Circulatory Dynamics across the
Stenotic Mitral Valve Orifice

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The configuration of the mitral valve orifice in mitral stenosis are most similar to the so-called rounded-edge orifice, and it is said that there will be a contraction of the issuing stream such that the cross-sectional area of the stream is some fraction less than that of the orifice itself. An attempt\(^1\) has been made to apply the hydraulic formula of fixed orifice to the stenotic valve orifice. But the Gorlin’s approach is not so complete method to estimate the size of valve orifice and understand the pressure-flow relationship through the orifice. The new approach has been applied in estimating the size of the stenotic valve orifice as a so-called pipe orifice in the hydraulic system.

Derivation of the hydrodynamic formula of the stenotic valve orifice.

R. Gorlin\(^1,2\) has derived the general hydraulic orifice formula as follows from two accepted equations:

\[
F = Cc \cdot A v \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (1)
\]

whereby with a fixed orifice\(^3\) its area, A, changes in flow, F, are associated with proportional changes in velocity, v. Cc is the coefficient of orifice contraction.

\[
v = C v \sqrt{2 gh} \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (2)
\]

whereby changes in velocity, v, through the orifice are proportional to the square root of the pressure in height, h. Pressure in height of a liquid above a given orifice is converted to velocity through the orifice as Torricelli’s principle in the open reservoir (Fig. 1), that is, pressure

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Do, Dc : diameter of orifice & orifice flow
\(\bar{D}_0\) : mean velocity in orifice
\(\bar{D}_0 = \sqrt{2 g (Y_1 - Y_2)} = \sqrt{2 gh}\)
Cc : velocity coefficient
Cc = \(\frac{D}{D_0}\) \(\frac{S_0}{S_e}\) : contraction coefficient

Fig.1. Torricelli’s Principle.

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energy is converted to kinetic energy. g is gravity acceleration in the centimeter-gram-second system. Cv is the coefficient of velocity whereby a certain fraction of pressure is converted to velocity, and the rest is dissipated as frictional energy loss. Gorlin's equation is solved for A from these two formulas:

\[ A = \frac{F}{Cc \sqrt{2gh}} = \frac{F}{44.5 \frac{L}{P1 - P2}} \]  

F is the rate of blood flow during the time the valve is open, h is the pressure gradient across the orifice, "loss of head", C is the discharge coefficient (empirical constant).

But considering the stream of the stenotic valve cross-sectional area as a pipe orifice in the hydraulic system (Fig. 2), the formula of orifice flow in the mitral valve model has been derived as follows: from the usual form of continuity equation:

\[ f_{cs} \sigma \cdot V \cdot ds = 0 \]  

\[ \dot{Q} = \frac{U_1 \pi D_1^2}{4} = \frac{U_2 \pi D_2^2 C}{4} = \frac{U_0 \pi D_0^2}{4} \]  

whereby there will be a contracted flow across the mitral orifice,

\[ \frac{\pi D_c^2}{4} = Cc \frac{\pi D_o^2}{4} \]  

\[ Cc = \frac{D_c}{D_o} \]  

D1, Do, and Dc are the diameter of a given pipe, orifice and contracted flow, S1, So, and Sc are their cross-section areas. The pressures at the upstream section and the downstream are actual pressures, but the velocities from Bernoulli's
Fig. 6. Relationship flow coefficient $C$ between Reynolds number of fluid.
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\[
C = \frac{a}{2\pi \int \frac{3b}{4} \frac{d\theta}{r} \sqrt{\frac{r}{b}}}
\]

\[
\rho = \frac{\mu}{\mu}
\]

\[
\frac{P_1}{\rho} + \frac{U_1^2}{2g} = \frac{P_2}{\rho} + \frac{U_2^2}{2g}
\]  
(8)

$\rho$ is the specific gravity of blood, $U_1$ and $U_2$ are mean velocities at the upstream and downstream section of the throat. Then the actual velocity is obtained by multiplying this by the velocity coefficient, $C$, with a loss of energy:

\[
U_1 = Cu \frac{C(D_0/D_1)^2 \cdot U = Cc \cdot Cu \cdot m \cdot \bar{U}_2}{...}(9)
\]

\[
U_2 = \frac{Cc \cdot Cu}{\sqrt{1-Cc^2 \cdot m^2}} \cdot \sqrt{2g(y_1-y_2) \cdot 2g(P_1-P_2)}
\]  
(10)

whereby $m$ is the throat area ratio, $(D_0/D_1)^2 = S_0/S_1$. The issuing flow across the orifice, $Q$ is solved from equation (5) and (10).

