MODELING OF CONSUMER CHOICE BEHAVIOR AS SEQUENTIAL DECISION PROCESS

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This study proposes a model of consumer choice behavior as sequential decision process. It is hypothesized that consumers form desirable product model and undesirable one through purchasing experience, by which they decide on whether a brand should be purchased or not. This study analyzes the dynamic process of formation of belief about brands and decision making through collecting information. We verified the ff.: how consumer belief is influenced by initial belief (2.3), why attributes with small utility can play an important role (2.4), how sellers improve their brands (3.1), when consumers can sacrifice correctness of choice for minimizing opportunity loss (3.2), and how choice environment is defined (3.3).

1. Introduction

Consumers collect information on alternatives to find desirable brands in the marketplace. Hence, it is a very important theme in marketing research to study consumer choice behavior from information processing perception. The studies on the theory of consumer information processing are divided roughly into two categories: that is, behavioral approach and economic approach (Ikeo, 1983, 1988).

In the former, the amount of collected information and its prescriptive factors have been studied (Bettman, 1979; Engel, Blackwell and Miniard, 1986; Assael, 1984; Wilkie, 1986). Since they aim to describe consumer behavior observed, these studies do not explain the motivation behind. How consumers integrate much information under limitation of processing capacity has also been studied (e.g., Staelin and Payne, 1976; Sakai, 1983). Various types of heuristics have been proposed to describe the way to integrate collected information, namely, affect referal, linear compensatory, conjunctive. These heuristics are a way of simplifying the choice task and adapting to limitation in processing capacity of consumers (Bettman, 1979). They explain how consumers integrate information and find desirable brands, but do not explain the dynamic process that consumers form their belief about brands through collecting information. In the latter, consumers are supposed to collect information to maximize expected utility under incomplete information. They have verified how many stores consumers go to on the condition of same quality of alternatives (Stigler, 1961), how many brands of various quality they search (Rosen, 1978; Weitzman, 1979). These studies, however, neither explain how much information is collected on each brand, nor how additional information influence consumer’s perception (Ikeo, 1988).

As shown above, the previous studies succeeded in describing each aspect of consumer choice behavior, but can not explain the whole process consisting of collecting information, forming belief about alternatives, and making a decision. The purpose of this study is to propose heuristics and to build an inclusive model of consumer choice behavior as sequen-

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tial decision process.

2. Choice rule

We define notations to be used in this paper;

\( G_1 \): desirable product model, defined by \( p_i \) \((i=1, 2, \ldots, I)\),
where \( I \) is the number of attributes,

\( G_2 \): undesirable product model, defined by \( p_i \) \((i=1, 2, \ldots, I)\),

\( p_i \) = probability that the \( i \)-th attribute of \( G_1 \) is satisfactory,

\( p_i \) = probability that the \( i \)-th attribute of \( G_2 \) is satisfactory,

where we suppose \( p_i \rightarrow p_2 \) at any attribute to simplify the problem,

\( q_i = 1 - p_i \),

\( q_i = 1 - p_i \),

\( \lambda_i = \begin{pmatrix} p_i & q_i \\ p_i & q_i \end{pmatrix} \), information matrix on the \( i \)-th attribute,

\( s_1 = \) ratio of brands of \( G_1 \) in the market,

\( s_2 = \) ratio of brands of \( G_2 \) in the market, or \( 1-s_1 \),

\( e_1 = \) consumer belief that a brand is identified as \( G_1 \),

\( e_2 = \) consumer belief that a brand is identified as \( G_2 \), or \( 1-e_1 \),

\( w_1 = \) loss incurred by passing over a brand of \( G_1 \),

\( w_2 = \) loss incurred by purchasing a brand of \( G_2 \), and

\( c = \) cost of collecting information.

2.1 Assumptions

Consumers cannot examine all attributes of all brands because their processing capacity is limited. They are proposed to have heuristics as shown below to reduce the choice problem to be simple one.

i) Consumers have formed \( G_1 \) and \( G_2 \) through past purchasing experience or information provided by sellers, friends etc,

ii) consumers judge attributes to be satisfactory or unsatisfactory,

iii) \( s_1 \) is known to consumers,

iv) consumers have to pay the cost for examination, and

v) consumers incur the loss by wrong decision.

