Angle estimation using combined Unitary ESPRIT-MUSIC algorithm for MIMO radar

Wei Wang^{a)}, Xianpeng Wang, Moxuan Li, and Xin Li

College of Automation, Harbin Engineering University, Harbin, 150001,
Heilongjiang P.R, China

a) chinaww2006@yahoo.com.cn

Abstract: A new Unitary ESPRIT-MUSIC algorithm for joint direction of departure (DOD) and direction of arrival (DOA) in bistatic MIMO radar is presented. The received data matrix is extended to a new matrix which is a centro-Hermitian matrix. A unitary transformation can be applied which converts a complex-valued matrix into a real-valued matrix by taking the advantage of the property of centro-Hermitian matrix. Then the DODs and DOAs are estimated by Unitary ESPRIT and Unitary Root-MUSIC, respectively. Compared with the combined ESPRIT-MUSIC method, the proposed method provides better performance and lower computational complexity. Some simulation results are presented to verify the efficiency of the proposed method.

Keywords: MIMO radar, bistatic, Unitary ESPRIT-MUSIC, angle estimation

Classification: Antennas and Propagation

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1 Introduction

Multiple-input multiple-output (MIMO) radar utilizes multiple antennas to simultaneously transmit diverse waveforms and receive the reflected signals in similar ways [1]. Angle estimation is very important aspect in MIMO radar. In allusion to this problem, an ESPRIT algorithm is presented in [2], which exploits the invariance property of both the transmit array and receive array to estimate DODs and DOAs. The DODs and DOAs are automatically paired. However, the received data of $N(M - 1)$ and $M(N - 1)$ elements is only used for DOD and DOA estimation, respectively, where M and N is the elements of transmit array and receive array, respectively, so it leads some loss of angle estimation performance. In [3], a combined ESPRIT-MUSIC approach which uses the ESPRIT method to estimate the DOD and the Root-MUSIC method to estimate the DOA is presented, and the angle performance is improved. When both of transmit array and receive array is larger, the heavy computational complexity also is a key problem. In this letter, a novel combined Unitary ESPRIT-MUSIC algorithm for joint DOD and DOA estimation is presented, which has following advantages: (1) owing to deal with real-valued processing, it provides lower computational complexity than combined ESPRIT-MUSIC; (2) owing to the extended data matrix which contains double the number of data samples, it provides better angle estimation performance than combined ESPRIT-MUSIC, especially in low data samples; (3) the DODs and DOAs are automatically paired.

2 Problem formulation

Consider a narrowband bistatic radar system with M closely spaced transmit antennas and N closely spaced receive antennas, both of which are half-wavelength spaced uniform linear arrays and all the elements are omnidirectional. It is assumed that the Doppler frequencies have almost no effect on the orthogonality of the waveforms and the variety of the phase within repetition intervals. Assume that there are P uncorrelated targets, and the p th target located at (φ_p, θ_p) , where φ_p denotes the direction of the p th target with respect to the transmit array (i.e. DOD) and θ_p denotes the direction of the p th target with respect to the receive array (i.e. DOA). For the P uncorrelated targets are located at the same range bin, the output of the matched filters at the receiver can be written as [2, 3]

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_P]$ is an $MN \times P$ matrix composed of the P transmit-receive steering vectors, $\mathbf{a}_p = \mathbf{a}(\varphi_p) \otimes \mathbf{a}(\theta_p)$ ($p = 1, 2, \dots, P$) is the transmit-receive steering vector of the p th target, $\otimes$ denotes the Kronecker product, $\mathbf{s}(t) = [\beta_1 e^{j2\pi f_{d1}t}, \dots, \beta_P e^{j2\pi f_{dP}t}]$, β is the reflection coefficient depending on the target radar cross section and f_d is the Doppler frequency, $\mathbf{a}(\theta) = [1, \exp(j\pi \sin \theta), \dots, \exp(j\pi(N - 1) \sin \theta)]^T$ and $\mathbf{a}(\varphi) = [1, \exp(j\pi \sin \varphi), \dots, \exp(j\pi(M - 1) \sin \varphi)]^T$ are the receive steering vector and the transmit steering vector, respectively, $\mathbf{n}(t)$ is an $MN \times 1$ complex Gaussian white noise

vector with zeros means and covariance matrix $\sigma^2 \mathbf{I}_{MN}$, where $\mathbf{I}_K$ denotes a $K \times K$ identify matrix.

3 Unitary ESPRIT-MUSIC for joint DOD and DOA estimation

We assume that the number of snapshots is L , and the received data matrix can be formulated as $\mathbf{X} = [\mathbf{x}(t_1), \mathbf{x}(t_2), \dots, \mathbf{x}(t_L)]$. The extended data matrix of the received data matrix is defined as $\mathbf{Y} = [\mathbf{X} \quad \mathbf{\Gamma}_{MN} \mathbf{X}^* \mathbf{\Gamma}_L]$, where $\mathbf{\Gamma}_k$ denotes the $K \times K$ exchange matrix with ones on its anti-diagonal and zeros elsewhere and $(\cdot)^*$ denotes complex conjugate of matrix or vector. It has been improved that the extend data matrix $\mathbf{Y}$ is a centro-Hermitian matrix [4]. The complex-valued extended data matrix is transformed into real-valued matrix by using unitary transformation, which is written as

$$\mathbf{Y}_{real} = \mathbf{U}_{MN}^H [\mathbf{X} \quad \mathbf{\Gamma}_{MN} \mathbf{X}^* \mathbf{\Gamma}_L] \mathbf{U}_{2L} \quad (2)$$

