Applications of Multipole Expansion

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With use of a homogeneous torso model, dipole and quadrupole sensitivity was determined for unipolar leads from many sites on the thoracic surface. Representatives of non-dipolar maps were obtained from the sensitivity of each of unipolar leads to the 5 rectangular components of the quadrupole. Over-all sensitivity to the quadrupole or the magnitude of the lead tensor was calculated from the measurements and mapped over the torso surface. This map indicates relative proximity of the thoracic surface and gives a theoretical basis for the selection of electrode locations in the surface mapping technique.

ADVANTAGES of taking multiple surface electrocardiograms are based on the existence of non-dipolar components of the cardiac generator. An authodox approach to non-dipolar informations is to consider multipole coefficients of the generator. Since the quadrupole is the first non-dipolar term, the sensitivity of the surface leads to components of the quadrupole may be related to representative patterns of isopotential lines. The over-all sensitivity to the quadrupole will be an index for non-dipolar informations contained in the lead. This study deals with these aspects of applications of the quadrupole term. Other applications of the quadrupole theory will be given elsewhere.

METHODS

A torso model of real size was used for the experimental determination of the lead tensor of unipolar leads from many thoracic sites. The 18 electrode system was constructed for the probe which consisted of small electrodes attached on the spherical surface with a radius of 4.0 cm. Six electrodes were placed at the 6 ends of 3 diameters in x, y and z directions and other 12 electrodes at the midpoints of the arcs between these 6 electrodes. The location of the electrodes is the same as the lead system for the isolated

Key Words:
- Torso model
- Quadrupole
- Non-dipolar components
- Lead tensor
- Lead proximity

Fig.1. Combinations of electrodes of 18 electrode system for the construction of rectangular dipole and quadrupole components. Upper 3 are for the dipole and other 6 are for the quadrupole components. Open circles and dots are positive and negative electrodes, respectively.

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heart described previously? Combinations of these electrodes shown in Fig. 1 correspond to rectangular components of dipole and quadrupole. In Fig 1, open circles are the positive electrodes and dots are the negative electrodes. Upper 3 are for the dipole components x, y and z. Lower 6 are quadrupole components, one of which is not independent because of the relation \((YY - ZZ) + (ZZ - XX) + (XX - YY) = 0\). Hence, the combination \(XX - YY\) was omitted from the measurements. Open circles and dots in quadrupole components are connected through equal resistors to give positive and negative terminals, respectively.

The torso was filled with tap water and the electrodes system was placed at the heart region. The center of the spherical probe was deviated 2.5 cm leftwards and 2.5 cm anteriorly from the geometrical center of the horizontal section of the torso. With 156 locations on the torso surface, unipolar leads were constructed with Wilson central terminal. Each of these leads was reciprocally energized with a constant current pulse of 10 to 20 msec duration. Measurements were made for the height of the pulse obtained with each of the combinations of electrodes in Fig. 1. Hence, 3 dipolar and 5 quadrupolar measurements were made for each of the locations on the torso.

Fig. 2. Sensitivity of the torso surface to rectangular components of the quadrupole shown in Fig. 1. Relative values are indicated by percentage to the maximal value of all the measurements.

Fig. 3. The magnitude of the lead tensor of unipolar leads from the thoracic surface. Attached numbers indicate the percentage to the maximal value.

RESULTS

Figure 2 shows relative sensitivity of various locations to 5 rectangular components of the quadrupole. Numbers indicate the percentage to the maximal value of all the quadrupole measurements. In general, each of the quadrupole components was associated with double positive and negative peaks. Patterns in Fig. 2 may be regarded as regular components of non-dipolar maps with respect to the conventional x, y and z coordinates.

