QUANTITATIVE ESTIMATION OF AORTIC REGURGITATION
BY ANALOG COMPUTER ANALYSIS OF RADIOCARDIOGRAM

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A new method for the quantitative estimation of aortic regurgitation is
reported in this paper. A mathematical model of the transport process of a
radioisotope in a circulatory system with aortic regurgitation is developed and
a simulation circuit of the transport process using analog computer is given.
On the mathematical model, the regurgitation at the aortic valve is expressed
as a back flow from the aorta with a transport delay equivalent to one cardiac
cycle.

The rate of regurgitant flow, as given by the ratio of regurgitant flow to the
total outflow from the left heart, is determined after curve fitting between an
actual radiocardiogram and a theoretical one produced by the simulation
circuit for a moderate or large regurgitation, in cases where a definite change
appears on the radiocardiogram. It is difficult to determine the rate of
regurgitant flow for a small regurgitation of less than 30%.

As a result of the simulation study of many radiocardiograms with aortic
regurgitation, the following is made clear: (1) Rates of regurgitant flow
obtained by the simulation are well correlated to those measured by left
ventricular angiography; (2) A linear relationship exists between the com-
puted mean left heart volume and the mean left ventricular volume by
angiography; (3) In 31 radiocardiograms with isolated aortic regurgitation,
the computed mean left heart volume increases linearly in proportion to the
regurgitant volume per beat; (4) Pulmonary blood volume shows a slight but
significant increase and it does not show any correlation with the grade of
regurgitation; (5) Mean right heart volume does not increase and the ratio
of mean right heart volume to mean left heart volume is inversely correlated
with the rate of regurgitant flow.

Key Words:
Radiocardiogram
Analog computer
Circulatory mathematical model
Aortic regurgitant flow
Mean left heart volume
Mean right heart volume
Pulmonary blood volume

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1972.

Fig. 1. Simplified model of the circulatory system.
The injection compartment, right heart and left heart are approximated by mixing chambers and are shown as rectangles. The bellows-like pipes represent transport delays. The pulmonary and systemic circulations are each approximated by a mixing chamber with a transport delay. A regurgitant flow into the left heart, $\alpha$ of the total outflow ($Q_f$), is provided with a transport delay equivalent to one cardiac cycle ($T_R$). The amount of isotope at time "t" and the mean blood flow in each compartment are shown. Intracardiac shunts are also taken into consideration.

Injection of a radioactive tracer into one of the peripheral veins, followed by precordial recording of its passage through the cardiac chambers, produces a double-peaked curve. This curve shows the transport process of the tracer in the right and left heart and is called a "radioangiogram". It is a non-invasive and safe method and is proven to be useful for the diagnosis of heart diseases because distinctive changes appear on the curve.

Kuwahara et al. proposed mathematical models of the transport process in the circulatory system and analog simulation circuits for the quantitative analysis of radioangiograms with or without intracardiac shunts. Most radioangiograms were properly analyzed by the above mentioned method and various circulatory parameters were obtained. These clinical results were reported by Saito et al. However, it is inadequate to analyze radioangiograms with moderate or large regurgitation by using these mathematical models and simulation circuits because the models do not take aortic regurgitation into consideration.

Therefore, a mathematical model and an analog simulation circuit in which regurgitation at the aortic valve is taken into consideration will be first developed in this paper. Next, the validity and the limits of the model will be discussed through application to radioangiograms obtained from patients with aortic regurgitation. Clinical results will be shown and discussed.

**Theoretical Consideration for the Analysis of the Radioangiogram**

As reported in the previous papers, the whole circulatory system is approximated by a closed loop of four compartments which are the right heart, lungs, left heart and body. The right heart and left heart are represented by a single mixing chamber, and the pulmonary and systemic circulations are represented by a mixing chamber with a transport delay. The assumption of constant flow in the circulatory system is adopted; therefore, pulsatile flow and volume change by contraction of the heart are not taken into account.