\[
Q = \frac{Cc \cdot Cu \cdot S_0}{\sqrt{1-Cc^2 \cdot m^2}} \cdot \sqrt{2g(y_1-y_2) \cdot 2g(P_1-P_2)}
\]  
(11)

\[
So = \frac{Q}{\sqrt{1-Cc^2 \cdot m^2}} \cdot \sqrt{2g(y_1-y_2) \cdot 2g(P_1-P_2)}
\]

\[
C = \frac{Cc \cdot Cu}{\sqrt{1-Cc^2 \cdot m^2}}
\]  
(12)

Discharge flow coefficient, $C$ is a function of Reynolds's Number, $Re = Ud\rho/\mu$ and throat area ratio, $m$. Vena contracta is caused by the inertia.

equation\textsuperscript{7} without a loss term are theoretical velocities.

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Fig. 8. Flow coefficient and the configuration of the mitral valve orifice. (I, II, III type)

Fig. 9. Relationship between C and Descent Rate.

Fig. 10. Relationship between flow coefficient and Reynolds number of fluid.

Fig. 11. Total pressure is decided by the angle, $\theta$ between the flow and the catheter in three-dimensional flow.

dilution technique and direct Fick’s method, and hemodynamic measurement were repeated from three times to twenty times at rest and after $\beta$-stimulator or $\beta$-blocker injections. Expired air was collected for two or three minutes in a Douglas Bag, and blood samples were withdrawn simultaneously from the brachial artery and pulmonary artery midway during the gas collection, and indicator was injected in the pulmonary artery and recorded in the peripheral artery with Cuvett Densitometer.

Theoretical cross-sectional areas of stenotic mitral valve were calculated by Gorlin’s formula; and the areas of each mitral valve orifices were actually measured in the operations by T. Yamada. These theoretical data were compared with actual data of the mitral valve throat. It was decided to effect final adjustment of these data by modifying the C factor. Another orifice flow experiments with plastic pipe such as figure 3 was performed in vitro. Blood and Mizuame were used as fluid, and Reynolds

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number and C factor were determined and investigated hydrodynamically.

**RESULTS**

Comparing the calculated theoretical cross-sectional area of the mitral valve orifice with the measured actual area on operated sixty patients, the various elements which are influenced upon the C factor on evaluating the orifice flow were analyzed. The greater part of the calculated areas of the smaller orifice than 0.8cm² were smaller than the actual area of the mitral orifice, on the contrary such a pattern was recognized in the larger orifices than 0.8 cm² (Fig. 4).

1) Relationship between flow coefficient and cross-sectional area of the mitral orifice.

C factors were investigated in the five cases whose mitral valve orifice were 0.16cm², 0.42cm², 0.66cm², 0.71cm² and 1.51cm² (Fig.5). Five parabolic curve are plotted from actual mitral valve flows and calculated flow coefficient C. The more increased mitral valve flow, the larger became discharge flow coefficient. The coefficients were larger in the small mitral valve orifice, and they were within 1.0 in the orifice of which area is 1.51cm². Velocity of blood flow across the stenotic mitral valve orifice was between 100cm/sec and 400cm/sec, and so Reynolds number \( \text{Re} = \rho U_0 D_0 / \mu \) was between \( 10^2 \) and \( 10^4 \), within allowable limit. These tendencies were recognized in the experiment of the orifice model shown in the figure 3. The sizes of the orifice were 2.8mm and 7.0mm of diameter, and throat area ratio, m were 0.0064 and 0.0424. There were allowable limit of the former between \( 10^3 \) and \( 10^4 \), allowable limit of the latter between \( 10^4 \) and \( 10^5 \); the former was larger than the latter, namely it is very interesting that the flow coefficient in the small orifice was larger than in the large orifice in the experiments as well as in the clinical cases.

2) Relationship between flow coefficient and the configuration of mitral valve orifice.

The configuration of the mitral orifice was similar to the so-called rounded edge orifice, and it was investigated whether the contraction of isising flow across the throat were affected by the differences in the shape of the mitral orifice. The configuration of the mitral orifice was classified in the three types. The first type, I (a=b) was similar to a sort of circle shape orifice, and there were 19 cases (33.9%) in this type. The second, II (a<2b<3a) were mild oval shape orifice, and there were most of all in this type, 29 cases (51.8%). The third (III) type was severe oval shape (0≤b≤a/2), and there were 6 cases (10.7%). The type IV was the other configuration of the mitral orifice, and there were 2 cases (3.6%), fewest of all (Fig. 7).

