Biochemical Studies on *Salmonidae*. VIII.**Studies on Synthetic Diets. II.**

Coefficient, maximum value and economical maximum
point of utilization of dietary protein.

By

H. SEKINE

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According to the results which had been worked out in the experiments reported in the former paper, compositions of diets may be due to different biological values of their contents, i. e. dietary protein, fat, carbohydrate minerals and etc..

In this paper, the writer have to consider further on the coefficient of the dietary protein which had been applied in the former study.

The coefficient in the different percentages of protein of real biological value (i. e. the proportion of absorbed nitrogen retained in a fish to the real applied nitrogen in the diet) are not equal, that is to say, as the results of the experiment, the diets which contain 16%, 36% and 64% of protein have been found to have the biological values 0.335, 0.204 and 0.109 respectively as shown in Tables I, II and III.

Table I.

Amounts of body protein accumulated by individual during the experiment.

	A			B			C		
	Initial	Final	Gain	Initial	Final	Gain	Initial	Final	Gain
Live weight (mg.)	1620	2770	1150	1630	2840	1210	1650	2550	900
Body protein (%)	12.9	14.9	—	12.9	15.4	—	12.9	14.8	—
Ditto weight (mg.)	199	413	214	210	437	227	213	377	165

Table II.

Amounts of assimilated dietary protein during the experiment.

	A	B	C
Amount of diet per a fish per day (mg.)	80	80	80
Dietary protein %	64	36	16
Ditto per day per a fish (mg.)	51.2	28.8	12.8
Total protein for 55 days (mg.)	2816	1580	704
Ditto assimilated (as 70%) (mg.)	1971	1109	493

Table III.
Coefficient of utilization of dietary protein.

	A	B	C
Accumulated protein in fish Assimilated protein in fish	0.107	0.204	0.335

Now, the most provable relation between the protein contents in the diets and their coefficients are obtained as follows :

Contents of dietary protein (%) are 16, 36, 64..... x

Ditto coefficients are 0.335, 0.204, 0.109..... A

Natural logarithmic values of A y

Then taking x on horizontal axis, y on vertical axis x, y may be in a lineal relation.

Fig. 1.

The equation of a straight line is

$$y + bx = a$$

which may be written $y = a - bx \dots (1)$

$$\log_e A = y = a - bx$$

$$\therefore A = e^{a-bx} \dots (2)$$

Substituting the points of P_1 and P_2 (see Fig. 1) in the equation (1)

$$-1.31 = a - b \times 25 \dots (1')$$

$$-2.11 = a - b \times 59 \dots (1'')$$

Solving these equations simultaneously will give the values of a and b ,

$$(1'') - (1') - 0.8 = -34b$$

$$\therefore b = \frac{0.8}{34} = 0.0235$$

Substituting the value of b in the equation (1')

$$\therefore a = -1.31 + 0.59 = -0.72$$

Therefore, the equation (2) may be written in the form

$$A = e^{-0.72 - 0.0235x} \dots (3)$$

Calculating the values of A from this equation, they are as shown in Table IV and the relation between x and A are shown in Fig. 2.

Table IV.
Values of A

x	A
10	0.385
20	0.303
30	0.240
40	0.190
50	0.150
60	0.118
70	0.093

Fig. 2.

According to this fact, the coefficient of dietary protein may be descended with increasing its content (%).

Therefore, in the case where a diet applying the equal amount for keeping fish, the real utilization of the dietary protein is expressed by Ax , and from the equation given, as shown in Table V and Fig. 3. And the maximum

Table V.
Values of Ax

x	Ax
10	3.85
20	6.08
30	7.40
40	7.60
50	7.50
60	7.08
70	6.51

Fig. 3. Curve of Ax

value of Ax is obtained as follows :

$$Ax = xe^{-0.72 - 0.0235x} \quad \frac{dAx}{dx} = e^{-0.72 - 0.0235x} - 0.0235xe^{-0.72 - 0.0235x}$$

$$e^{-0.72 - 0.0235x}(1 - 0.0235x) = 0. \quad x = \frac{1}{0.0235} = 42.5$$

Then the maximum value of Ax is in the case $x=42.5$.