In the case of aortic insufficiency, a part of the outflow flows back into the left heart from the aorta. This regurgitant blood is mixed with blood from the other chambers in the left heart and flows out into the aorta again after one cardiac cycle. From this point of view, a new model is developed modifying the basic model, as shown in Figure 1. Equivalent mean volume (V ml), concentration of the radioisotope (c(t) $\mu$Ci/ml), and flow in each chamber are shown. $\tau$ is the transport delay. Subscripts $r$, $p$, $l$, $b$ and $R$ refer to the right heart, lung, left heart, body and regurgitation, respectively. Aortic regurgitation can be expressed by a short circuit which starts from the aorta and comes back directly into the left heart with a transport delay of $\tau_R$ seconds. $\tau_R$ is defined by 60/(heart rate). The flow in the left heart is represented by F ml/second. As $k$ of the left heart flow (kF) leaks into the right heart in the presence of left to right shunt, the outflow from the left heart toward the body compartment is (1-k)F. In the presence of aortic regurgitation, $\alpha$ of the outflow, $\alpha(1-k)$F, flows back into the left heart. The flow in the body compartment, is, therefore, (1-$\alpha$)(1-k)F. The flow in the right heart is the sum of flows from the body and from the left heart through the left to right shunt and is expressed by (1-$\alpha$-k)F. If combined by right to left shunt, $k'$ of the right heart and left heart.
heart flow, $k' (1-\alpha+\alpha k) F$, leaks into the left heart, and the pulmonary blood flow is represented by $(1-\alpha+\alpha k) (1-k') F$. The flow into the left heart, $F$, is the sum of the pulmonary flow, the shunt flow from the right heart, and the regurgitant flow through the aortic valve.

In order to preserve safety and simplicity, the radiisotope is injected into a peripheral vein. Since the injection process and the transport process of the tracer from the injection site to the right heart have a large influence on the pattern of the radiodiagram, the injection compartment is expressed by a small mixing chamber into which a known amount of the radiisotope is injected at a constant speed. The transport process in this chamber can be expressed as follows:

$$V_t c_t(t) = F_0 i(t) dt - F_0 \int_0^t c_t(t) dt$$

$$i(t) = \begin{cases} \frac{1}{\tau}, & 0 \leq t \leq \tau \\ 0, & \tau < t \end{cases}$$

$$\int_0^\tau i(t) dt = 1$$

where

- $I$: total amount of the injected radiisotope ($\mu$Ci)
- $i(t)$: rate of injection ($\mu$Ci/sec)
- $\tau$: time required for injection (sec)
- $V_t$: volume of the injection compartment (ml)
- $c_t(t)$: concentration of radiisotope in the injection compartment ($\mu$Ci/ml)
- $F_0$: blood flow in the injection compartment (ml/sec)

Recirculation of the tracer into the injection compartment during the injection period is omitted, as the injection time is short enough in comparison with the transit time in the whole circulation.

The transport process of the tracer in the right heart can be expressed by the following equation:

right heart:

$$V_t c_t(t) = F_0 i(0) c_t(t) dt + (1-\alpha) (1-k) F_0^t c_b(t-\tau_b) dt$$

$$+ k F_0 c_1(t) dt - (1-\alpha+\alpha K) (1-k') F_0^t c_t(t) dt$$

$$- (1-\alpha+\alpha K) k' F_0^t c_t(t) dt$$

(2)

The basic concept of the equation is that the amount of radiisotope in the right heart at time "t", $V_t c_t(t)$, is the difference of the amount that flowed in and flowed out from this chamber during the t seconds. $F_0^t c(t) dt$ is the amount of the inflowed tracer from the injection compartment; $(1-\alpha)(1-k) F_0^t c_b(t-\tau_b) dt$ is the amount which returns back from the body compartment with the transport delay of $\tau_b$ seconds; $k F_0 c_1(t) dt$ is the amount which flows from the left heart through the left to right intracardiac shunt; $(1-\alpha+\omega K)(1-k') F_0^t c_t(t) dt$ and $(1-\alpha+\omega K) k' F_0^t c_t(t) dt$ are the amount of tracer flowing out from the right heart to the lung and from the right heart to the left heart through the right to left intracardiac shunt, respectively.

