Analysis of active control of noise in duct by means of impedance

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In this paper, the principle of the active control of noise in duct is analyzed by means of impedance. It is shown that the principle is equivalent to making a short circuit on a transmission line by the equivalent impedance of the active system. From the result of the analysis, general equations and their equivalent circuits of multi-microphone and multi-loudspeaker system are deduced, and also general requirements for control without acoustical feedback which makes the system unstable, are made clear. For the practical use, some suitable combinations of microphones and loudspeakers which control the noise without acoustical feedback are shown. In addition, an electrical method by which the acoustical feedback is canceled is proposed.

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1. INTRODUCTION

Since Paul Leug¹⁾ proposed the concept of active control of sound wave using the antiphase signal in 1934, a number of reports based on his idea have been published in this field. Recently the researches in the active noise control have become more and more extensive, but almost no one has succeeded in perfect control of noise over wide frequency range based on this principle and few method have been put to practical use.

It is difficult to analyze the active control system strictly only on the principle of noise reduction by the wave with an antiphase of the original one. Then we should reexamine the basic principle of active control.

In this paper, the principle of active control of noise in duct is analyzed by means of impedance. And suitable configurations of system and the requirements for control without acoustic feedback are shown.

2. PRINCIPLE AND METHODS OF ACTIVE CONTROL OF NOISE IN DUCT

2.1 Principle

When noise in duct travels down to the outlet of the duct where noise is radiated into the room, the radiated noise poses a problem. In order to suppress a travelling sound in duct, an electro-acoustic transducer (loudspeaker) is mounted on the duct wall as shown in Fig. 1. Assume that the walls of the duct are frictionless, the sound field in the duct is one-dimensional and the effective diameter of the transducer diaphragm is small compared with the wave length.

The notation used in the figure is as follows: Z is the characteristic impedance of the duct, R_r is the radiation resistance of the outlet of the duct, z_s is the acoustic impedance of the transducer, p_s is the driving force of the transducer which is represented by sound pressure, and l_a is the equivalent length between the transducer and the outlet of the duct which includes the radiation reactance of the duct. z_s is the mechanical impedance of the transducer divided by

Fig. 1 System configuration.

Fig. 2 Equivalent circuit.

S_s^2 and p_s is the driving force divided by S_s , which is the effective area of the transducer diaphragm. The impedance looking back into the outlet of the duct from the plane "a" where the transducer is mounted, is

$$z_a = R_a + jX_a = Z \frac{R_r \cos kl_a + jZ \sin kl_a}{Z \cos kl_a + jR_r \sin kl_a} \quad (1)$$

where k is the wave number.

Let p_a be the sound pressure at "a," and let U_a and U_s be the volume velocity flowing into "a" from upstream duct and that flowing into the transducer respectively. The equivalent circuit is shown in Fig. 2 where z_{seq} is the equivalent impedance, that is

$$z_{seq} = \frac{p_a}{U_s} \quad (2)$$

The reflection coefficient of sound pressure at "a" is given as follows:

$$\Gamma = \frac{\frac{z_{seq}z_a}{z_{seq} + z_a} - Z}{\frac{z_{seq}z_a}{z_{seq} + z_a} + Z} = \frac{z_{seq}(z_a - Z) - z_a Z}{z_{seq}(z_a + Z) + z_a Z} \quad (3)$$

When the pressure of the incident progressive sound wave at the plane "a" is p_{in} and the reflecting one is p_r , p_a is represented by the following equation.

$$p_a = p_{in} + p_r = \frac{2z_{seq}z_a}{z_{seq}(z_a + Z) + z_a Z} p_{in} \quad (4)$$

And the travelling sound power transmitted downward is

$$W = \left| \frac{p_a}{z_a} \right|^2 R_a \quad (5)$$

When $p_s = 0$, $z_{seq} = z_s$ and the pressure at "a" is

$$p_{a0} = \frac{2z_s z_a}{z_s(z_a + Z) + z_a Z} p_{in} \quad (6)$$

and

$$p_a = p_{a0} + p_{as} \quad (7)$$

where p_{as} is the sound pressure radiated from the transducer.

