Correction and Remark to "Continuity of the mean values of BMO functions and Calderón-Zygmund properties of certain singular integrals"

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In the above paper there were some inaccuracies and should be modified as follows:

Page 2. Line 5 ↓ : Add $K(x, y)$ between "function" and "on".

Page 3. Line 1 ↓ : $\log \frac{b-a}{d-c}$ should be read as $\left(\log \frac{b-a}{d-c} + 1 \right)$.

Page 3. Line 2 ↓ : $a < c < d < b$ should be read as $a \leq c < d \leq b$.

Next we note that the following singular integrals are Calderón-Zygmund singular integrals: Let $A(x)$ be a real valued function with $A'(x) \in \text{BMO}(\mathbf{R})$ and consider

$$T_1 f(x) = \text{p.v.} \int_{-\infty}^{\infty} \frac{1}{x-y} \exp\left(i \frac{A(x)-A(y)}{x-y}\right) f(y) dy, \text{ and}$$

$$T_2 f(x) = \text{p.v.} \int_{-\infty}^{\infty} \frac{f(y) dy}{x-y+i(A(x)-A(y))}.$$

These are singular integrals of Calderón type, related to the Cauchy integral. Then, quite recently, Murai [2] has shown that T_1 and T_2 are bounded on $L^2(\mathbf{R})$. On the other hand, as in our former paper [3], one can easily check that the kernels in the above singular integrals satisfy the desired conditions (3.1), (3.2) and (3.3) in [3]. Hence they are Calderón-Zygmund singular integral operators.

Finally we note that, because of the Calderón-Zygmund property, weighted norm inequalities hold for them, i.e., if $1 < p < \infty$ and $w(x) \in A_p(\mathbf{R})$, then

$$\left(\int_{-\infty}^{\infty} |T_j f(x)|^p w(x) dx \right)^{1/p} \leq C(j, p) \left(\int_{-\infty}^{\infty} |f(x)|^p w(x) dx \right)^{1/p},$$

where $A_p(\mathbf{R})$ is the Muckenhoupt weight class A_p , (see, for example, [1] concerning A_p). (One can easily show that the arguments in Journé [1, pp. 47-58] hold in our case [3], too.)

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References

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- [2] T. Murai, Boundedness of singular integral operators of Calderón type, II, Preprint series No. 1 Department of Mathematics, College of General Education, Nagoya University, 1983.
- [3] K. Yabuta, Continuity of the mean values of BMO functions and Calderón-Zygmund properties of certain singular integrals, Bull. Fac. Sci., Ibaraki Univ., Math., No. 15 (1983), 1-8.