The transport process of the tracer in other compartments can be expressed by the following equations:

lung:

$$V_p c_p(t) = (1-\alpha+\omega K)(1-k') F_0^t c_t(t) dt$$

$$- (1-\alpha+\omega K)(1-k') F_0^t c_p(t) dt$$

(3)
left heart:

\[
V_i c_1(t) = (1-\alpha+\alpha k)(1-k') F f_0^t c_p(t-\tau_p) dt \\
+ (1-\alpha+\alpha k) k F f_0^t c_1(t) dt \\
+ \alpha(1-k) F f_0^t c_1(t-\tau_R) dt \\
- (1-k) F f_0^t c_1(t) dt - k F f_0^t c_1(t) dt
\]

body:

\[
V_b c_b(t) = (1-\alpha)(1-k) F f_0^t c_1(t) dt \\
- (1-\alpha)(1-\alpha k) F f_0^t c_b(t) dt
\]

In equation (4), \(\alpha(1-k) F f_0^t c_1(t-\tau_R) dt\) shows the amount of the tracer that flows back into the left heart through the incompetent aortic valve.

In these equations systemic blood flow, pulmonary blood flow, aortic regurgitant flow, left to right shunt flow and right to left shunt flow are shown by \((1-\alpha)(1-k) F\), \((1-\alpha+\alpha k)(1-k') F\), \(\alpha(1-k) F\), \(k F\) and \((1-\alpha+\alpha k) k F\), respectively. These equations are based on the assumption that complete mixing of the tracer takes place in each compartment.

Figure 2 shows a block diagram of the circulatory system, calculating transfer functions of the compartments by operating Laplace transformation on equations (1) to (5).

Time constants, marked \(T\)'s in the block diagram, can be expressed as follows:

\[
T_r = V_i/(1-\alpha+\alpha k) F \\
T_p = V_p/(1-\alpha+\alpha k)(1-k') F \\
T_1 = V_1/F
\]

\[T_b = V_b/(1-\alpha)(1-k) F\]

\[T_i = V_i/F_i\]

Based on their transfer functions, mean transit times in the input compartment, right heart, and left heart are represented by \(T_r\), \(T_p\), and \(T_1\), and mean transit times of the lung and body are represented by \(T_1\) and \(T_b\), respectively.

To obtain an analog simulation circuit of the circulatory system, we should note that the total amount of isotope injected during \(t\) seconds equals the sum of the isotope in the four compartments. This relation, necessary to obtain a stable operation of the analog simulation circuit, can be expressed as follows:

\[
F f_0^t c_i(t) dt = V_i c_i(t) + V_p c_p(t) + (1-\alpha+\alpha k)(1-k') F [f_0^t c_p(t) dt - f_0^t c_p(t-\tau_p) dt] + V_1 c_1(t) + \alpha(1-k) F [f_0^t c_1(t) dt - f_0^t c_1(t-\tau_R) dt] + V_b c_b(t) + (1-\alpha)(1-k) F [f_0^t c_b(t) dt - f_0^t c_b(t-\tau_b) dt]
\]

A radioangiogram can be defined as a dynamic representation of the counting rate in the right and the left heart at the same counting efficiency and is represented by the following equation:

\[r(t) = \gamma [V_i c_i(t) + V_1 c_1(t)]
\]

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where \( r(t) \) is the relative counting rate at time \( t \) and \( \gamma \) is the counting efficiency in each compartment. The radiocardiograms that satisfy equation (7) can be recorded under the special conditions that were discussed in the previous paper.\(^{13}\)