When the transducer is not mounted on the wall, z_s is infinite and the pressure at "a" is given as

$$p_{a00} = \frac{2z_a}{z_a + Z} p_{in} \quad (8)$$

and the sound power transmitted downstream is

$$W_{00} = \left| \frac{p_{a00}}{z_a} \right|^2 R_a \quad (9)$$

The insertion loss due to the second source, the transducer system, is described as

$$IL = 10 \log \frac{W_{00}}{W} = 20 \log \left| \frac{p_{a00}}{p_a} \right| \\ = 20 \log \left| 1 + \frac{z_a Z}{z_{seq}(z_a + Z)} \right| \text{ dB} \quad (10)$$

If IL is positive, the sound power travelling downstream, namely the power radiating from the outlet of the duct, could be reduced. Especially, if $p_a = 0$, IL could be infinite. The condition: $p_a = 0$, means that $p_{as} = -p_{a0}$. As shown by P. Leug, the principle of the active control of noise is to cancel the noise by the addition of the antiphase noise using the second source. It must be noticed, however, that the sound pressure to be canceled is not p_{a00} which is one before the transducer is mounted but p_{a0} which is one after the transducer without driving force is mounted.

Viewed from a different standpoint, making p_a equal zero or IL equal infinite, is making the equivalent impedance z_{seq} equal zero. In other words, the principle of the active control of noise in duct is to make short-circuit on the transmission line. If z_{seq} can not be perfectly equal to zero, the characteristics of IL are influenced by the characteristics of z_{seq} and z_a .

When z_{seq} is not the impedance of an active system but that of a resonator, it becomes the resonator-type muffler. The resonator-type muffler makes a short-circuit in the duct at the frequency of resonance.

2.2 Methods of Active Control

There are three methods by which the input signal to the transducer is picked out,²⁾ that is,

- (1) independent electric source.
- (2) sound pressure in the field.

- (3) vibration of the moving part of the transducer (motional feedback).

On the first method, the electric signal for control can not follow the change of the boundary condition. T. Kosaka and S. Yamada³⁾ controlled the fundamental frequency of the second source on the basis of the mechanical vibration of the noise source and also controlled the amplitude and the phase of harmonics on the basis of the residual noise detected by a microphone. The second method is the most popular one. In the third method, any microphones are not required, so that the system can be used in severe condition such as in high temperature, or in ducts with a high speed air current, or with a dusty current.⁴⁾

In case where a microphone is used, the acoustic feedback from the loudspeaker to the microphone becomes a serious problem. To reduce the feedback, various systems with multiple microphones or multiple loudspeakers have been proposed.⁵⁻¹²⁾ But in these systems, the impedances z_s and z_a were neglected. As mentioned above, z_s and z_a can not be neglected compared with Z . To control the noise over the wide frequency range, it is necessary to take z_s and z_a into account.

3. MULTI-MICROPHONE AND MULTI-LOUDSPEAKER SYSTEM

3.1 Configuration of System and Assumptions

Assumptions in the analysis are as follows:

- The duct is frictionless and the sound field in the duct is one dimensional.
- All microphones have the same characteristics and they do not disturb the sound field because of their small size and high impedance.
- All loudspeakers have the same characteristics and the effective diameter of their diaphragm is sufficiently small compared with the wavelength.

The configuration of the system is shown in Fig. 3 where the characteristic impedance of the duct is Z and the acoustic impedance of the moving part of

Fig. 3 Configuration of system with m microphones and n loudspeakers.

the loudspeaker which includes the motional impedance is z_s . m microphones are mounted in the duct where Δl_m is the spacing of each microphone. Let the sound pressure at the i -th microphone be p_{Mi} ($i=1, 2, \dots, m$). The gain and the phase of the output of each microphone are controlled by a network with transfer function G_{Mi} where $G_{M1}=1$, and then they are added up. As shown in Fig. 3, this output is denoted by the sound pressure p_M which is represented in terms of p_{M1} and it is supplied to each loudspeaker through the power amplifier G_i ($i=1, 2, \dots, n$). n loudspeakers are also mounted on the duct wall where the spacing is Δl . The distance between the first microphone and the first loudspeaker is l . Let z_a be the impedance looking back into the outlet of the duct from the n -th loudspeaker. Sound pressures and volume velocities are defined as follows:

- p_i : the sound pressure at the i -th loudspeaker.
- p_{si} : the driving forth through G_i which is represented in terms of sound pressure.
- U_i : the volume velocity flowing into the section of the i -th loudspeaker, this is divided to U_{si} and volume velocity going down to the outlet.
- U_{si} : the volume velocity flowing into the i -th loudspeaker.
- p_{in} : the pressure of incident progressive sound wave flowing in the section of the first loudspeaker.
- p_r : the pressure of the progressive sound wave reflected at the section of the first loudspeaker.

$$p_i = p_{in} + p_r, \quad ZU_i = p_{in} - p_r \quad (11)$$

3.2 Loudspeaker System

Figure 4 shows the section of the duct on which the i -th and the $(i+1)$ -th loudspeakers are mounted with Δl spacing and its equivalent circuit.

