq. (32) coincides with the hysteresis loss factor defined by Eq. (5). The use of Eq. (32) shows that the area of the ellipse given by the stress-strain curve equals the energy lost as heat during one cycle of the wall vibration.\(^3\) This lost energy is then given as \( 2\pi \eta_{wb} \) times the vibratory energy in one cycle of the vibration. According to our notation based on the power, we can thus define \( \eta_{wb} \) by Eq. (5) because the ratio of energies equals that of powers.

Although a theory on solid-borne loss has been already proposed,\(^5\) the determination of loss factor \( \eta_{wb} \) or complex modulus \( E^* \) is exclusively carried out by various experimental methods. It is because the physical processes that produce the energy dissipation in solids are intricate and not yet fully under-
stood. The experimental values of $\eta_{\text{wb}}$ will then be adopted in our numerical calculation of the overall $Q$-value (cf. Table 2 in Sec. 8). We must further notice that such experimental values for various materials only indicate the order of magnitude.

5. WALL RADIATION

Another appreciable dissipation of the vibratory energy of the pipe wall is caused by sound radiation from the external wall surface. Its radiative properties are described in terms of the radiation efficiency $\sigma$ or the radiation loss factor $\eta_{\text{rad}}$. These are related by

$$\eta_{\text{rad}} = \frac{\rho_0 c_0 \sigma}{\omega N \rho_w t_w} \quad (33)$$

in usual cases.\(^{10,11}\)

However, we must notice that Eq. (33) is only valid when the vibratory energy of radiators takes the form of kinetic energy or when the angular frequency of wall vibration $\omega N \gg$ the angular ring frequency $\omega_r$. But, in our underwater and aerial organ pipes $\omega N \ll \omega_r$ as discussed in Sec. 3 as far as the mode number $N$ is not too large. Under such a stiffness-controlled condition, our radiation loss factor $\eta_{\text{wr}}$ concerning the $N$th mode resonance of the column must be expressed as follows:

$$\eta_{\text{wr}} = \eta_{\text{rad}} (\omega N / \omega_r)^2 = (\rho_0 c_0 / \rho_w t_w \omega_r^2) \sigma \omega_N \quad \text{for} \quad \omega N \ll \omega_r, \quad (34)$$

where Eq. (33) is employed. Equation (34) gives the loss factor defined by Eq. (6).

Next, we must express $\sigma$ explicitly. The radiation efficiency generally indicates how much less power a given actual object radiates than a piston with the same area, the same vibrational velocity, and ideal radiation impedance $\rho_0 c_0$. We can thus define our $\sigma$ as

$$\sigma = \Pi_{\text{wr}} / \Pi_{\text{p}} \quad (35)$$

where $\Pi_{\text{wr}}$ denotes the actual power radiated from the pipe wall of unit length and $\Pi_{\text{p}}$ the power radiated from the above ideal piston.

According to the definition of ideal piston, we can easily express $\Pi_{\text{p}}$ as

$$\Pi_{\text{p}} = (\rho_0 c_0 \bar{v}) (2 \pi a_0), \quad (36)$$

where $\bar{v}$ denotes the root-mean-square normal velocity averaged over the radiating surface. On the other hand, $\Pi_{\text{wr}}$ is given by\(^{12}\)

$$\Pi_{\text{wr}} = \pi^2 a_0^2 \rho_0 \bar{v}^2 \omega_N, \quad (37)$$

Hence, we can express $\sigma$ of Eq. (35) from Eqs. (36) and (37) as follows:

$$\sigma = \left( \frac{\pi}{2} \right) (\omega N a_1 / c_0) \quad \text{cf. (38)}$$

Introducing Eq. (38) into Eq. (34) and using Eqs. (9) and (11), we get

$$Q_{\text{wr}} = \left( \frac{2}{\pi} \right) \left( \frac{\rho_w t_w}{\rho_0 a_0} \right) (\omega N / \omega_r)^2 = \left( \frac{2}{\pi} \rho_0 a_0^2 \right) \frac{1}{\left[ 1 + (a_0 / t_w)^2 \right]} \left( E / \rho_0 c_0^2 \right) \quad (39)$$

provided that $\omega N \ll \omega_r$. Moreover, using Eq. (30), we can approximately rewrite Eq. (39) as

$$Q_{\text{wr}} \approx \left[ \rho_0 a_0^2 / 4 D \omega_N^2 \right]^{-1} \quad (40)$$

provided that $a_1 \approx a_0$ and $\mu \approx 0$. These Eqs. (39) and (40) tell us that we can not ignore the wall radiation in water compared to that in air because the $Q_{\text{wr}}$ is inversely proportional to the medium density.