Figure 3 shows an analog circuit computer set up from equations (1) to (7). By changing the values of potentiometers and transport delay units of the computer, the influence of each parameter on the radiocardiogram can be easily observed. The effects of parameters \( \tau \), \( T_t \), \( T_r \), \( T_p \), \( T_{\tau} \), \( T_b \), \( \tau_b \), \( k \) and \( k' \), have already been discussed.\(^ {13}\) Here, the effects of the rate of aortic regurgitant flow (\( \alpha \)) and of the transport delay in the regurgitant circuit (\( \tau_R \)) on the radiocardiogram will be shown. Figure 4 shows the effect of \( \alpha \) on a radiocardiogram. With its increase the left heart wave gets higher and its descending slope is prolonged although no change appears in the right heart wave. If the backflow is 100% of cardiac output (\( \alpha=1.0 \)), the left heart wave reaches a plateau, showing that no tracer flows out into the systemic circulation. Regurgitation of less than 30% shows little change on the curve and it may not be helpful for the diagnosis of small regurgitation. Figure 5 shows the effect of \( \tau_R \). With the shortening of \( \tau_R \), the left heart wave gets higher and its descending slope steeper, if regurgitation rate and other parameters in the simulation circuit are fixed. From Figures 4 and 5, if radiocardiograms with different heart rates show the same pattern, the curve with the faster heart rate suggests a larger regurgitation.

Theoretical radiocardiograms can be obtained by adjusting various parameters of the circuit. Operation of the computer is tried again and again until the actual radiocardiogram obtained from a patient is successfully fitted with a theoretical curve. Then, values of the parameters \( T_r \), \( T_p \), \( T_{\tau} \), \( T_b \), \( \tau_b \), \( k \), \( k' \), and \( \alpha \) give information relating to the circulatory system. Figure 6 is a radiocardiogram of a 32-year-old male with isolated aortic regurgitation. The solid line is a theoretical curve by the analog computer and it is superimposed on the actually recorded curve.

Mean transit time multiplied by mean blood flow gives mean blood volume of each compartment:

right heart volume: \( V_r = (1-\alpha+\alpha k)FT_r \)
pulmonary blood volume:
\[
V_p = (1-\alpha+\alpha k')F(T_p+\tau_p)
\]
left heart volume: \( V_l = FT_t \)
body blood volume:
\[
V_b = (1-\alpha)(1-k)F(T_b+\tau_b)+\alpha(1-k)FT_R
\]

Circulating blood volume, \( V \) (ml), can be calculated from the following equation:
\[
V = I/c(\infty)
\]
where \( c(\infty) \) is the concentration of radioisotope in the equilibrium state and is measured by a well-type counter. As circulating blood volume
Fig. 6. Radiocardiogram of a 32-year-old male with isolated aortic regurgitation. A simulated curve by the analog computer (solid line) is superimposed on the recorded curve. Clinical data obtained by the two different methods are as follows.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>RCG computer data</th>
<th>LV-raphy &amp; Fick method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cardiac output</td>
<td>5.86 L/min</td>
<td>4.47 L/min</td>
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<tr>
<td>Mean right heart volume</td>
<td>120 ml/m²</td>
<td>——</td>
</tr>
<tr>
<td>Mean left heart volume</td>
<td>434 ml/m²</td>
<td>——</td>
</tr>
<tr>
<td>Pulmonary blood volume</td>
<td>323 ml/m²</td>
<td>——</td>
</tr>
<tr>
<td>LV end-diastolic volume</td>
<td>——</td>
<td>342 ml/m²</td>
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<tr>
<td>LV end-systolic volume</td>
<td>——</td>
<td>169 ml/m²</td>
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<td>Total stroke volume</td>
<td>325 ml/beat</td>
<td>249 ml/beat</td>
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<tr>
<td>Effective stroke volume</td>
<td>97 ml/beat</td>
<td>73 ml/beat</td>
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<tr>
<td>Regurgitation volume</td>
<td>227 ml/beat</td>
<td>176 ml/beat</td>
</tr>
<tr>
<td>Regurgitation rate</td>
<td>70 %</td>
<td>70 %</td>
</tr>
<tr>
<td>Heart rate</td>
<td>60 beat/min</td>
<td>61 beat/min</td>
</tr>
</tbody>
</table>

