A hexagonal phase grating based on the quadratic-residue sequence

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A ceiling design based on the quadratic-residue sequence for enhancing the early lateral reflections is described. The design has a hexagonal star pattern which can blend very well with contemporary architecture. Measurements confirm the expected excellent scattering pattern over large angles.

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1. INTRODUCTION

Subjective preference of sound diffusion in concert halls has been extensively investigated by a number of authors.¹⁻⁴⁾ These studies have led to the conclusion that early lateral reflections are highly desirable for enhanced biaural perception. It has been specifically noted that sound reflected from ceiling and arrived in the median plane of a listener is characterised by a high degree of biaural similarity and hence has a low acceptance rating in subjective evaluations. Thus the ceiling shape and surface structure to keep early median-plane sound away from the listener's ears is an important consideration in concert hall design.

In the search for surface structures that give excellent sound diffusion over large wavelength ranges with minimum specular reflection, Schroeder has proposed reflection phase-gratings based on the primitive-roots series and on the quadratic-residue series.⁵⁻⁷⁾ Both grating designs have been tested on 1- and 2-dimensional square patterns and shown to be better scatterers than random surfaces. In this paper a quadratic-residue diffusion is constructed with hexagonal packed pattern. Such hexagonal pattern may be esthetically preferred over the square pattern. Moreover, the 6-fold symmetry of the scattered sound field could be expected to provide desirable sound distribution for some concert hall

architecture.

2. HEXAGONAL ARRANGEMENT

The 2-dimensional hexagonal pattern may be regarded as a lattice with a basis structure given by

$$C^2 = a^2 + b^2 + ab \quad (1)$$

where a and b are integer basis vectors subtending an angle of 60° between them as shown in Fig. 1. Alternatively the basis vectors may be chosen with the term ab negative, in which case the angle subtended between the basis vectors is 120° . Equation (1) is fortuitous in that it is a simple quadratic expression and could therefore be employed to generate the quadratic-residue series

$$S_{a,b}^2 = [C^2]_{\text{mod } N} \quad (2)$$

where $[C^2]_{\text{mod } N}$ is the remainder of C^2 modulo some prime number N . The array of numbers $S_{a,b}$ generated exhibit 6-fold symmetry. The case for $N=17$ is illustrated in Fig. 2 where for brevity only 1/6 of the hexagonal pattern is shown.

In exact analogy to the 1- and 2-dimensional square patterns of Schroeder the autocorrelation sequence of the exponential sequence.

$$r_{a,b} = \exp i2\pi S_{a,b}/N \quad (3)$$

is zero except for a zero shift.^{8,9)} Hence the spectrum of the sequence is theoretically flat. A hard reflecting surface constructed in accordance to the sequence of

Fig. 1 Configuration of the basis vectors.

Fig. 2 The 2-dimensional array of numbers generated by quadratic-residue sequence with $N=17$.

Fig. 3 Photograph of the scale model of the hexagonal phase grating based on quadratic-residue array.

Eq. (3) may be expected to produce smooth scatterings over a range of large angles. In practice a reflecting surface so constructed presents a formidable diffraction problem which could not be easily solved.

The quadratic-residue phase grating of $N=17$ is constructed from solid aluminium hexagonal pieces cut to lengths to give the depths $d_{a,b} = S_{a,b} \lambda_0 / 2N$ and width $w = 0.137 \lambda_0$ where λ_0 is the longest wavelength for which the grating will scatter efficiently. A scale model of the diffusor is built for the design frequency of $f_0 = 3.72$ kHz. A photograph of the scale model is shown in Fig. 3.

3. RESULTS AND DISCUSSIONS

Measurements were carried out using pulse technique in an anechoic room with Bruel and Kjaer Types 4155 and 2807 microphone and preamplifier. The gating was provided by a Bruel and Kjaer Type

Fig. 4 (a) Scattering pattern of the hexagonal diffusor for the frequency $f_0 = 3.72$ kHz at normal angle of incidence along the direction A.

Fig. 4 (b) Scattering pattern of the hexagonal diffusor for the frequency $f_0 = 3.72$ kHz at normal angle of incidence along the direction B.

4440 and the pure tone was generated from Philips PM 5190 frequency synthesizer. A nearly plane wave pulse was incident normally at the diffusor while the reflected pulse was measured on a storage oscilloscope.

The result of measurement along the two major directions of A and B is shown in Figs. 4 (a) and 4 (b) respectively. In both directions, there are narrow regions of dip in scatterings around the reflection angles of $\pm 30^\circ$. The central peak due to specular reflection is seen to be on the average 3 dB higher than the large angle scatterings. From the scattering patterns the standard deviations of the energy fluxes may be deduced. For the two different wavelengths of λ_0 and $\lambda_0/2$ the following standard deviations were obtained.

	Direction A	Direction B
λ_0 :	$\sigma = 3.2$ dB	$\sigma = 1.9$ dB
$\lambda_0/2$:	$\sigma = 5.6$ dB	$\sigma = 3.7$ dB

The values for the direction B is seen to be comparable to the values (1.7 dB for λ_0 and 3.3 dB for $\lambda_0/2$)

Fig. 5 Scattering pattern of the hexagonal diffusor for the frequency $2f_0 = 7.44$ kHz at normal angle of incidence along the direction B.

Fig. 6 Reflection of the hexagonal diffusor as a function of frequencies of the incident wave at a fixed angle of reflection of 45° .

obtained by Schroeder for the case of 2-dimensional square pattern. While the larger standard deviation for direction A is due to the dips around 30° , the scattering of energy fluxes into large angles is seen to be pronounced and remarkably uniform. This large-angle scatterings from the ceiling provide the desirable early lateral sound.

It has been shown that for frequencies that are integer multiples of f_0 the new sequence is a permutation the original sequence^{8,9)} and hence it has in principle the same 2-valued autocorrelation. In practice, the constraint on the high frequency scattering is given by $\lambda > 2w$. Experimental results for scattering at $2f_0 = 7.44$ kHz along the direction B is shown in Fig. 5, and as a function of frequency at a fixed angle of reflection is shown in Fig. 6. It is seen that there is a large number of peaks at higher frequencies up to around 20 kHz where the condition imposed by $\lambda > 2w$ set in. For frequencies below $f_0 = 3.7$ kHz the fraction of the incident wave scattered

into large angles is small compared to the specular reflection. This observed trend on the frequency dependence of scattering is seen to conform to the anticipation from the generalised analysis of diffusing walls by Ando.¹⁰⁾

4. CONCLUSION

The hexagonal phase grating has been found to exhibit excellent diffusion of sound over all large angles of scattering. At smaller angles the scattering is found to be slightly less uniform, but this is compensated by the fact that early lateral reflections are dominantly large angle scatterings from the ceiling. Biaural dissimilarity may be further enhanced by combining the quadratic diffusor with other geometrical factors of a concert hall. Acoustically, the hexagonal design offers a viable alternative to the square design and architecturally the 6-fold star shape is an intriguing and beautiful pattern for the ceiling.

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