6. WALL BOUNDARY EFFECTS

In the previous sections we have assumed the particle velocity uniformly distributed over the entire cross-section of the column. But, in the real situations the particle velocity is rapidly reduced in the viscous boundary layer between the column and the wall, and falls null at the wall surface. The viscous loss to the wall then comes about. Moreover, the thermal loss also takes place in the boundary layer because the adiabatic fluctuation associated with the acoustic fluctuation diminishes near the wall of practically constant temperature. Energy dissipation due to these viscous and thermal losses, which is designated as wall boundary effects, overwhelmingly surpasses the dissipation in the column medium itself for our low frequency cases.

Theory\(^{13}\) gives the following expression of the $Q$ due to wall boundary effects:

$$Q_b = a_1 \sqrt{ \omega N \left[ \sqrt{2 \nu (\gamma - 1) / \sqrt{2 \chi} } \right]^{-1}, \quad (41)$$

where $\nu$, $\chi$, and $\gamma$ are the dynamical viscosity, thermal diffusivity, and ratio of specific heats of column medium respectively.

7. END RADIATION

Energy dissipation in organ pipes is further caused by sound radiation from the pipe end openings. To a first approximation, the mouth opening takes the same strength of radiation as the far end opening. Therefore we get the radiated power as\(^{13}\)

$$\Pi_e = (\pi a_0^2) \left( \omega N^2 / 2 \pi \rho_0 c_0^2 \right) \bar{p}^2 \quad \text{cf. (42)}.$$
Hence, from Eqs. (8), (9), and (25) we get the following expression of the $Q$ due to the open end radiation:

$$Q = (\pi/2)(c_e c_a^{1/3})(N/\omega_v^2). \quad (43)$$

### 8. NUMERICAL CALCULATION

The result of numerical calculation on $Q$s derived in previous sections is shown in this section (cf. Tables 7-11). As examples we consider four kinds of pipe geometry given in Table 1. As the wall materials we consider iron, aluminium, acryl, wood, and hard rubber whose physical properties are listed in Table 2. Note that the experimental value of hysteresis loss factor $\eta_{wh}$ is included in Table 2 by regarding it as a property inherent in a solid. Moreover, as the acoustical mediums we consider water and air whose physical properties are listed in Table 3.

Other important quantities governing the $Q$s are ring frequency $f_r$, sound velocity in the column $c$, and resonant frequency of the column $f_N$. These calculated values are listed in Tables 4, 5, and 6 respectively. Note that values of fundamental resonant frequency $f_1$ listed in Table 6 are estimated from the experimental values (cf. Tables 1 and 4 in Ref. 1) by using the proportionality between $f_1$ and $c$ for the same pipe length. Also note that $f_N$ of higher mode is approximately given by $N f_1$.

The result of numerical calculation on $Q$s shows the followings:

1) The most decisive mechanism determining the overall $Q$ in underwater organ pipes is the wall vibration associated with the column resonance (cf. Tables 7 and 11).

2) Wall vibrations in aerial organ pipes are almost completely negligible except those in pipes made of rubber (cf. Table 7).

3) The wall radiation is negligible in usual cases, particularly it has almost no effect on the sound radiation from aerial organ pipes (cf. Table 8 and Sec. 9.2).