is equal to the sum of blood volume in each compartment, \( V \) is expressed as follows:

\[
V = (1-\alpha + \alpha k)FT_a + (1-\alpha + \alpha k)(1-k')F(T_p + \tau_p)
+ FT_a + (1-\alpha)(1-k)F(T_b + \tau_b) + \alpha(1-k)F \tau_R
\]

Consequently,

\[
F = V/((1-\alpha + \alpha k)T_a + (1-\alpha + \alpha k)(1-k')T_p + \tau_p)
+ T_a + (1-\alpha)(1-k)(T_b + \tau_b) + \alpha(1-k)\tau_R \]  \(8\)

All values of the parameters in the right side of equation (8) can be determined by the curve fitting of a theoretical curve on the recorded radiocardiogram. \( F(\text{ml/sec}) \), thus calculated, gives mean blood flow in the left heart. Effective flow from the left heart to the systemic circulation can be expressed as \( (1-\alpha)(1-k)F \), and aortic regurgitant flow as \( \alpha(1-k)F \). The percentage of aortic regurgitant flow when compared to the total outflow from the left heart is given by \( \alpha \times 100\% \).

**MATERIALS AND METHODS**

A radiocardiogram and an input curve were recorded by the procedure and with the equipment described in the previous report.\(^{14}\)

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TABLE 1  AORTIC INSUFFICIENCY: RESULTS OF SIMULATION ANALYSIS OF RADIOCARDIOGRAPHS

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Age (yr)</th>
<th>BSA (m²)</th>
<th>HR (b/min)</th>
<th>LHV (ml)</th>
<th>PBV (ml/m²)</th>
<th>PTT (sec)</th>
<th>RHV (ml/m²)</th>
<th>Eff. SV (ml/beat)</th>
<th>Reg. V (ml/beat)</th>
<th>% Reg.</th>
<th>RHV/LHV (%)</th>
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BSA = body surface area,  HR = heart rate,  LHV = mean left heart volume,  PBV = pulmonary blood volume,  PTT = mean pulmonary transit time,  RHV = mean right heart volume,  Eff. SV = effective stroke volume,  Reg. V = regurgitant volume per beat,  % Reg. = rate of regurgitant flow,  RHV/LHV = ratio of mean right heart volume to mean left heart volume.

To verify the validity of the present method, some of the radiocardiographic data were compared with those obtained by cardiac catheterization (Fick procedure and left ventriculography). Studies were made on eleven patients with aortic regurgitation (from cases No.1 to No.11 in Table I and Table II). Aortic stenosis was combined in three cases (case No.3, No.10 and No.11). Patients with aortic regurgitation combined with mitral valvular disease were excluded from this

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study. The radiocardiogram was recorded within four days before cardiac catheterization, and the difference of heart rate between the two examinations was less than 10 beats per minute in 9 cases, and 17 beats in 2 case. Opacification of the left ventricular cavity was performed by injection of the contrast material (80% Angioconray®) into the left ventricle or into the ascending aorta. Serial 30 films were taken at the rate of 6 films per second. Left ventricular end-diastolic volume (LVEDV) and end-systolic volume (LVESV) were calculated by the area-length method of Sandler and Dodge. Films during and one beat after premature contraction were discarded. The subtraction of LVESV from LVEDV gives the left ventricular stroke volume (LVSV). Effective stroke volume (Eff.SV) was obtained by the direct Fick method, which was performed just before the angiography. If the difference of heart rate was more than 5 beats per minute between the left ventriculography and the Fick procedure, the patient was excluded from this study. Aortic regurgitant volume per beat (Reg.V) and the rate of aortic regurgitant flow (%Reg) were calculated as follows:

\[ \text{Reg.V} = \text{LVSV} - \text{Eff.SV} \]
\[ \% \text{Reg} = \left( \frac{\text{Reg.V}}{\text{LVSV}} \right) \times 100 \]

The rate (%Reg) was compared with that obtained by the computer analysis of the radiocardiogram. Mean left ventricular volume (MLVV) was calculated as follows: \(^{18}\)

\[ \text{MLVV} = \text{LVEDV} - \frac{\text{LVSV}}{3} \]

and compared with mean left heart volume by radiocardiogram.