We give an example of the purchase of bags, which have attributes such as color, size, figure, material etc. When they have used up a bag, consumers evaluate the bag by the performance of attributes during the durable years. The performance depends on the varied condition under which the bag is used. For example, even if the figure was satisfactory when it was examined, it is not satisfactory all the time when it is used. That applies well especially in the impulsive purchase. Consumers know only a small part of the performance when they examine the bag. They have to infer the performance from the result of examination by means of the relation between them based on purchasing experience or information provided by friends. They need the criterion of performance to judge whether the bag examined is desirable one or undesirable one. The criterion is the product model, \( G_1 \) or \( G_2 \). The relation between \( G_1 \) or \( G_2 \) and the result of examination of the \( i \)-th attribute are \( p_i \) or \( p_i \), respectively.
Consumers examine a bag to know whether attributes are satisfactory or not, considering the cost and the loss. They make their belief that the bag is identified as $G_1$ or $G_2$ through examination. When the bag is identified as $G_1$, it is purchased. Otherwise, consumers switch to another one until they end up with the bag identified as $G_1$.

2.2 Definition of two product models

Utility is defined here as the value of performance. The utility consumers recognize by examination is assumed to be a random variable of which expected value is the utility consumers recognize when they have used up the brand. Let $u$ and $u_a$ be the former utility and the latter one, respectively. $u$ is described as

$$u = x_1u_1 + x_2u_2 + \cdots + x_iu_i + \cdots + x_iu_i,$$  \hspace{1cm} (2.1)

here $x_i$ is a random variable as

$$P(x_i = 1) = p^i,$$

$$P(x_i = 0) = 1 - p^i.$$

It is assumed to be independent each other. $p^i$ is the probability that the $i$-th attribute of the brand is satisfactory. $u_i$ is the inherent utility consumers enjoy on the $i$-th attribute. It is assumed to be constant on all brands. The linear compensatory model has intuitive appeal and has been widely used in marketing (Hagerty and Aaker, 1984). $u_a$ is described as

$$u_a = E(u)$$

$$= E(x_1)u_1 + E(x_2)u_2 + \cdots + E(x_i)u_i + \cdots + E(x_i)u_i$$

$$= \sum_{i=1}^{\infty} p^i u_i.$$  \hspace{1cm} (2.2)

When they have used up the brand, consumers recognize the utility of the $i$-th attribute, or $p^i u_i$. They are assumed to know the distribution density of $p^i u_i$ or $p^i$, because $u_i$ is constant on all brands. When they examine the brand, consumers know whether attributes are satisfactory or not, but do not know $p^i$. Let $u_*$ be the critical utility which divide brands into $G_1$ and $G_2$. Consumers calculate the critical probability, $p_*$, which satisfies the condition as

$$u_* = \sum_{i=1}^{\infty} p_* u_i.$$  \hspace{1cm} (2.3)

$p_1$ and $p_2$ are defined as

$$p_1 = E(p^i \mid p^i \geq p_*),$$  \hspace{1cm} (2.4)

$$p_2 = E(p^i \mid p^i < p_*).$$  \hspace{1cm} (2.5)

The utility of $G_1$ and $G_2$ are respectively

$$u_1 = \sum_{i=1}^{\infty} p_1 u_i,$$  \hspace{1cm} (2.6)

$$u_2 = \sum_{i=1}^{\infty} p_2 u_i.$$  \hspace{1cm} (2.7)

The difference of utility between the two product models is
This is the opportunity loss which consumers incur by wrong choice. It is $w_2$. The product models are determined by $p_*$ and the distribution density of $p'$. We have the three cases of their change. These are: i) the critical point can shift with the distribution density fixed, ii) the distribution density changes while the critical point remains fixed, and iii) both of them can change. i) is the case that the level at which consumers are satisfied with the attribute gets higher or lower. ii) is the case that sellers supply new brands in the market or they improved their brands. iii) is the most usual case that change of consumer preference induces supply of new brands and the supply also influences consumer preference. Moore and Lehman (1980) found that consumers collect more information in the more complicated and changeable market. This seems because the product models are unstable.