4) However, we can not thoroughly neglect the wall-borne hysteresis substantially determines the overall $Q$ in underwater organ pipes but its effect is not so decisive in aerial ones because their wall vibrations

### Table 1 Geometries of model organ pipes.

<table>
<thead>
<tr>
<th>Model</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe length $l$ (mm)</td>
<td>550</td>
<td>1050</td>
<td>628</td>
<td>1443</td>
</tr>
<tr>
<td>Inner radius $a_0$ (mm)</td>
<td>15</td>
<td>27</td>
<td>15</td>
<td>35</td>
</tr>
<tr>
<td>Wall thickness $t_w$ (mm)</td>
<td>6</td>
<td>3</td>
<td>3.5</td>
<td>5</td>
</tr>
</tbody>
</table>

### Table 2 Physical properties of pipe wall materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Iron</th>
<th>Aluminium</th>
<th>Acryl</th>
<th>Wood</th>
<th>Rubber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus $E$ (N/m$^2$)</td>
<td>$200 \times 10^8$</td>
<td>$72 \times 10^9$</td>
<td>$3 \times 10^9$</td>
<td>$2 \times 10^9$</td>
<td>$10^9$</td>
</tr>
<tr>
<td>Poisson's ratio $\mu$</td>
<td>0.30</td>
<td>0.34</td>
<td>0.35</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>Density $\rho_w$ (kg/m$^3$)</td>
<td>$7.8 \times 10^3$</td>
<td>$2.7 \times 10^2$</td>
<td>$1.2 \times 10^3$</td>
<td>$0.7 \times 10^3$</td>
<td>$1.05 \times 10^3$</td>
</tr>
<tr>
<td>Loss factor $\quad \eta_{wh}$</td>
<td>$4 \times 10^{-4}$</td>
<td>$10^{-4}$</td>
<td>$2 \times 10^{-2}$</td>
<td>$10^{-2}$</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 3 Physical properties of mediums.

<table>
<thead>
<tr>
<th>Medium</th>
<th>Water</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $\rho$ (kg/m$^3$)</td>
<td>$1.0 \times 10^3$</td>
<td>1.2</td>
</tr>
<tr>
<td>Sound velocity $c_s$ (m/s)</td>
<td>1468</td>
<td>345</td>
</tr>
<tr>
<td>Dynamical viscosity $\nu$ (m$^2$/s)</td>
<td>$1.1 \times 10^{-4}$</td>
<td>$1.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>Thermal diffusivity $\chi$ (m$^2$/s)</td>
<td>$1.4 \times 10^{-7}$</td>
<td>$1.9 \times 10^{-5}$</td>
</tr>
<tr>
<td>Ratio of specific heats $\gamma$</td>
<td>1.007</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Note: Temperature of water $= 15^\circ C$, temperature of air $= 20^\circ C$.

### Table 4 Ring frequency $f_r$ (evaluated in kHz) defined by Eq. (11).

<table>
<thead>
<tr>
<th>Model</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron</td>
<td>56.3</td>
<td>31.3</td>
<td>56.3</td>
<td>24.1</td>
</tr>
<tr>
<td>Aluminium</td>
<td>58.3</td>
<td>32.4</td>
<td>58.3</td>
<td>25.0</td>
</tr>
<tr>
<td>Acryl</td>
<td>17.9</td>
<td>10.0</td>
<td>17.9</td>
<td>7.7</td>
</tr>
<tr>
<td>Wood</td>
<td>18.3</td>
<td>10.2</td>
<td>18.3</td>
<td>7.8</td>
</tr>
<tr>
<td>Rubber</td>
<td>5.1</td>
<td>2.8</td>
<td>5.1</td>
<td>2.2</td>
</tr>
</tbody>
</table>
### Table 5
Sound velocity in the column $c$ (evaluated in m/s) defined by Eq. (28).