TABLE I  RELATIVE DIPOLE AND QUADRUPOLE SENSITIVITIES AND THEIR RATIO OF CONVENTIONAL LEADS. LEAD I IS NORMALIZED AS UNITY

<table>
<thead>
<tr>
<th></th>
<th>Dipole Sensitivity</th>
<th>Quadrupole Sensitivity</th>
<th>Q/D ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>II</td>
<td>1.35</td>
<td>1.11</td>
<td>0.82</td>
</tr>
<tr>
<td>III</td>
<td>1.53</td>
<td>1.33</td>
<td>0.87</td>
</tr>
<tr>
<td>V₁</td>
<td>1.48</td>
<td>3.72</td>
<td>2.51</td>
</tr>
<tr>
<td>V₂</td>
<td>1.81</td>
<td>5.67</td>
<td>3.13</td>
</tr>
<tr>
<td>V₃</td>
<td>2.02</td>
<td>4.89</td>
<td>2.41</td>
</tr>
<tr>
<td>V₄</td>
<td>1.92</td>
<td>3.91</td>
<td>2.04</td>
</tr>
<tr>
<td>V₅</td>
<td>1.48</td>
<td>2.44</td>
<td>1.65</td>
</tr>
<tr>
<td>V₆</td>
<td>1.15</td>
<td>1.55</td>
<td>1.35</td>
</tr>
</tbody>
</table>

To express the quadrupole sensitivity as the lead tensor of the 2nd rank as

\[
\begin{pmatrix}
  A & F & E \\
  F & B & D \\
  E & D & C \\
\end{pmatrix}
\]

elements A to F are related to the measured values as

\[ A = \frac{1}{3} [- (YY - ZZ) - 2(ZZ - XX)] \]
\[ B = \frac{1}{3} [2(YY - ZZ) + (ZZ - XX)] \]
\[ C = \frac{1}{3} [- (YY - ZZ) + (ZZ - XX)] \]
\[ D = \frac{1}{2} (YZ) \]
\[ E = \frac{1}{2} (ZX) \]
\[ F = \frac{1}{2} (XY) \]

where \( A + B + C = 0 \). The over-all quadrupole sensitivity can be defined from the moment of the tensor Q:

\[ Q^2 = \frac{1}{2} (A^2 + B^2 + C^2 + D^2 + E^2 + F^2) \]

The Q value was calculated for each of the leads and mapped in Fig. 3, where the relative tensor magnitude was shown as the percentage to the maximal value. The map in Fig. 3 is the proximity map of the thoracic surface in the first approximation. It can be seen that non-dipolar informations are concentrated, as expected, to the precordial area near the heart.

Table I shows relative values of the lead vector, lead tensor of the 2nd rank and their ratio. Values of lead I were normalized as unity. The sensitivity to non-dipolar components was the largest in \( V₂ \) and was about 3 times as in lead I.

DISCUSSION

Only the first 2 terms of multipole expansion were used in this study. Higher order terms may have influences on the leads from the vicinity of the heart. In the surface potential, however, the number of independent informations are limited, as pointed out by Dr. Abildskov in this symposium, because of the measurement errors and noise. Other authors\(^3,4\) also have shown that the time course of the potential consists of several major components. Available data suggest that 2 or 3 components are predominant and additional several components seem to constitute most of the residual potentials. Hence, quadrupole components seem to be major elements of non-dipolarity. Patterns in Fig. 2 may be regarded as representatives of non-dipolar components. Similar maps have appeared in the literature\(^5\). But Fig. 2 is based on the rectangular components of the quadrupole in Fig. 1 and their geometrical meaning is clearer.

The proximity and the lead tensor have been studied by Brody and his associates\(^6,7\). They constructed approximates of flow lines of certain orthogonal leads. In the present study, relative proximity of different locations on the thoracic surface was of main interest and many unipolar leads were examined, although higher order terms were truncated. In order to pick up informations efficiently from the body surface, an
adequate selection of the number and locations of electrodes is necessary. The proximity map in Fig. 4 implies that the electrodes should be distributed in proportion to the proximity indicated. The map will give a practical guideline based on the theoretical background.

Unipolar leads were constructed with Wilson central terminal in this study. Although the central terminal is not an ideal reference, the results obtained here are reasonable at least for regions with greater proximity. For some of the components such as YY–ZZ, residual potential may be present at lower part of the body because of the errors of the central terminal. Other methods such as analytical calculations are possible for the determination of the lead tensor. But these methods are also susceptible to various inherent errors when applied to the complex configuration and structures of the human body. Hence, we adopted the experimental approach with Wilson terminal, which would be of practical significance.

REFERENCES
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