2.3 Bayesian information processing

Consumers collect information and update their current belief about brands they examine. The processing is described by:

\begin{align*}
    e_1(n+1) = & \frac{e_1(n)p_1(n)}{e_1(n)p_1(n) + (1 - e_1(n))p_2(n)}, \quad \text{when the attribute is satisfactory,} \\
    e_1(n+1) = & \frac{e_1(n)q_1(n)}{e_1(n)q_1(n) + (1 - e_1(n))q_2(n)}, \quad \text{when it is unsatisfactory,}
\end{align*}

where $e_1(n)$ is their belief when they examine an attribute in the $n$-th order. $p_1(n)$, $p_2(n)$, $q_1(n)$, and $q_2(n)$ are elements of the information matrix of the attribute. $e_1(1)$ is initial belief and is equivalent to $s_1$, which is given as Prob. ($u \geq u_*$).

From (2.9) and (2.10),

\begin{equation}
    e_1(n) = \frac{1}{1 + \frac{1 - e_1(1)}{e_1(1)} \prod_{j=1}^{m} p_2(j) \prod_{k=1}^{n} q_2(k)}, \quad \text{for } n \geq 2,
\end{equation}

where $l$ and $m$ are frequency counts of satisfaction and dissatisfaction, respectively. Putting $L$ and $B$ as

\begin{equation}
    L = \sum_{j=1}^{l} \log \frac{p_1(j)}{p_2(j)} + \sum_{k=1}^{m} \log \frac{q_1(k)}{q_2(k)}, \quad B = \frac{1 - e_1(1)}{e_1(1)},
\end{equation}

we have a logistic curve as

\begin{equation}
    e_1(n) = \frac{1}{1 + B \cdot 2^{-L}}
\end{equation}

The locus of the curve is determined by consumer’s initial belief. We should note that initial belief plays a significant role in determining the value of $e_1(n)$, when the absolute value of $L$ is small.
Fig. 2.1 Flowchart of choice behavior

start

choose a brand to be examined

set initial belief; \( e_1(1) = s_1 \)

examine an attribute and revise belief; \( e_1(n) = \frac{1}{1 + B \cdot 2^{-l}} \)

calculate maximum value of information; \( V_\lambda(n) \)

NO

\( V_\lambda(n) \leq c \)

Yes

NO

\( e_1(n) \geq \frac{w_2}{w_1 + w_2} \)

Yes

buy the brand

end
2.4 Decision procedure

Consumers are to minimize the opportunity loss. The value of information, defined as the expected reduction of incurred loss before and after examination of an attribute, is

$$V_i = \begin{cases} 
  e_i(w_i p_i^1 + w_2 p_i^2) - w_2 p_i^2 & \text{for } \frac{w_2 p_i^1}{w_1 p_i^1 + w_2 p_i^2} < e_i < \frac{w_2}{w_1 + w_2}, \\
  w_2 q_i^1 - e_i(w_1 q_i^1 + w_2 q_i^2) & \text{for } \frac{w_2}{w_1 + w_2} < e_i < \frac{w_2 q_i^1}{w_1 q_i^1 + w_2 q_i^2}, \\
  0 & \text{for other cases.}
\end{cases}$$

$w_1$ and $w_2$ are constant over all attributes. The difference of value is dependent only on information matrix. Since $\frac{\partial V_i}{\partial p_i} > 0$ and $\frac{\partial V_i}{\partial p_i} < 0$, the information with large $p_i$ and small $p_i$ plays an important role in decision making even for the attribute with small inherent utility. Generally, consumers with more experience or more distinctive preference have more attributes to judge by, and hence large value of information. Moore and Lehman (1980) stated that the more experience they have, the less information they collect.

Putting $V_\ast(n)$ as the maximum value over the remaining attributes unexamined up to the time $n$, we have the decision rule. When $V_\ast(n) > c$, they should collect more information. Otherwise, they should stop and take action:

buy the brand, if $e_i(n) \geq w_2/(w_1 + w_2)$, and
leave it and examine another, if $e_i(n) < w_2/(w_1 + w_2)$.