<table>
<thead>
<tr>
<th>Medium</th>
<th>Water</th>
<th>Air</th>
<th>Water</th>
<th>Air</th>
<th>Water</th>
<th>Air</th>
<th>Water</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron</td>
<td>1419</td>
<td>345</td>
<td>1313</td>
<td>345</td>
<td>1388</td>
<td>345</td>
<td>1343</td>
<td>345</td>
</tr>
<tr>
<td>Aluminium</td>
<td>1344</td>
<td>345</td>
<td>1123</td>
<td>345</td>
<td>1271</td>
<td>345</td>
<td>1179</td>
<td>345</td>
</tr>
<tr>
<td>Acryl</td>
<td>619</td>
<td>345</td>
<td>345</td>
<td>345</td>
<td>489</td>
<td>345</td>
<td>389</td>
<td>345</td>
</tr>
<tr>
<td>Wood</td>
<td>521</td>
<td>345</td>
<td>296</td>
<td>345</td>
<td>416</td>
<td>345</td>
<td>333</td>
<td>345</td>
</tr>
<tr>
<td>Rubber</td>
<td>176</td>
<td>344</td>
<td>90</td>
<td>342</td>
<td>132</td>
<td>344</td>
<td>103</td>
<td>343</td>
</tr>
</tbody>
</table>

### Table 6
Fundamental frequency $f_1$ (evaluated in Hz) estimated from the measured values (underlined numerals) and Table 5.

<table>
<thead>
<tr>
<th>Model</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium</td>
<td>Water</td>
<td>Air</td>
<td>Water</td>
<td>Air</td>
</tr>
<tr>
<td>Iron</td>
<td>1101</td>
<td>268</td>
<td>579</td>
<td>152</td>
</tr>
<tr>
<td>Aluminium</td>
<td>1043</td>
<td>268</td>
<td>495</td>
<td>152</td>
</tr>
<tr>
<td>Acryl</td>
<td>480</td>
<td>268</td>
<td>152</td>
<td>152</td>
</tr>
<tr>
<td>Wood</td>
<td>404</td>
<td>268</td>
<td>130</td>
<td>152</td>
</tr>
<tr>
<td>Rubber</td>
<td>136</td>
<td>267</td>
<td>40</td>
<td>151</td>
</tr>
</tbody>
</table>

### Table 7
$Q_w$-value due to the wall vibration defined by Eq. (27).

<table>
<thead>
<tr>
<th>Model</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium</td>
<td>Water</td>
<td>Air</td>
<td>Water</td>
<td>Air</td>
</tr>
<tr>
<td>Iron</td>
<td>195</td>
<td>$275 \times 10^4$</td>
<td>80</td>
<td>$97 \times 10^4$</td>
</tr>
<tr>
<td>Aluminium</td>
<td>79</td>
<td>$99 \times 10^4$</td>
<td>40</td>
<td>$36 \times 10^4$</td>
</tr>
<tr>
<td>Acryl</td>
<td>15</td>
<td>$42 \times 10^4$</td>
<td>18</td>
<td>$15 \times 10^5$</td>
</tr>
<tr>
<td>Wood</td>
<td>14</td>
<td>$28 \times 10^5$</td>
<td>15</td>
<td>$9 \times 10^5$</td>
</tr>
<tr>
<td>Rubber</td>
<td>13</td>
<td>$28 \times 10^5$</td>
<td>18</td>
<td>$10 \times 10^5$</td>
</tr>
</tbody>
</table>

Note: The $Q_w$-value does not theoretically depend on mode number $N$.

### Table 8
$Q_{wr}$-value due to the wall radiation defined by Eq. (39) for the 1st mode.

<table>
<thead>
<tr>
<th>Model</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium</td>
<td>Water</td>
<td>Air</td>
<td>Water</td>
<td>Air</td>
</tr>
<tr>
<td>Iron</td>
<td>3710</td>
<td>$5230 \times 10^4$</td>
<td>1450</td>
<td>$1750 \times 10^4$</td>
</tr>
<tr>
<td>Aluminium</td>
<td>1530</td>
<td>$1940 \times 10^4$</td>
<td>740</td>
<td>$650 \times 10^4$</td>
</tr>
<tr>
<td>Acryl</td>
<td>300</td>
<td>$810 \times 10^4$</td>
<td>330</td>
<td>$270 \times 10^4$</td>
</tr>
<tr>
<td>Wood</td>
<td>260</td>
<td>$500 \times 10^4$</td>
<td>270</td>
<td>$170 \times 10^4$</td>
</tr>
<tr>
<td>Rubber</td>
<td>260</td>
<td>$570 \times 10^4$</td>
<td>330</td>
<td>$190 \times 10^4$</td>
</tr>
</tbody>
</table>

Note: The $Q_{wr}$-value theoretically depends on mode number $N$ as proportional to $1/N^2$. 