Radiocardiograms, recorded from 31 patients with aortic regurgitation, were analyzed by the present analog computer. Mean right heart volume and left heart volume, pulmonary blood volume, transit times of right and left heart, pulmonary transit time, effective systemic flow and effective stroke volume, regurgitant flow, and rate of regurgitant flow were obtained. Although catheterization studies were not made in 20 patients, any combination of mitral valvular disease or congenital heart diseases with intracardiac shunts was carefully ruled out by chest X-ray films, electrocardiograms, phonocardiograms, echocardiograms and vectorcardiograms.

**Results**

Clinical data obtained by the simulation analysis of 31 radiocardiograms are summarized in Table 1. Table II shows the results of the catheterization study which was performed in cases No.1 to No.11.

In Figure 7 the rates of regurgitant flow computed by the simulation analysis of radiocardiograms are compared with those measured by...
the left ventriculography and Fick method. A linear relationship was found between them and the coefficient of correlation was 0.95 (p<0.01). A good agreement was found especially in 8 cases with regurgitation of larger than 40% (simulation method/angiography-Fick method = 0.96±0.12 (mean±SD)).

A linear relationship was found between computed mean left heart volumes by the present method and mean left ventricular volumes calculated from left ventriculograms, as shown in Figure 8. The coefficient of correlation was 0.92 (p<0.01), and the regression equation was y=0.91x−117.

Computed mean left heart volume was found to range from 125 to 566 ml/m² of body surface area in the 31 radiocardiograms. The normal value is 147±29 ml/m² (mean±SD). As shown in Figure 9, mean left heart volume showed a linear increase in proportion to the regurgitant volume per beat. The coefficient of correlation was 0.80 (p<0.01) and the regression equation was y=0.37x−41. Case No.1 was omitted in Figure 9 and in the following studies for reasons discussed later.

Pulmonary blood volume, computed from 30 radiocardiograms of aortic regurgitation, ranged from 213 to 496 ml/m² (337±50 ml/m² (mean±SD)), a significant (p<0.01) increase over the normal (290±43 ml/m²). Mean pulmonary transit time was 5.8±1.2 seconds (mean±SD) and was significantly (p<0.001) longer than the normal (4.2±0.6 sec.). However, neither the increase nor the prolongation correlated with the rate of regurgitant flow. Pulmonary blood flow was 3.64±0.71 L/min/m² (mean±SD) and was slightly (p<0.01) under the normal (4.19±0.54 L/min/m²).

Mean right heart volume (162±26 ml/m² (mean±SD)) did not show a significant increase in comparison with the normal (144±25 ml/m²) and did not correlate with the rate of regurgitation.

As shown in Figure 10, a linear relationship was found between the rate of aortic regurgitant flow and the ratio of mean right heart volume to mean left heart volume. The coefficient of correlation was −0.85 (p<0.01) and the regression equation was y=−76x +92.

**DISCUSSION**

Characteristic changes appear on the radiocardiogram of a moderate or large aortic regurgitation — a normal right heart wave and a high left heart wave with prolonged descending slope. This configuration suggests the retarded disappearance of the radioisotope from the left heart by its to and fro movement between the left ventricle and the aorta. Simulation analysis of the abnormal hemodynamics was performed successfully using the new model. The same concept can be useful in simulating the regurgitation at the pulmonary
valve, although its clinical usefulness has not been studied yet. Because of the simple expression of the heart in the model with no separation of the atrium and the ventricle, it is impossible to extend the model to regurgitation from the ventricle into the atrium.