3. Strategic analysis

3.1 Seller's strategies

The expected probabilities that the $i$-th attribute is satisfactory or not are $e_i p_i^1 + e_2 p_i^2$ and $e_i q_i^1 + e_2 q_i^2$, respectively. Hence, from (2.9) and (2.10), $e_i(n + 1)$ is equivalent to $e_i(n)$. That is, the expected consumer belief remains the same after examination of the attribute. However, probability, $p'$, of a brand supplied by sellers is not always identical to the product models, $p_i^1$, $p_i^2$. So the true expected belief after examination is

$E\{e_i\} = p' \frac{e_i p_i^1}{e_i p_i^1 + e_2 p_i^2} + q' \frac{e_i q_i^1}{e_i q_i^1 + e_2 q_i^2}$.  \hfill (3.1)

[Proposition 1] Let $\Delta E = E\{e_i\} - e_i$. $\Delta E$ can be regarded as the effect on consumer belief, which is caused by the difference between the product models and a brand examined actually. Then,

i) $\Delta E \geq 0$ for $p' \geq p_i$,
ii) $\Delta E \geq 0$ for $p_i \leq p' < p_i$ and $p' \geq e_i p_i^1 + e_2 p_i^2$,
iii) $\Delta E < 0$ for $p_i \leq p' < p_i$ and $p' < e_i p_i^1 + e_2 p_i^2$.

Proof: Obvious from

$$\Delta E = \frac{e_1 e_2 (p_i - p_i^2) (p' - (e_i p_i^1 + e_2 p_i^2))}{(e_i p_i^1 + e_2 p_i^2) (e_i q_i^1 + e_2 q_i^2)}.$$  \hfill (3.2)

Interpretation: Sellers try to make consumers identify their brands as $G_1$. If they can control the order of examination, they show positively the attribute to consumers provided
\( p^i \geq p^i \). They conceal it provided \( p^i < p^i \). Things depend on consumer belief provided \( p^i \leq p^i < p^i \).

**Proposition 2**

\[
\varepsilon = \frac{\partial \Delta E}{\partial p^i} = e_1 \left( \frac{p^i}{e_1 p^i + e_2 p^i} - \frac{q^i}{e_1 q^i + e_2 q^i} \right) = \frac{e_1 e_2 (p^i - p^i)}{(e_1 p^i + e_2 p^i)(e_1 q^i + e_2 q^i)} > 0. \quad (3.3)
\]

**Interpretation:** \( \Delta E \) is a positive linear function of \( p^i \). This positive marginal effect, \( \varepsilon \), of \( p^i \) upon \( \Delta E \) is controlled only by consumers.

**Proposition 3**

As a function of \( p^i \) and \( p^2 \), \( \varepsilon \) has the following properties:

<table>
<thead>
<tr>
<th>( p^i )</th>
<th>( p^i \geq 1/2 )</th>
<th>( p^i &lt; 1/2 ) and ( p^i &lt; \rho )</th>
<th>( p^i &lt; 1/2 ) and ( p^i \geq \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon(p^i) )</td>
<td>convex,</td>
<td>concave,</td>
<td>convex,</td>
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<tr>
<td></td>
<td>monotonously increasing</td>
<td>monotonously increasing</td>
<td>monotonously increasing</td>
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<tr>
<td></td>
<td>( p^i \leq 1/2 )</td>
<td>( p^i &gt; 1/2 ) and ( p^i &lt; \tau )</td>
<td>( p^i &gt; 1/2 ) and ( p^i \geq \tau )</td>
</tr>
<tr>
<td>( \varepsilon(p^i) )</td>
<td>convex,</td>
<td>concave,</td>
<td>convex,</td>
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<tr>
<td></td>
<td>monotonously decreasing</td>
<td>monotonously increasing</td>
<td>monotonously increasing</td>
</tr>
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</table>

where,

\[
\rho = \frac{1}{e_1} \left( \frac{D}{1+D} - e_2 p^i \right), \quad D = (p^i/q^i)^{1/3},
\]

\[
\tau = \frac{1}{e_1} \left( \frac{E}{1+E} - e_1 p^i \right), \quad E = (p^i/q^i)^{1/3}.
\]