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Table 9  $Q_b$-value due to the wall boundary effects defined by Eq. (41) for the 1st mode.

<table>
<thead>
<tr>
<th>Medium</th>
<th>Model I</th>
<th>Model II</th>
<th>Model III</th>
<th>Model IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water</td>
<td>Air</td>
<td>Water</td>
<td>Air</td>
</tr>
<tr>
<td>Iron</td>
<td>840</td>
<td>77</td>
<td>1100</td>
<td>105</td>
</tr>
<tr>
<td>Aluminium</td>
<td>820</td>
<td>77</td>
<td>1010</td>
<td>105</td>
</tr>
<tr>
<td>Acryl</td>
<td>550</td>
<td>77</td>
<td>560</td>
<td>105</td>
</tr>
<tr>
<td>Wood</td>
<td>510</td>
<td>77</td>
<td>520</td>
<td>105</td>
</tr>
<tr>
<td>Rubber</td>
<td>300</td>
<td>77</td>
<td>290</td>
<td>105</td>
</tr>
</tbody>
</table>

Note: The $Q_b$-value theoretically depends on mode number $N$ as proportional to $\sqrt{N}$.

Table 10  $Q_r$-value due to the open end radiation defined by Eq. (43) for the 1st mode.

<table>
<thead>
<tr>
<th>Medium</th>
<th>Model I</th>
<th>Model II</th>
<th>Model III</th>
<th>Model IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water</td>
<td>Air</td>
<td>Water</td>
<td>Air</td>
</tr>
<tr>
<td>Iron</td>
<td>300</td>
<td>290</td>
<td>310</td>
<td>280</td>
</tr>
<tr>
<td>Aluminium</td>
<td>320</td>
<td>290</td>
<td>370</td>
<td>280</td>
</tr>
<tr>
<td>Acryl</td>
<td>700</td>
<td>290</td>
<td>1200</td>
<td>280</td>
</tr>
<tr>
<td>Wood</td>
<td>830</td>
<td>290</td>
<td>1400</td>
<td>280</td>
</tr>
<tr>
<td>Rubber</td>
<td>2480</td>
<td>290</td>
<td>4570</td>
<td>280</td>
</tr>
</tbody>
</table>

Note: The $Q_r$-value theoretically depends on mode number $N$ as proportional to $1/N$.

Table 11  Overall $Q$-value of $N$th mode resonance defined by Eq. (10).

1st mode

<table>
<thead>
<tr>
<th>Medium</th>
<th>Model I</th>
<th>Model II</th>
<th>Model III</th>
<th>Model IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water</td>
<td>Air</td>
<td>Water</td>
<td>Air</td>
</tr>
<tr>
<td>Iron</td>
<td>104</td>
<td>61</td>
<td>60</td>
<td>76</td>
</tr>
<tr>
<td>Aluminium</td>
<td>58</td>
<td>61</td>
<td>35</td>
<td>76</td>
</tr>
<tr>
<td>Acryl</td>
<td>13</td>
<td>61</td>
<td>15</td>
<td>76</td>
</tr>
<tr>
<td>Wood</td>
<td>13</td>
<td>61</td>
<td>14</td>
<td>76</td>
</tr>
<tr>
<td>Rubber</td>
<td>2</td>
<td>53</td>
<td>2</td>
<td>50</td>
</tr>
</tbody>
</table>

2nd mode

<table>
<thead>
<tr>
<th>Medium</th>
<th>Model I</th>
<th>Model II</th>
<th>Model III</th>
<th>Model IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water</td>
<td>Air</td>
<td>Water</td>
<td>Air</td>
</tr>
<tr>
<td>Iron</td>
<td>79</td>
<td>63</td>
<td>51</td>
<td>72</td>
</tr>
<tr>
<td>Aluminium</td>
<td>50</td>
<td>63</td>
<td>51</td>
<td>72</td>
</tr>
<tr>
<td>Acryl</td>
<td>12</td>
<td>63</td>
<td>14</td>
<td>72</td>
</tr>
<tr>
<td>Wood</td>
<td>12</td>
<td>63</td>
<td>13</td>
<td>72</td>
</tr>
<tr>
<td>Rubber</td>
<td>2</td>
<td>54</td>
<td>2</td>
<td>48</td>
</tr>
</tbody>
</table>

(to be continued)
are quite weak (cf. Tables 2 and 11).