Fig. 4 Equivalent circuit of i -th section, (a) configuration of i -th section (b) equivalent circuit.

$$p_{i+1} = \cos x \cdot p_i - jZ \sin x \cdot (U_i - U_{si}) \quad (12)$$

$$U_{i+1} = -j \frac{1}{Z} \sin x \cdot p_i + \cos x \cdot (U_i - U_{si}) \quad (13)$$

$$p_i = p_{si} + z_s U_{si} \quad (14)$$

$$p_{si} = G_i p_M \quad (15)$$

where

$$x = k\Delta l \quad (16)$$

From these equations

$$p_{i+1} = Ap_i - BU_i - \frac{1}{z_s} BG_i p_M \quad (17)$$

$$U_{i+1} = -Cp_i + DU_i + \frac{1}{z_s} DG_i p_M \quad (18)$$

where

$$\left. \begin{aligned} A &= \cos x + j \frac{Z}{z_s} \sin x, & B &= jZ \sin x \\ C &= j \frac{1}{Z} \left(\sin x - j \frac{Z}{z_s} \cos x \right), & D &= \cos x \end{aligned} \right\} \quad (19)$$

For p_n

$$p_n = p_{sn} + z_s U_{sn} \quad (20)$$

$$p_n = z_a (U_n - U_{sn}) \quad (21)$$

p_n can be described as following two equations, using p_{in} and p_r

$$p_n = \left(A_n - \frac{B_n}{Z} \right) p_{in} + \left(A_n + \frac{B_n}{Z} \right) p_r - G_{Bn} p_M \quad (22)$$

and

$$\left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) p_n = (D_n - ZC_n) p_{in} - (D_n + ZC_n) p_r + G_{Dn} p_M \quad (23)$$

where

$$\left. \begin{aligned} A_i &= A \cdot A_{i-1} + B \cdot C_{i-1} \\ B_i &= A \cdot B_{i-1} + B \cdot D_{i-1} \\ C_i &= C \cdot A_{i-1} + D \cdot C_{i-1} \\ D_i &= C \cdot B_{i-1} + D \cdot D_{i-1} \end{aligned} \right\} \quad (24)$$

$$A_i D_i - B_i C_i = 1 \quad (25)$$

$$\left. \begin{aligned} A_1 &= D_1 = 1, & B_1 &= C_1 = 0 \\ A_2 &= A = \cos x + j \frac{Z}{z_s} \sin x, & B_2 &= B = jZ \sin x \\ C_2 &= C = j \frac{1}{Z} \left(\sin x - j \frac{Z}{z_s} \cos x \right), & D_2 &= D = \cos x \end{aligned} \right\} \quad (26)$$

$$\left. \begin{aligned} G_{Bn} &= \frac{1}{z_s} \sum_{i=1}^n B_{n+1-i} G_i \\ G_{Dn} &= \frac{Z}{z_s} \sum_{i=1}^n D_{n+1-i} G_i \end{aligned} \right\} \quad (27)$$

3.3 Microphone System

The sound pressure at each microphone is given as

$$p_{Mi} = e^{jkl} e^{-j(i-1)y} p_{in} + e^{-jkl} e^{j(i-1)y} p_r \quad (28)$$

where

$$y = k\Delta l_M \quad (29)$$

hence the output of the microphone system which is represented in terms of sound pressure is given as

$$p_M = e^{jkl} G_{im} p_{in} + e^{-jkl} G_{rm} p_r \quad (30)$$

where

$$\left. \begin{aligned} G_{im} &= \sum_{i=1}^m e^{-j(i-1)y} G_{Mi} \\ G_{rm} &= \sum_{i=1}^m e^{j(i-1)y} G_{Mi} \end{aligned} \right\} \quad (31)$$

3.4 General Equation of Total System and Its Equivalent Circuit

From Eq. (30), Eqs. (22) and (23) yield to the equations as follows:

$$\begin{aligned} p_n &= \left\{ \left(A_n - \frac{B_n}{Z} \right) - e^{jkl} G_{im} G_{Bn} \right\} p_{in} \\ &\quad + \left\{ \left(A_n + \frac{B_n}{Z} \right) - e^{-jkl} G_{rm} G_{Bn} \right\} p_r \\ \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) p_n &= \{ (D_n - ZC_n) + e^{jkl} G_{im} G_{Dn} \} p_{in} \\ &\quad - \{ (D_n + ZC_n) - e^{-jkl} G_{rm} G_{Dn} \} p_r \end{aligned} \quad (32)$$