**Proof:** We have

\[
\frac{\partial \varepsilon}{\partial p^i} = e_1 e_2 \left( \frac{p^i}{(e_1 p^i + e_2 p^i)^2} + \frac{q^i}{(e_1 q^i + e_2 q^i)^2} \right) > 0, \quad (3.4)
\]

\[
\frac{\partial^2 \varepsilon}{\partial p^i^2} = \frac{2e_1^2 e_2^2 (e_1 p^i + e_2 p^i)^3 - p^i (e_1 q^i + e_2 q^i)^3}{(e_1 p^i + e_2 p^i)(e_1 q^i + e_2 q^i)^3}. \quad (3.5)
\]

But, putting

\[
h(p^i) = q^i (e_1 p^i + e_2 p^i)^3 - p^i (e_1 q^i + e_2 q^i)^3,
\]

we obtain

\[
\frac{dh(p^i)}{dp^i} = 3e_1 q^i (e_1 p^i + e_2 p^i)^2 + p^i (e_1 q^i + e_2 q^i)^2 > 0. \quad (3.6)
\]

Since

\[
h(p^i) = q^i p^i - p^i q^i = p^i q^i (2p^i - 1),
\]

we finally have

\[
\frac{\partial^2 \varepsilon}{\partial p^i^2} \geq 0 \text{ for } p^i \geq 1/2,
\]

\[
\frac{\partial^2 \varepsilon}{\partial p^i^2} < 0 \text{ for } p^i < 1/2 \text{ and } p^i < \rho,
\]

\[
\frac{\partial^2 \varepsilon}{\partial p^i^2} \geq 0 \text{ for } p^i < 1/2 \text{ and } p^i \geq \rho.
\]
In the same manner, we obtain
\[ \frac{\partial^2 \varepsilon}{\partial p_i^2} \geq 0 \text{ for } p_i \leq 1/2, \]
\[ \frac{\partial^2 \varepsilon}{\partial p_i^2} < 0 \text{ for } p_i > 1/2 \text{ and } p_i > r, \]
\[ \frac{\partial^2 \varepsilon}{\partial p_i^2} \geq 0 \text{ for } p_i > 1/2 \text{ and } p_i \leq r. \]

**Interpretation:** The marginal effect is large, when \( p_i \) is large and \( p_i \) is small. Sellers improve their brands on the attribute with large \( p_i \) and small \( p_i \), if the marginal cost of improvement is constant. It is also a good strategy to induce the shift of critical point \( p_i^* \) through advertising.

### 3.2 Order of examination of attributes

[**Proposition 4**] Let \( V_i \) and \( V_j \) be the values of an arbitrary pair of information. Inequalities as below are the necessary and sufficient condition that \( V_i \geq V_j \) for all \( e_i, w_1 \), and \( w_2 $$:

\[ \frac{p_i}{p_j} \leq \frac{p_i}{p_j}, \quad \frac{q_i}{q_j} \leq \frac{q_i}{q_j}. \]

**Proof:** When \( V_i \) and \( V_j \) have the relation shown in Fig 3.1, we have the conditions as

\[ \frac{w_2 p_i}{w_1 p_i^1 + w_2 p_i^2} \leq \frac{w_2 p_j}{w_1 p_j^1 + w_2 p_j^2}, \quad \frac{w_2 q_i}{w_1 q_i^1 + w_2 q_i^2} \geq \frac{w_2 q_j}{w_1 q_j^1 + w_2 q_j^2} \]

That is,

\[ \frac{p_i}{p_j} \leq \frac{p_i}{p_j}, \quad \frac{q_i}{q_j} \leq \frac{q_i}{q_j}. \]

**Interpretation:** When the inequalities hold for all pairs of information, the order of examination is unique. Otherwise, it is dependent on consumer belief.