Discharge flow coefficients were investigated in these three types except IV type. Coefficient C was larger than 1.0 in the greater part of the first type (84.2%) except 3 cases (15.8%), but it was smaller than 1.0 in the greater part of the II type (83.3%) except one case. C factor was smaller than 1.0 in 20 cases (68.9%), and was larger than 1.0 in 9 cases (31.1%). Namely, the more sever oval is the configuration of the orifice, the more extreme contraction there occurs in the issuing stream across the orifice (Fig.8). Because the mechanism of a contraction of blood flow is converted to the inertia of fluid.

3) Relationships between C and mobility of mitral valve.

The mobility of anterior leaflet of mitral valve was investigated with Ultrasoundcardiography (UCG), and diastolic descent rate of anterior leaflet was determined. Coefficient C was calculated as small values in larger diastolic descent rate, and calculated as large values in smaller descent rate (Fig.9). Almost all C factor was larger than 1.0 in the descent rate less than 15mm/sec, and was smaller than 1.0 in the larger descent rate than 15mm/sec. The relationship between flow coefficient and diastolic descent rate describes a hyperbolic curve. The mitral valve orifice was descent situated from the annulus, and it is considered a sort of nozzled shape orifice.

**DISCUSSION**

The square-edged orifice in a pipe (Fig.2) causes a contraction of the jet down stream from the orifice opening in the hydraulic system. It is said that discharge flow coefficient C describe a sort of parabolic curve within \( 10^5 \) of Reynolds number, and are constant in the allowable limit beyond \( 10^5 \) (Fig.10). Flow coefficient C is smaller than 1.0 in the vena contracta. Does vena contracta phenomenon occur in the issuing stream through the stenotic mitral valve orifice? If the vena contracta is actually brought about in mitral stenosis, all flow coefficients should be under 1.0 such as Gorlin's formula. But actually flow coefficients are larger than 1.0 in about fifty-one percent of the patients. C factor is constant in mitral stenosis and also mitral regurgitation by Gorlin's formula, but it is not constant.
actually. C is a function between reynolds number and throat area ratio, that is, values of the coefficient are calculated larger as the mitral valve flow increase, and calculated smaller as the size of the mitral valve orifice is smaller. This clinical results as well as our experiments in vitro were different from the results in the so-called thin-edge orifice flow. Vena contracta is caused by the inertia of fluid and the configuration of orifice, and orifice flow brings about energy loss in hemodynamic system. Probably vena contracta will be not caused by stenotic mitral valve orifice when C is larger than 1.0. The reasons why no vena contracta is caused by the mitral valve configuration are considered as follows, the stenotic mitral valve orifices are situated downward from the annulus, and considered so-called nozzled shape. There is little accuracy of measurement of blood flows and intracardiac pressures (Fig.11) mitral valve orifices have various shapes; blood is a non-newtonian liquid, and so it is necessary to measure the apparent viscosity of blood (μa). But it is difficult to investigate all factors of hydrodynamics of non-newtonian fluid.

There is a positive correlation between diastolic descent rate and the cross-section area of the stenotic mitral valve orifice by T. Yamada by Gustafson, A. and there is a hyperbolic correlation between diastolic descent rate of anterior leaflet and flow coefficient C such as Fig.9. Vena contracta increase as the values of diastolic descent rate are larger, that is, the mobility of anterior leaflet of mitral valve is larger. But vena contracta does not probably occur in the small mitral valve orifice and in the small mobility of mitral leaflet.

It is generally considered that vena contracta is brought out in the stenotic valve orifice such as pulmonic stenosis, aortic stenosis, and mitral stenosis since Gorlin's papers. But actually vena contracta is not recognized in over fifty percent of the valvular stenosis, because jet stream through the orifice is caused by the funnel shaped orifice or a sort of nozzle-shaped orifice. On the other hand from a different point of view Gunner, R. M. reported the modified Gorlin's formula by left atrial pressure.

SUMMARY

Considering the configuration of the stenotic mitral valve orifice as a sort of pipe orifice in the hydraulic system, the above-mentioned formula has been derived from the usual form of continuity equation and energy equation. Vena contracta has not been caused by the configuration of mitral orifice in about fifty-one percent of mitral stenosis. This result is different from Gorlin's one. The reasons why vena contracta was not brought about in the small mitral valve orifice as follow; the greater part of the mitral valve orifices were situated downward from the annulus and considered as so-called nozzled shape; there are little precision in the measurement of blood flow and pressure; mitral valve orifices themselves have various configurations. And so flow coefficient C is not constant such as Gorlin's formula, but a function of Reynolds number and throat area ratio. This coefficient is attributable to the configuration of the mitral orifice, the velocity of mitral valve flow and the viscosity of blood flow.

REFERENCES