(33)

hence

$$\begin{aligned} \frac{\Delta p_n}{p_{in}} &= 2 + \left\{ \left(A_n + \frac{B_n}{Z} \right) e^{jkl} G_{im} \right. \\ &\quad \left. - \left(A_n - \frac{B_n}{Z} \right) e^{-jkl} G_{rm} \right\} G_{Dn} \\ &\quad - \{ (D_n + ZC_n) + e^{jkl} G_{im} \\ &\quad + (D_n - ZC_n) e^{-jkl} G_{rm} \} G_{Bn} \end{aligned} \quad (34)$$

$$\begin{aligned} \frac{\Delta p_r}{p_{in}} &= \left\{ (D_n - ZC_n) - \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \left(A_n - \frac{B_n}{Z} \right) \right\} \\ &\quad + e^{jkl} G_{im} \left\{ G_{Dn} + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) G_{Bn} \right\} \end{aligned} \quad (35)$$

Fig. 5 Block diagram for total system.

$$\Delta = \left\{ (D_n + ZC_n) + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \left(A_n + \frac{B_n}{Z} \right) \right\} - e^{jkl} G_{rm} \left\{ G_{Dn} + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) G_{Bn} \right\} \quad (36)$$

The sound pressures p_n , p_r and p_m are represented by p_{n0} , p_{r0} and p_{m0} which are the pressure when $p_{si} = 0$ ($G_{Bi} = G_{Di} = 0$), and p_{ns} , p_{rs} and p_{ms} which are produced by p_{si} .

$$p_n = p_{n0} + p_{ns}, \quad p_r = p_{r0} + p_{rs}, \quad p_m = p_{m0} + p_{ms} \quad (37)$$

The block diagram which shows the operation can be represented in Fig. 5. Where

$$\left. \begin{aligned} H_1 &= \frac{p_{n0}}{p_{i0}}, \quad H_M = \frac{p_{m0}}{p_{i0}}, \quad \mu = \frac{p_{ns}}{p_m}, \quad \beta = \frac{p_{ms}}{p_{ns}} \\ H_2 &= H_M \frac{\mu}{1 - \mu\beta} = \frac{p_{ns}}{p_{i0}} \end{aligned} \right\} \quad (38)$$

3.5 Requirement for Control Attended with no Feedback

Substitution of Eq. (37) in Eq. (35) results in the equation as follows:

$$\begin{aligned} \frac{p_{rn}}{p_{in}} &= \frac{G_{Dn} + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) G_{Bn}}{\Delta \left\{ (D_n + ZC_n) + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \left(A_n + \frac{B_n}{Z} \right) \right\}} \\ &\times \left[\left\{ (D_n + ZC_n) + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \cdot \left(A_n + \frac{B_n}{Z} \right) \right\} e^{jkl} G_{im} \right. \\ &+ \left. \left\{ (D_n - ZC_n) - \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \cdot \left(A_n - \frac{B_n}{Z} \right) \right\} e^{-jkl} G_{rm} \right] \end{aligned} \quad (39)$$

Substitution of Eq. (37) in Eq. (30) results in the equation as follows:

$$p_{ms} = e^{-jkl} G_{rm} \cdot p_{rs} \quad (40)$$

The condition in which there is no feedback ($\beta = 0$) is given by the following equation.

$$G_{rm} = 0 \quad (41)$$

or

$$G_{Dn} + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) G_{Bn} = 0 \quad (42)$$

When $n=1$, the condition of complete cancelation ($p_1=0$) is given as follows:

$$\frac{Z}{z_s} G_1 = - \frac{2}{e^{jkl} G_{im} - e^{-jkl} G_{rm}} \quad (43)$$

When $n \geq 2$, the condition of complete cancelation corresponding to Eq. (41) can be represented by the equation as follows:

$$\left(A_n + \frac{B_n}{Z} \right) G_{Dn} - (D_n + ZC_n) G_{Bn} = - \frac{2e^{-jkl}}{G_{im}} \quad (44)$$

and the equation corresponding to Eq. (42), is represented by the following equation,

$$\begin{aligned} \frac{2}{G_{Bn}} &= \left\{ (D_n + ZC_n) + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \cdot \left(A_n + \frac{B_n}{Z} \right) \right\} e^{jkl} G_{im} \\ &+ \left\{ (D_n - ZC_n) - \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \cdot \left(A_n - \frac{B_n}{Z} \right) \right\} e^{-jkl} G_{rm} \end{aligned} \quad (45)$$

3.6 Input Acoustic Impedance of the Loudspeaker System

Substituting Eq. (11) and Eq. (35) in $z_{in} = p_1/U_1$ that is the input acoustic impedance of the loudspeaker system; z_{in} results in the equation as follows:

$$\begin{aligned} z_{in} &= Z \left[2 \left\{ D_n + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \frac{B_n}{Z} \right\} \right. \\ &+ (e^{jkl} G_{im} + e^{-jkl} G_{rm}) \cdot \left\{ G_{Dn} + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) G_{Bn} \right\} \\ &\left. / \left[2 \left\{ ZC_n + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) A_n \right\} \right. \right. \\ &+ (e^{jkl} G_{im} - e^{-jkl} G_{rm}) \cdot \left. \left. \left\{ G_{Dn} + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) G_{Bn} \right\} \right] \right] \end{aligned} \quad (46)$$

Especially when $n=1$ and $P_1=0$,

$$z_{in} = 0 \quad (47)$$

In this case, the radiating sound power from the original noise source becomes only reactive one.