**Fig. 3.1** Value of an arbitrary pair of information
[Proposition 5] Let $H$ be the entropy of the uncertainty of consumer belief;

$$H = -(e_1 \log e_1 + e_2 \log e_2).$$

From (2.9) and (2.10), the entropy changes as;

when the attribute is satisfactory,

$$H' = \left\{ \frac{e_1 p_1}{e_1 p_1 + e_2 p_2} \log \frac{e_1 p_1}{e_1 p_1 + e_2 p_2} + \frac{e_2 p_2}{e_1 p_1 + e_2 p_2} \log \frac{e_2 p_2}{e_1 p_1 + e_2 p_2} \right\}. \quad (3.7)$$

when it is unsatisfactory,

$$H'' = -\left\{ \frac{e_1 q_1}{e_1 q_1 + e_2 q_2} \log \frac{e_1 q_1}{e_1 q_1 + e_2 q_2} + \frac{e_2 q_2}{e_1 q_1 + e_2 q_2} \log \frac{e_2 q_2}{e_1 q_1 + e_2 q_2} \right\}. \quad (3.8)$$

The expected entropy after examination is

$$E\{H\} = H' \cdot (e_1 p_1 + e_2 p_2) + H'' \cdot (e_1 q_1 + e_2 q_2)$$

$$= -\left\{ \frac{e_1 p_1}{e_1 p_1 + e_2 p_2} \log \frac{e_1 p_1}{e_1 p_1 + e_2 p_2} + \frac{e_2 p_2}{e_1 p_1 + e_2 p_2} \log \frac{e_2 p_2}{e_1 p_1 + e_2 p_2} \right\}$$

$$- \left\{ \frac{e_1 q_1}{e_1 q_1 + e_2 q_2} \log \frac{e_1 q_1}{e_1 q_1 + e_2 q_2} + \frac{e_2 q_2}{e_1 q_1 + e_2 q_2} \log \frac{e_2 q_2}{e_1 q_1 + e_2 q_2} \right\}. \quad (3.9)$$

The expected reduction of entropy is

$$I = H - E\{H\}$$

$$= e_1 \left\{ p_1 \log \frac{p_1}{e_1 p_1 + e_2 p_2} + q_1 \log \frac{q_1}{e_1 q_1 + e_2 q_2} \right\}$$

$$+ e_2 \left\{ p_2 \log \frac{p_2}{e_1 p_1 + e_2 p_2} + q_2 \log \frac{q_2}{e_1 q_1 + e_3 q_3} \right\}. \quad (3.10)$$

This is the contribution of information to correctness of choice. Putting $I_i$ and $I_j$ as the contribution of an arbitrary pair of information, we have: for inequalities in proposition 4, $I_i \geq I_j$ and $V_i \geq V_j$ for all $e_1$, $w_1$, and $w_2$, otherwise, $V_i \geq V_j$ and $I_i \leq I_j$ for a set of $e_1$, $w_1$, and $w_2$.

Proof: Let $b_1^i$, $b_2^i$, $b_1^j$, and $b_2^j$ be $e_i$ after examination of the pair of attributes. The inequalities in proposition 4 are equivalent to $b_1^i \leq b_2^i \leq b_1^j \leq b_2^j$. It is a necessary condition that $I_i \geq I_j$, for all $e_i$ as shown in Fig 3.2. Otherwise, there exists a set of $w_1$ and $w_2$, which makes $V_i \geq V_j$ and $I_i \leq I_j$ for certain $e_1$ as shown in Fig. 3.3.

Interpretation: For the inequalities, minimization of the opportunity loss is equivalent to minimization of the uncertainty. Otherwise, consumers can sacrifice minimization of the uncertainty for minimization of the opportunity loss.

3.3 Choice environment

[Proposition 6] Let $a_{ij}$ be the probability that consumers erroneously identify brands of $G_i$ as $G_j$. Consumers are supposed to know $a_{ij}$ through purchasing experience. Putting $C_a$ as the average unit cost of one series of examination necessary for decision, the expected charge which consumers must pay in the choice problem is

$$\frac{C_a + w_2 s_2 a_{21}}{s_1 a_{11} + s_2 a_{21}} \quad (3.11)$$

When there are so many brands that $s_1$ may be fixed, consumers must prepare to pay the
charge at the beginning of every examination. Then, the charge is also equivalent to \( w_i \).