6) The wall boundary effects are decisive for determining the overall $Q$ of low mode number resonances in aerial organ pipes, while insignificant in underwater ones (cf. Tables 9 and 11).

7) The pipe end radiation is decisive for determining the overall $Q$ of high mode number resonances in aerial organ pipes, while is not so decisive in underwater ones (cf. Tables 10 and 11).

8) The overall $Q$ in underwater organ pipes strongly depends on their wall material, while that in aerial ones does not depend on it except rubber (cf. Table 11).

9) The overall $Q$ in underwater organ pipes gradually decreases with ascending mode number.

9. DISCUSSION

9.1 Comparison of Theoretical and Experimental Overall $Q$s in Underwater Organ Pipes

Table 12 shows the result of this comparison. Experimental $Q$-values are given in Table 4 in Ref. 1). Experimental models AL-I, AL-II, AC-I, and AC-II correspond to aluminium pipes with the sizes of I and II, and acryl pipes with the sizes of III and IV in this paper respectively.

We obtain a considerably good agreement between theory and experiment on acryl pipes. But, the experimental $Q$-values in aluminium pipes are about two times the theoretical ones. This discrepancy may be attributed to our assumption that the energy of wall vibrations is completely transmitted to the external systems. Remember that our theory defines the theoretical minimum of the overall $Q$. The degree of discrepancy probably corresponds to the degree of the vibratory energy still remaining in the wall.

9.2 Wall Radiation vs. End Radiation

Acoustic output power of organ pipes is given by the radiations from pipe ends and wall surface. We can compare these strengths quantitatively from Eqs. (4), (6), (8), (29), (31), and (43):

$$
\frac{\Pi_{wz}}{\Pi_z} = \frac{2\pi Q_z/Q_{wz}}{\xi z} = \frac{2\pi (c_0/c)Q_{wz}}{E z}.
$$

Moreover, using Eqs. (29), (31), and the approximate relation $D \approx 2a_0/z$, we can rewrite Eq. (44) as

$$
\frac{\Pi_{wz}}{\Pi_z} = \frac{2\pi (c_0/c)Q_{wz}}{E z}.
$$

Table 13 shows the calculated result of level difference $10 \log \left( \frac{\Pi_{wz}}{\Pi_z} \right)$ defined by Eq. (45) for the first
mode, where Tables 3, 5, and 7, and Eq. (21) are employed. Table 13 tells us the following:

1) In underwater organ pipes, we cannot ignore the sound radiation from the wall surface at all. Particularly, it surpasses the radiation from the end openings when the pipes are made of considerably distensible materials such as acryl, wood, and rubber.

2) In aerial organ pipes, we can practically ignore the radiation from the wall surface, though more distensible wall material appreciably makes the level difference smaller. However, it may be rather hard to tell the difference in tone colour by hearing sounds of an individual woodwind instrument made of different wall material.15)

10. UNDERWATER MEASUREMENT OF WALL VIBRATIONS

10.1 Experimental Procedure

In order to know the actual amplitude of wall vibrations in underwater organ pipes, the author measured their acceleration. And, to experimentally estimate the $Q_w$-value due to these wall vibrations from the acceleration measurement, the author furthermore measured the radiated sound pressure at an axial distance of 1 m from the pipe end simultaneously.

An accelerometer was secured by means of an accessory stud screwed into the hole tapped in the accelerometer base and a hole tapped on the external wall surface. This hole on the wall was positioned near the acoustical centre of the column corresponding to the pressure loop of first mode resonance. As our accelerometer a miniature piezoelectric one (Brüel & Kjaer type 4344; 7.0 mm in diameter, 10.2 mm in height, and 2 g in weight) was selected to exclude the effect of inertia as much as possible. But, as shown below, such a selection made the voltage sensitivity lower in low frequencies because of small capacity of the accelerometer and finite input resistance of the amplifier.