For $n \geq 2$, $z_{in} \neq 0$ in general and

$$z_{in} = R_{in} + jX_{in} \quad (48)$$

The power radiating from the original noise source is represented by the following equation.

$$W = |U_1|^2 R_{in} \quad (49)$$

When $p_n = 0$, the power W is consumed by each loudspeaker except the n -th one. Namely

$$W = \sum_{i=1}^{n-1} |U_{si}|^2 R_s \quad (50)$$

where

$$z_s = R_s + jX_s \quad (51)$$

Especially, when the condition Eq. (42) is valid

$$z_{in} = Z \frac{D_n + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \frac{B_n}{Z}}{ZC_n + \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) A_n} \quad (52)$$

In this case, z_{in} is equal to the input impedance at $p_{st} = 0$.

4. ARRANGEMENT OF MICROPHONE AND LOUDSPEAKER

There are two conditions to cancel the sound in the duct, that is $\beta = 0$ and $p_n = 0$, so that only two variables are required. As $G_{m1} = 1$, the numbers m and n must satisfy the equation; $m + n = 3$, that is $m = 1$ and $n = 2$ or $m = 2$ and $n = 1$. The arrangement of microphones and loudspeakers are shown in Fig. 6 in which those of $m = n = 1$ are also shown.

4.1 $m = n = 1$

In this case, the equation of loudspeaker system; Eq. (23) is reduced to the equation as follows:

$$\left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) p_1 = p_{in} - p_r + \frac{Z}{z_s} G_1 p_M \quad (53)$$

- a) In case where a microphone is above a loudspeaker, as shown in Fig. 6 (a)

p_M is given as

$$p_M = e^{jkl} p_{in} + e^{-jkl} p_r \quad (54)$$

hence

$$\frac{p_1}{p_{in}} = \frac{2 \left(1 + j \frac{Z}{z_s} \sin kl \cdot G_1 \right)}{\left(1 + \frac{Z}{z_s} + \frac{Z}{z_a} \right) - e^{-jkl} \frac{Z}{z_a} G_1} \quad (55)$$

$$\beta = \frac{p_{Ms}}{p_{1s}} = e^{-jkl} \quad (56)$$

$$\mu\beta = \frac{p_{Ms}}{p_M} = \frac{Z}{z_s} G_1 \frac{e^{-jkl}}{1 + \frac{Z}{z_s} + \frac{Z}{z_a}} \quad (57)$$

Fig. 6 Arrangements of microphones and loudspeakers.

When $\mu\beta = 1$, the system becomes unstable. Therefore, in order that the system is stable

$$G_1 \neq \left(1 + \frac{z_s}{Z} + \frac{z_s}{z_a} \right) e^{jkl} \quad (58)$$

In order that $p_1 = 0$, G_1 must be given by the equation as follows:

$$G_1 = - \frac{1}{j \frac{Z}{z_s} \sin kl} \quad (59)$$

If the microphone is mounted very near the loudspeaker, $l = 0$, then Eqs. (56), (58) and (59) reduced to the following equations.

$$\beta = 1 \quad (60)$$

$$G_1 \neq \left(1 + \frac{z_s}{Z} + \frac{z_s}{z_a} \right) \quad (61)$$

$$G_1 = - \frac{1}{j \frac{Z}{z_s} (kl)_{l \rightarrow 0}} \quad (62)$$

In this case, the characteristics required for G_1 seems to become simple. But the sound field near the loudspeaker can not be considered as a one-dimensional one in the actual condition, so that some corrections are required in Eqs. (60) ~ (62) and they are not simple. When the microphone is mounted very near the loudspeaker, the condition is always the same as one

mentioned above.

