On the Spectrum of the Space of Bounded Holomorphic Functions on Strongly Pseudoconvex Domains in C^n

Kenzō ADACHI*

Let D be a bounded strongly pseudoconvex domain in C^n with C^1 -class boundary. Let $H^\infty(D)$ be the space of bounded holomorphic functions on D . We denote the spectrum of $H^\infty(D)$ by $\Sigma H^\infty(D)$. Then the following theorem holds.

THEOREM. D is open in $\Sigma H^\infty(D)$.

REMARK. If S is a unit polydisc in C^n , then S is open in $\Sigma H^\infty(S)$ (see [1]). If S is a ball or a special analytic polyhedron, then S is open in $\Sigma H^\infty(S)$ (see [4]).

LEMMA 1. Let F be the sheaf of germs of bounded holomorphic functions on D . Then $H^p(\bar{D}, F) = 0$ for $p \geq 1$.

PROOF. This is the result of Lieb [3].

LEMMA 2. Let $a \in D$ and $f \in H^\infty(D)$. Then there exists $f_1, \dots, f_n \in H^\infty(D)$ such that $f(z) = \sum_{k=1}^n f_k(z)(z_k - a_k)$ if $f(a) = 0$.

PROOF. The proof is the same as the proof of the theorem 1 of Kerzman and Nagel [2].

PROOF OF THE THEOREM.

For any $\phi \in H^\infty(D)$, we define

$$\pi(\phi) = \phi(z) = (\phi(z_1), \dots, \phi(z_n)).$$

Received October 16, 1972.

*Department of Mathematics, Faculty of Science, Ibaraki University, Mito.

Then π is continuous. For $\lambda \in D$, we define $\phi_i(f) = f(\lambda)$ for all $f \in H^\infty(D)$. Then we have $\phi_i \in \Sigma H^\infty(D)$ and $\pi(\phi_i) = \phi_i(z) = \lambda$. Suppose that $\pi(\phi) = \lambda \in D$ and $f \in H^\infty(D)$. If f vanishes at λ , then from Lemma 2, there exists $f_1, \dots, f_n \in H^\infty(D)$ such that $f(z) = \sum_{k=1}^n (z_k - \lambda_k) f_k(z)$. Then we have

$$\phi(f) = \sum_{k=1}^n (\phi(z_k) - \lambda_k) \phi(f_k(z)) = 0.$$

This shows that π is one to one over D . Let $\pi^{-1}(D) = \Delta$, then π maps Δ homeomorphically onto D . Therefore our theorem is proved.

References

- [1] K. Hoffman, Banach spaces of analytic functions, Prentice-Hall, INC., Englewood Cliffs, N. J., (1962).
- [2] N. Kerzman and A. Nagel, On finitely generated ideals in certain function algebras, J. Funct. Anal., **7** (1971), 212-215.
- [3] I. Lieb, Die Cauchy-Riemannschen Differentialgleichungen auf streng pseudoknvexen Gebieten, Math. Ann., **190** (1970), 6-44.
- [4] E. L. Stout, Some remarks on varieties in polydiscs and bounded holomorphic functions, Pacific J. Math., **32** (1970), 813-820.