After mounting the accelerometer on the wall, a sufficient water-proofing of it and the cable was done by smearing the silicone rubber. An organ pipe was then immersed in water and driven by the water jet. The outputs of the accelerometer and external hydrophone were amplified and filtered, then the frequency and amplitude of the resultant signals were measured by a counter and an oscilloscope.

The calibration of the accelerometer voltage sensitivity (its nominal value $V_0 = 2.37 \text{ mV/g}$, where the symbol g denotes the gravitational acceleration or $9.81 \text{ m/s}^2$) including the cable was carried out by...
using a vibration exciter. The calibration was limited below 800 Hz because the exciter had many mechanical resonances above 1.5 kHz. We got a linear calibration curve between 50 and 500 Hz, whose slope was about \(1.6 \times 10^{-3}\). The cut-off frequency \(f_c\) was then estimated as \(2.37/(1.6 \times 10^{-3})=1.48\) kHz. The following calculated calibration curve was thus adopted for the frequency range of our interest:

\[
V = V_0[(f/f_c)/\sqrt{1+(f/f_c)^2}],
\]

where \(V\) and \(f\) denote the calibrated voltage sensitivity and the frequency respectively.

10.2 Result

Figure 1 shows examples of non-filtered wave forms of the above radial acceleration and radiated pressure signals. The acceleration signal contains strong components of higher harmonics. As inferred from Fig. 1 the wall vibration faithfully follows the temporal variation of acoustic pressure in the resonant column.

The waveforms on the fundamental component are then obtained by band-pass filtering the raw waveforms such as Fig. 1. The magnitude of acceleration measured by reading out the peak voltage value is converted into that of displacement \(\xi_1\) by dividing by \((2\pi f_1)^2\). The magnitude of radiated pressure is represented by the logarithmic level in “dB re 1 \(\mu\)Pa” and denoted as \(L_{out}\). These results on fundamental component are listed in Table 14.

Sound level of internal pressure \(L_{in}\) near the column centre corresponding to \(L_{out}\) can be estimated from the relation between them obtained by the previous experiment [cf. Table 2 in Ref. 1]. The magnitude of internal pressure \(p_1\) of first mode resonance is thus given by the relation \(L_{in}\)

\[=20 \log(\hat{p}_1/\sqrt{2})+120.\] These values are also listed in Table 14.

The experimental \(Q_w\)-value is obtained as follows: From Eq. (17) we get the radial displacement of wall vibration

\[\hat{\xi}_1 = a_1 e_{1v} = \alpha'(a_1^2 t_{1E})\hat{p}_1,\]

where \(\alpha'=1+(t_{1e}/a_{0})/[1+(t_{1e}/2a_{0})].\) Using Eq. (47), we can rewrite Eq. (27) as

\[Q_w^{(E)} = 2\pi(\alpha'/a)\hat{p}_1/p_{0w}c_2(a_1/\hat{\xi}_1).\]

The \(Q_w\) determined by Eq. (49) involving the experimental quantities \(\hat{p}_1\) and \(\hat{\xi}_1\) is denoted by \(Q_w^{(E)}\) to