b) In case where a loudspeaker is mounted above a microphone, as shown in Fig. 6 (b)

p_M is given as

$$p_M = \left(\cos kl_d - j \frac{Z}{z_a} \sin kl_d \right) p_1 \quad (63)$$

hence

$$\frac{p_1}{p_{in}} = \frac{2}{\left(1 + \frac{Z}{z_s} + \frac{Z}{z_a} \right) - \left(\cos kl_d - j \frac{Z}{z_a} \sin kl_d \right) \frac{Z}{z_s} G_1} \quad (64)$$

$$\beta = \cos kl_d - j \frac{Z}{z_a} \sin kl_d \quad (65)$$

In order that the system is stable

$$G_1 \neq \frac{1 + \frac{z_s}{Z} + \frac{z_s}{z_a}}{\cos kl_d - j \frac{Z}{z_a} \sin kl_d} \quad (66)$$

and in order that $p_1 = 0$, G_1 must be infinite.

$$G_1 \rightarrow \infty \quad (67)$$

In the system of one microphone and one loudspeaker, the frequency range of noise control must be limited in a very narrow one by the reason of the restriction; $\mu\beta \neq 1$.

If $p_1 \neq 0$, IL is influenced by z_s and z_a as expressed in Chap. 2.

4.2 $m=2, n=1$

There are three cases.

c) In case where the system is shown in Fig. 6 (c) From Eq. (41) the condition of $\beta=0$ is given by the following equation.

$$G_{M1} = -e^{-jk\Delta l_M} \quad (68)$$

and from Eq. (43) it follows that in order for p_1 to be zero,

$$G_1 = -\frac{e^{-jk(l-\Delta l_M)}}{j \frac{Z}{z_s} \sin k\Delta l} \quad (69)$$

d) In case where the system is shown in Fig. 6 (d) The sound pressure at each microphone is given as

$$p_{M1} = e^{jkl} p_{in} + e^{-jkl} p_r \quad (70)$$

$$p_{M2} = \left(\cos kl_d - j \frac{Z}{z_a} \sin kl_d \right) p_1 \quad (71)$$

hence

$$\begin{aligned} \frac{p_1}{p_{in}} = & 2 \left(1 + j \frac{Z}{z_s} \sin kl \cdot G_1 \right) \\ & / \left[\left(1 + \frac{Z}{z_s} + \frac{Z}{z_a} \right) e^{jkl} \right. \\ & \left. - \left\{ 1 + \left(\cos kl_d - j \frac{Z}{z_a} \sin kl_d \right) e^{jkl} G_{M2} \right\} \frac{Z}{z_s} G_1 \right] \end{aligned} \quad (72)$$

$$\beta = \frac{p_{M2}}{p_{1s}} = 1 + \left(\cos kl_d - j \frac{Z}{z_a} \sin kl_d \right) e^{jkl} G_{M2} \quad (73)$$

In order that $\beta=0$

$$G_{M2} = -\frac{e^{-jkl}}{\cos kl_d - j \frac{Z}{z_a} \sin kl_d} \quad (74)$$

and in order that $p_1=0$

$$G_1 = -\frac{1}{j \frac{Z}{z_s} \sin kl} \quad (75)$$

e) In case where the system is shown in Fig. 6 (e) $p_M=0$ on the condition of $\beta=0$. Therefore this situation is meaningless.

4.3 $m=1, n=2$

In this case, Eqs. (22) and (23) are reduced to the following equations:

$$p_2 = \left(A_2 - \frac{B_2}{Z} \right) p_{in} - \left(A_2 + \frac{B_2}{Z} \right) p_r - G_{B2} p_M \quad (76)$$

$$\left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) p_2 = (D_2 - ZC_2) p_{in} - (D_2 + ZC_2) p_r + G_{D2} p_M \quad (77)$$

f) In case where the system is shown in Fig. 6 (f)

In order that $\beta=0$, Eq. (42) is reduced to the equation as follows:

$$G_2 = -E(k\Delta l) G_1 \quad (78)$$

where

$$E(x) = \cos x + j \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \sin x \quad (79)$$

Also from Eq. (45), in order that $p_2=0$, G_1 must be represented by the equation as follows:

$$G_1 = \frac{1}{j \frac{Z}{z_s} \sin k\Delta l \left[E\{k(l+\Delta l)\} + j \frac{Z}{z_s} E(k\Delta l) \sin kl \right]} \quad (80)$$

g) In case where the system is given in Fig. 6 (g) p_M is given as

$$p_M = \cos kl \cdot p_2 + jZ \sin kl \cdot U_2 \quad (81)$$

hence

$$\frac{p_2}{p_{in}} = 2 \left(1 + j \frac{Z}{Z_s} \sin kl \cdot G_2 \right) / \left\{ \left(1 + j \frac{Z}{Z_s} \sin kl \cdot G_2 \right) \Delta_0 - \frac{Z}{Z_s} E(kl) \cdot \left[G_1 + \left\{ e^{jk(l+l_d)} + j \frac{Z}{Z_s} \sin k(l+l_d) \right\} G_2 \right] \right\} \quad (82)$$