From this result, it may be seen that in a synthetic diet the amount of protein content exceeding 43% goes to waste.

And the second time, the writer have something further to consider on an economical utilization of dietary protein. Although a coefficient of utilization of dietary protein is decreasing on the increasing of its amount, values of its

real utilization increase to the maximum point (42.5%), then a ratio of increase of Ax to that of x is not equal at every point, i. e. values of

$$\frac{Ax}{(Ax)_{\max}} - \frac{x}{(Ax)_{\max}} \text{ or } \frac{Ax}{7.67} - \frac{x}{42.55}$$

increase, at first, gradually and then descending to zero at the end (Fig. 4 and Table VI). There is, therefore, the maximum point of this value at any amount of x , and on this point, the dietary protein should be most economically utilized (Fig. 5 and Table VII). The writer designated that point "Economical Maximum Point of Utilization of Dietary Protein".

Fig. 4.

Fig. 5.

Table VI.

x	$\frac{Ax}{7.67}$	$\frac{x}{42.55}$
10	0.493	0.235
15	0.670	0.352 +
20	0.793	0.469
22	0.830	0.517
30	0.935	0.705

Table VII.

x	$\frac{Ax}{7.67} - \frac{x}{42.55}$
10	0.257
15	0.317
16	0.319
17	0.324
18	0.326
19	0.324
20	0.323
30	0.231

This point can be obtained by the following solution :

$$\begin{aligned} \text{Put } \frac{d}{dx} \left(\frac{Ax}{(Ax)_{\max}} - \frac{ax}{(Ax)_{\max}} \right) &= \frac{d}{dx} \left(\frac{Ax}{7.67} - \frac{0.18x}{7.67} \right) \\ &= \frac{e^{-0.72-0.0235x} - 0.0235x e^{-0.72-0.0235x}}{7.67} - \frac{0.18}{7.67} = 0 \end{aligned}$$

$$\text{Then } e^{-0.72-0.0235x} (1-0.0235x) = 0.18$$

$$1-0.0235x = 0.18 e^{0.72+0.0235x}$$

Then the maximum value of x is about 18 by graphical solution (Fig. 6 and Table VIII).

Fig. 6

Table VIII.

x	$1-0.0235x$	$0.18e^{0.72+0.0235x}$
10	0.765	—
15	0.644 ₅	0.525
20	0.530	0.593
30	0.295	0.747

Conclusion

Dietary protein (for use in trout fry culture) can be utilized for growth to 42.5% and about 18% of it, in case of dietary or supplied body or assimilated protein are of the same value, would be at an economical loss.

Summary

A coefficient of biological value of utilization (A) and an amount ($x\%$) of dietary protein are in the relation as the following equation:

$$A = e^{-0.72-0.0235x}$$

A value of A is, therefore, gradually decreased with increasing of x .

Ax indicates a value of real utilization of dietary protein and it shows the maximum value when $x=42.5\%$.

Then, although the value of utilization (Ax) should be ascended until $x=42.5$, a ratio of increase of Ax to that of x is not equal at every point, i. e. values of

$$\frac{Ax}{(Ax)_{\max}} - \frac{ax}{(Ax)_{\max}} \quad \text{or} \quad \frac{Ax}{7.67} - \frac{x}{42.5}$$

increase, at first, and then descend to zero on that point, and the maximum point of these values is in the case of $x \div 18$.

The author designated that point "Economical Maximum Point of Utilization of Dietary Protein".

From the results, an economical protein content of an artificial diet use in feeding trout-fry may be 20%~30%.

In conclusion I desire to express my thanks to Mr. Sato and Mr. Hoshino for their heapful advice in this work.