<table>
<thead>
<tr>
<th>Model (f_1) (Hz)</th>
<th>(\xi_1) ((\mu)m)</th>
<th>(L_{out}) (dB)</th>
<th>(L_{in}) (dB)</th>
<th>(\hat{p}_1) (kPa)</th>
<th>(Q_w^{(E)})</th>
<th>(Q_w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1) (Hz)</td>
<td>(\xi_1) ((\mu)m)</td>
<td>(L_{out}) (dB)</td>
<td>(L_{in}) (dB)</td>
<td>(\hat{p}_1) (kPa)</td>
<td>(Q_w^{(E)})</td>
<td>(Q_w)</td>
</tr>
<tr>
<td>952</td>
<td>0.015</td>
<td>146</td>
<td>206</td>
<td>28</td>
<td>80</td>
<td>79</td>
</tr>
<tr>
<td>960</td>
<td>0.020</td>
<td>150</td>
<td>210</td>
<td>45</td>
<td>95</td>
<td>40</td>
</tr>
<tr>
<td>970</td>
<td>0.022</td>
<td>152</td>
<td>212</td>
<td>56</td>
<td>109</td>
<td>17</td>
</tr>
<tr>
<td>442</td>
<td>0.094</td>
<td>144</td>
<td>203</td>
<td>11</td>
<td>34</td>
<td>18</td>
</tr>
<tr>
<td>458</td>
<td>0.13</td>
<td>148</td>
<td>208</td>
<td>36</td>
<td>33</td>
<td>17</td>
</tr>
<tr>
<td>480</td>
<td>0.15</td>
<td>151</td>
<td>211</td>
<td>50</td>
<td>43</td>
<td>18</td>
</tr>
<tr>
<td>420</td>
<td>0.72</td>
<td>144</td>
<td>210</td>
<td>45</td>
<td>34</td>
<td>40</td>
</tr>
<tr>
<td>532</td>
<td>1.3</td>
<td>148</td>
<td>214</td>
<td>71</td>
<td>33</td>
<td>17</td>
</tr>
<tr>
<td>440</td>
<td>2.6</td>
<td>151</td>
<td>218</td>
<td>110</td>
<td>43</td>
<td>18</td>
</tr>
<tr>
<td>158</td>
<td>0.50</td>
<td>132</td>
<td>193</td>
<td>6.3</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>163</td>
<td>0.81</td>
<td>136</td>
<td>197</td>
<td>10</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>170</td>
<td>0.95</td>
<td>138</td>
<td>199</td>
<td>13</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
distinguish from the theoretical one of Eq. (27).

Theoretical and experimental $Q_w$-values seem to substantially agree each other as shown in Table 14. For more precise evaluation of $Q_w(\text{E})$ we need a small accelerometer of much higher sensitivity. The above agreement is not full but enough to confirm our theory developed in Sec. 3.

11. CONCLUSION

Fundamental processes of energy dissipation in underwater and aerial organ pipes have been theoretically formulated. Acoustic driving energy of the jet is lost from the resonant column by the following processes: (1) pipe wall vibrations associated with the column resonance, (2) wall-borne hysteresis, (3) sound radiation from the wall surface, (4) wall boundary effects, and (5) sound radiation from the end openings. The overall $Q$ of resonance is defined as the ratio of $2\pi$ times the acoustic power finally stored in the column to the sum total of powers lost by the above processes.

We have further examined which processes are predominant by factorizing the overall $Q$ into the individual components corresponding to the above dissipation processes. As the result of numerical calculation on them, we can conclude the following: (1) The wall vibration is the most predominant in underwater organ pipes, while it is thoroughly insignificant in aerial ones. (2) Wall boundary effects and open end radiation are predominant in aerial organ pipes, while they are almost insignificant in underwater ones. (3) The wall-borne hysteresis substantially determines the overall $Q$ in underwater organ pipes provided that they are made of rubber, while it only makes the overall $Q$ a little lower in such aerial ones because of the smallness of their wall vibrations.

Additionally, we can state that (4) the wall radiation becomes comparable to the open end radiation in underwater organ pipes made of considerably distensible materials such as acryl, wood, and rubber. On the other hand, the wall radiation is not appreciable in aerial ones almost independent of their wall material. Hence, it can scarcely contribute to tone colour in wind instruments.

Experimental results on underwater organ pipes confirm our present theory. First, we get a relatively good agreement between experimental and theoretical $Q$-values. Second, we also get a better agreement between experimental and theoretical $Q$-values due to wall vibrations faithfully following the column resonance.

Summing up, our study on the mechanisms of energy dissipation brings out in sharp contrast the difference between underwater and aerial organ pipes.

ACKNOWLEDGEMENTS

The author would like to thank Prof. M. Okujima and Asst. Prof. S. Ohtuki of Research Laboratory of Precision Machinery and Electronics in Tokyo Institute of Technology for their sincere encouragement and helpful discussions.

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10) Ref. 8), pp. 461-463.
14) Ref. 8), pp. 208, 214, 216.