$$\frac{p_{Ms}}{p_{in}} = \frac{2E(kl)}{\Delta_0 \Delta} \cdot \frac{\Delta_0 - \left(1 + j \frac{Z}{Z_s} \sin kl \cdot G_2 \right) \Delta}{1 + j \frac{Z}{Z_s} \sin kl \cdot G_2} \quad (83)$$

where

$$\left. \begin{aligned} \Delta_0 &= \left(1 + \frac{Z}{Z_s} + \frac{Z}{Z_a} \right) e^{jk(l+l_d)} + \frac{Z}{Z_s} E(kl) \\ \Delta &= \Delta_0 - \frac{Z}{Z_s} E(kl) \frac{\left\{ e^{jk(l+l_d)} + j \frac{Z}{Z_s} \sin k(l+l_d) \right\} G_2}{1 + j \frac{Z}{Z_s} \sin kl \cdot G_2} \end{aligned} \right\} \quad (84)$$

In order that $\beta=0$ ($p_{Ms}=0$)

$$G_1 = - \left\{ e^{jk(l+l_d)} + j \frac{Z}{Z_s} \sin k(l+l_d) - j \frac{\sin kl}{E(kl)} \Delta_0 \right\} G_2 \quad (85)$$

In order that $p_2=0$

$$G_2 = - \frac{1}{j \frac{Z}{Z_a} \sin kl} \quad (86)$$

h) In case where the system is shown in Fig. 6 (h) p_M is given as

$$p_M = \left(\cos kl_d - j \frac{Z}{Z_a} \sin kl_d \right) p_2 \quad (87)$$

In order that $\beta=0$ ($p_{Ms}=0$)

$$p_{2s}=0 \quad (88)$$

This situation is meaningless.

4.4 $m+n \geq 4$

Possible combinations of G_{Mi} and G_i are numberless in order that $\beta=0$ and $p_n=0$. But any other combinations are less advantageous compared with the case of $m+n=3$. If complexity is not considered, the acoustical feedback may be decreased by the two ways of insufficient conditions in which $\beta=0$, Eqs. (41) and (42).

5. ELECTRICAL CONSTRUCTION OF G_i AND CANCELING THE ACOUSTICAL FEEDBACK BY AN ELECTRICAL FEEDBACK

5.1 Electrical Construction of G_i

G_i is given by the ratio of p_{si} to p_M and it contains the sensitivity of the microphone and the loudspeaker. Assume that a condenser microphone and a dynamic loudspeaker are used in this system. Figure 7 shows the block diagram of the control system, where

K_M : voltage sensitivity of microphone.

μ_M : amplification factor of microphone amplifier.

μ_{ei} : voltage amplification factor of the power amplifier for each loudspeaker, in which the suitable phase shift is contained.

z_{eo} : output impedance of the power amplifier.

z_{esi} : electrical impedance of each loudspeaker which contains the motional impedance

I_i : electric current of each loudspeaker

A_s : force factor of loudspeaker

In Fig. 7

$$G_i p_M = p_{si} = \frac{A_s I_i}{S_s} = \frac{A_s}{S_s} \cdot \frac{K_M \mu_M \mu_{ei}}{z_{eo} + z_{esi}} p_M \quad (89)$$

hence

$$\mu_{ei} = \frac{S_s (z_{eo} + z_{esi})}{A_s K_M \mu_M} G_i \quad (90)$$

Generally z_{esi} of each loudspeaker is not equal. But as the motional impedance is usually small compared with the pure electric impedance z_{es} , μ_{ei} can be denoted approximately as follows:

$$\mu_{ei} \doteq \frac{S_s (z_{eo} + z_{es})}{A_s K_M \mu_M} G_i \quad (91)$$

If the polarity of the loudspeaker is reversed, a negative sign must be added to μ_{ei} .

Figure 8 shows two examples of the control system. The characteristics required in electric circuits which should satisfy the equations, μ_{ei} and G_{M2} , are not simple and it is difficult to realize the characteristics over the wide frequency range only by the

Fig. 7 Block diagram of control system.

Fig. 8 Examples of control system.

analog technique. They require the digital technique.