**Proof:** When they make decision, consumers stand at four situations.

- \( T_1 \): situation that \( G_1 \) is identified as \( G_2 \)
- \( T_2 \): situation that \( G_2 \) is identified as \( G_2 \)
- \( T_3 \): situation that \( G_1 \) is identified as \( G_1 \)
- \( T_4 \): situation that \( G_2 \) is identified as \( G_1 \)

If they stand at \( T_3 \) or \( T_4 \), they buy the brand. Otherwise, they leave it and examine others until they reach \( T_3 \) or \( T_4 \). This procedure can be expressed by absorptive Markov chain.
with the state space \( \{T_1, T_2, T_3, T_4\} \). Let \( P \) be transition matrix as
\[
P = \begin{pmatrix}
S_{11} & S_{12} & S_{13} & S_{14} \\
S_{21} & S_{22} & S_{23} & S_{24} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
Then, from Morimura and Takahashi (1979),
\[
\lim_{n \to \infty} P^n = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0
\end{pmatrix}
\]
with
\[
Q = \begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix},
R = \begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix}
\]
Let \( \pi(n) \) be the probability distribution at the \( n \)-th examination. From \( \pi(1) = (S_{11} S_{12} S_{22} S_{11} S_{22}) \), we have
\[
\lim_{n \to \infty} \pi(n) = \pi(1) \cdot \lim_{n \to \infty} P^n = \begin{pmatrix}
0 & 0 & S_{11} & 0 \\
S_{11} & S_{12} & 0 & 0 \\
0 & 0 & S_{22} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
Let
\[
\tau = (I - Q)^{-1} \xi \quad \text{with } \xi = (1, 1)^T.
\]
Each element of \( \tau \), \( \tau_i \), is average step counts that consumers starting at \( T_i \) need to be absorbed into \( T_3 \) or \( T_4 \). That is, \( \tau_1 = \tau_2 = (S_{11} + S_{22})^{-1} \). We have three cases of consumer choice behavior; i) consumers buy brands at first examination of brand, ii) they need \((1 + \tau_1)\) times of examination, and iii) they need \((1 + \tau_2)\) times of examination. Putting \( N \) as the expected counts of examination consumers need, we have
\[
N = 1 \cdot (S_{11} + S_{22}) + \left(1 + \frac{1}{S_{11} + S_{22}}\right) \cdot S_{11} + \left(1 + \frac{1}{S_{11} + S_{22}}\right) \cdot S_{22} = \frac{1}{S_{11} + S_{22}}
\]
The probability of wrong choice is \( S_{22} / (S_{11} + S_{22}) \). Hence the expected charge in the choice problem is
\[
N \cdot C_a + w_2 \cdot \frac{S_{22}}{S_{11} + S_{22}} = \frac{C_a + w_2 S_{22}}{S_{11} + S_{22}}
\]
4. Conclusion
We hypothesized that consumers form two product models, and treated consumer choice behavior as sequential decision process. We analyzed three processes combined: collection of information, formation of belief, and decision making. We succeeded in explaining some of consumer behavior and in describing seller's strategies as shown below.

i) Consumer belief is updated on the logistic curve. The locus of the curve is determined by consumer's initial belief. Initial belief plays an important role in formation of belief, when the absolute value of \( L \) is small.

ii) The value of information depends much on the information matrix. Attributes
with large $p_1$ and small $p_2$ have large value of information, even if they have small utility. Thanks to such attributes, consumers can easily find desirable brands, while the product has many attributes.

iii) The effect of seller's improvement on consumer belief is controlled only by consumers. Its marginal effect is large, when $p_1$ is large and $p_2$ is small. Sellers improve their brands on such attributes, if the marginal cost of improvement is constant.

iv) When the inequalities, $p_1/p_2 \geq p_1/p_2$, $q_1/q_2 \leq q_1/q_2$, hold for all pairs of information, the order of examination of attributes is unique and is determined in advance. Minimization of the opportunity loss is equivalent to minimization of the uncertainty of consumer belief. Otherwise, the order depends on consumer belief and consumers can sacrifice maximizing correctness of choice for minimizing the opportunity loss.

v) In the choice problem, consumers must pay the expected charge as $(C_a + w_2S_2A_2)/ (S_1A_1 + S_2A_2)$, which is an indicator of choice environment.

There remain some problems to be considered: i) how we make sure whether the assumption that consumers form two product models is reasonable, ii) how we describe the effect of correlation between the condition of attributes, which diminishes the value of information, and iii) how we describe more strictly the game between consumers and sellers.

**References**


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