The analysis of the actual radiocardiogram recorded from a patient is made by the curve fitting technique based on the visual comparison of the actual and the theoretical curves. Therefore, it is to be desired that simulation data obtained independently by two operators from the same actual radiocardiogram show satisfactory reproducibility. The value of $\alpha$ in 14 radiocardiograms selected at random showed a good agreement, the coefficient of correlation being 0.96 and the ratio of $\alpha$ by the two operators being $0.99 \pm 0.13$ (mean$\pm$SD). The reproducibility

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**Fig. 9.** Relation between mean left heart volume and aortic regurgitant volume per beat.

**Fig. 10.** Relation between the rate of regurgitant flow and the ratio of mean right heart volume to mean left heart volume.

with regard to other parameters is discussed in
the previous paper.\textsuperscript{3}

A rate of regurgitation larger than about 30% can be detected by the present method. Smaller regurgitation makes trivial changes in the radiocardiogram, and it is impossible to determine the rate of regurgitant flow. If the ratio of right heart volume to left heart volume is in the normal range, it suggests slight or little enlargement of the left ventricle and the grade of regurgitation must be small.

The simple expression of the heart in the present model is responsible for the discrepancy between the computed mean left heart volume by radiocardiograms and the mean left ventricular volume by ventriculograms. As the left heart is approximated by a mixing chamber including the atrium and the ventricle in one, each separate volume cannot be calculated. The mean left heart volume gives the sum of left atrial and ventricular volumes. Although the flow in the left heart is expressed by $F$ in this model, the actual flow in the left atrium and the left ventricle should be $(1-\alpha)F$ and $F$ respectively, if the aortic regurgitant flow is $\alpha F$. For these reasons, the computed mean left heart volume was larger than the mean left ventricular volume as in Fig. 8.

A linear relationship between mean left heart volumes and regurgitant volumes was found in subjects who did not have a history of heart failure and could be compared with the relation between left ventricular end-diastolic volumes and regurgitant volumes.\textsuperscript{9} However, catheterization study of Case No.1 showed the severe increase of both end-diastolic and end-systolic volumes of the left ventricle. The total left ventricular stroke volume and the regurgitant volume were relatively small in spite of the large percent regurgitation. This suggests the presence of left ventricular failure and Case No.1 was discarded in the studies about the relation between the flow and volume.

The ratio of mean right heart volume to mean left heart volume can be an index of the grade of aortic regurgitation if it is not associated by other congenital and valvular heart diseases or left heart failure, because the right heart volume shows no significant increase while the left heart volume increases with linear relation to the grade of regurgitation.

The normal value of pulmonary blood volume obtained by the present method shows a fairly good agreement with those reported by other workers\textsuperscript{20–23} Moreover, the slight but significant increase of pulmonary blood volume in the present study of aortic regurgitation is consistent with other reports\textsuperscript{20,24} and is accompanied by the prolongation of mean pulmonary transit time and slight decrease of pulmonary blood flow.

A rapid increase and retarded decrease in the counting rate of the left heart can be observed not only in aortic regurgitation but also in left to right shunt.\textsuperscript{4} The differentiation can be made as follows: in patients with shunt, the pulmonary blood flow increases in proportion to the size of the shunt flow and the pulmonary transit time is shortened. This results in a short peak to peak time on the radiocardiogram. This finding has never been observed in the case of aortic regurgitation.

This noninvasive and safe procedure can be performed on any severely ill patient without changing the performance of the heart. Repeated examination is not hazardous and long-term observation is easily made. By a single shot of the tracer, information about not only the regurgitation but also the left heart, right heart, pulmonary and systemic circulation, and the intracardiac shunt can be obtained. Due to these merits, this method is used in our hospital as a routine examination for cardiac patients.

REFERENCES