5.2 Canceling the Acoustical Feedback by an Electrical Feedback

For the system shown in Fig. 6 (a), sound pressures and transfer functions in Fig. 5 are denoted as follows:

$$p_n = p_1, \quad p_n = p_{10}, \quad p_{ns} = p_{1s} \quad (92)$$

and

$$\left. \begin{aligned} H_1 = \frac{p_{10}}{p_{10}} = 1, \quad H_M = \frac{p_M}{p_{10}} = \cos kl \\ + j \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \sin kl \\ \beta = \frac{p_{Ms}}{p_{1s}} = e^{-jkl}, \quad \mu = \frac{p_{1s}}{p_M} = \frac{1}{1 + \frac{z_s}{Z} + \frac{z_s}{z_a}} \end{aligned} \right\} \quad (93)$$

and from equivalent circuit, Fig. 9, the relation between p_{s1} and p_{1s} is described as

$$p_{1s} = \frac{1}{1 + \frac{z_s}{Z} + \frac{z_s}{z_a}} p_{s1} \quad (94)$$

An electrical feedback loop is added to cancel the acoustical feedback as shown in Fig. 10. In which

$$\beta = \frac{p_{Ms}}{p_{1s}} = \frac{p_{Ms}}{p_{s1}} \cdot \frac{p_{s1}}{p_{1s}} = \frac{p_{Ms}}{p_{s1}} \left(1 + \frac{z_s}{Z} + \frac{z_s}{z_a} \right) \quad (95)$$

$$G_\beta = -\frac{p_{Ms}}{p_{1s}} = -\frac{e^{-jkl}}{1 + \frac{z_s}{Z} + \frac{z_s}{z_a}} \quad (96)$$

Figure 11 shows the electric system with feedback. Where E_{M0} and E_{Ms} are the output of μ_M caused by p_{M0} and p_{Ms} respectively.

$$E_{Ms} = K_M \mu_M p_{Ms} \quad (97)$$

$$p_{s1} = \frac{A_s I_1}{S_s} \quad (98)$$

Hence, the gain of the electrical feedback loop which cancels the acoustical feedback is

Fig. 9 Equivalent circuit for p_{s1} and p_{1s} , (a) configuration (b) its equivalent circuit.

Fig. 10 Configuration of system in which an electrical feedback is added, (a) configuration (b) block diagram.

Fig. 11 Electric construction with a feedback loop.

$$\begin{aligned} \mu_{e\beta} &= -\frac{E_{Ms}}{I_1 R} = -\frac{A_s K_M \mu_M}{S_s R} \cdot \frac{p_{Ms}}{p_{1s}} \\ &= -\frac{A_s K_M \mu_M}{S_s R} \frac{e^{-jkl}}{1 + \frac{z_s}{Z} + \frac{z_s}{z_a}} \end{aligned} \quad (99)$$

As the equation $p_1 = 0$ means that $p_{1s} = -p_{10}$, the gain of the power amplifier which cancels the noise under the condition of Eq. (99) is deduced as follows:

$$\begin{aligned} G_1 &= \frac{p_{s1}}{p_{M0}} = \frac{p_{1s}}{p_{M0}} \cdot \frac{p_{s1}}{p_{1s}} = -\frac{p_{10}}{p_{M0}} \cdot \frac{p_{s1}}{p_{1s}} \\ &= -\frac{1 + \frac{z_s}{Z} + \frac{z_s}{z_a}}{\cos kl + j \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \sin kl} \end{aligned} \quad (100)$$

and by substituting Eq. (100) to Eqs. (90) and (91), the required amplification factor of the electric power amplifier is described as follows:

$$\mu_{e1} = -\frac{S_s (z_{e0} + R + z_{e1})}{A_s K_M \mu_M} \frac{1 + \frac{z_s}{Z} + \frac{z_s}{z_a}}{\cos kl + j \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \sin kl} \quad (101)$$

$$\doteq -\frac{S_s(z_{00}+R+z_{0s})}{A_s K_M \mu_M} \frac{1 + \frac{z_s}{Z} + \frac{z_s}{z_a}}{\cos kl + j \left(\frac{Z}{z_s} + \frac{Z}{z_a} \right) \sin kl} \quad (102)$$

By use of the electric system shown in Fig. 11, in which μ_{e1} and $\mu_{e\beta}$ satisfy Eq. (101) or (102) and Eq. (99) respectively, a microphone and a loudspeaker system is able to control the noise in duct in avoidance of acoustical feedback. If the polarity of the loudspeaker is reversed, the minus sign in μ_{e1} and $\mu_{e\beta}$ must be removed.

6. CONCLUSION

The principle of active control of noise in duct is explained and it is made clear in this paper what the pressure to be canceled out should be. And it is also made clear that canceling the original sound wave by the sound wave with the antiphase is equivalent to making the equivalent impedance of the active control system be zero, and there are three methods of picking up the canceling signal in active control.

We also showed the general equations of multi-microphone and multi-loudspeaker system and the conditions of control without acoustical feedback by means of impedance. Four suitable configurations and their conditions of control without acoustical feedback were deduced from the results of analysis. In addition, an electrical method which can cancel the acoustical feedback is proposed.

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