Where $\mathbf{Y}_{real}$ is a real-valued data matrix which contains double the number of data samples, $(\cdot)^H$ denotes the Hermitian transpose, $\mathbf{U}_K$ is a unitary matrix which is defined as

$$\mathbf{U}_{2K} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_K & j\mathbf{I}_K \\ \mathbf{\Gamma}_K & -j\mathbf{\Gamma}_K \end{bmatrix} \quad (3)$$

and

$$\mathbf{U}_{2K+1} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_K & 0 & j\mathbf{I}_K \\ \mathbf{0}^T & \sqrt{2} & \mathbf{0}^T \\ \mathbf{\Gamma}_K & 0 & -j\mathbf{\Gamma}_K \end{bmatrix} \quad (4)$$

According to equation (2), the ML estimation of the real-valued covariance matrix of $\mathbf{Y}_{real}$ for $2L$ snapshots is written as $\mathbf{R}_{real} = 1/(2L) \mathbf{Y}_{real} \mathbf{Y}_{real}^H$. Let $\mathbf{E}_s$ be the $MN \times P$ signal subspace matrix which is the eigenvectors corresponding to the P largest eigenvalues of $\mathbf{R}_{real}$. Let $\mathbf{E}_n$ be the $MN \times (MN - P)$ noise subspace matrix which are the eigenvectors corresponding to the smallest eigenvalues of $\mathbf{R}_{real}$. After using unitary transformation, the relationship between real-valued transmit-receive steering vector and complex-valued transmit-receive steering vector is $\mathbf{g}_p = \mathbf{U}_{MN}^H \mathbf{a}_p$, ($1 \leq p \leq P$) [4]. The real-valued transmit-receive steering matrix can be defined as $\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_P]$. The real-valued noise subspace $\mathbf{E}_n$ is orthogonal to the real-valued transmit-receive steering vector [5], i.e. $\|\mathbf{E}_n^H (\mathbf{U}_{MN}^H [\mathbf{a}(\varphi) \otimes \mathbf{a}(\theta)])\|^2$, which can be expressed as

$$\mathbf{a}(\theta)^H [\mathbf{a}(\varphi) \otimes \mathbf{I}_N]^H \mathbf{U}_{MN} \mathbf{E}_n \mathbf{E}_n^H \mathbf{U}_{MN}^H [\mathbf{a}(\varphi) \otimes \mathbf{I}_N] \mathbf{a}(\theta) = 0 \quad (5)$$

According to expression (5), the DOD and DOA can be estimated separately in real-valued space. Firstly, we determine the DOD by using the Unitary ESPRIT. Define $\mathbf{A}_{t1} = \mathbf{\Pi}_1 \mathbf{A}$ and $\mathbf{A}_{t2} = \mathbf{\Pi}_2 \mathbf{A}$, where $\mathbf{A}_{t1}$ and $\mathbf{A}_{t2}$ are the $N(M-1) \times P$ submatrices of $\mathbf{A}$ consisting of the first and the last $N(M-1)$ rows of $\mathbf{A}$, $\mathbf{\Pi}_1 = [\mathbf{I}_{N(M-1)} \quad \mathbf{F}]$ and $\mathbf{\Pi}_2 = [\mathbf{F} \quad \mathbf{I}_{N(M-1)}]$ are selection matrices, $\mathbf{F}$ is $N(M-1) \times N$ matrix with zeros. Then $\mathbf{A}_{t2} = \mathbf{A}_{t1} \mathbf{\Phi}_t$, where $\mathbf{\Phi}_t$ is a diagonal matrix with diagonal elements $\gamma_p = \exp(j\pi \sin \varphi_p)$ for $p = 1, 2, \dots, P$, which represents the complex-valued shift invariance relation.

Due to the centrosymmetric array configuration of bistatic MIMO radar, the complex-valued shift invariance relation can be turned into the real-valued shift invariance relation [4]:

$$\mathbf{D}_2 \mathbf{G} = \mathbf{D}_1 \mathbf{G} \mathbf{\Psi}_t \quad (6)$$

where $\mathbf{D}_1 = \text{Re}\{\mathbf{U}_{N(M-1)} \mathbf{\Pi}_2 \mathbf{U}_{MN}\}$ and $\mathbf{D}_2 = \text{Im}\{\mathbf{U}_{N(M-1)} \mathbf{\Pi}_2 \mathbf{U}_{MN}\}$ are real-valued matrices, $\mathbf{\Psi}_t$ is real-valued diagonal matrix with diagonal elements $\lambda_p = \tan(\pi \sin \varphi_p / 2)$ for $p = 1, 2, \dots, P$. Let $\mathbf{E}_{s1} = \mathbf{D}_1 \mathbf{E}_s$ and $\mathbf{E}_{s2} = \mathbf{D}_2 \mathbf{E}_s$, then the diagonal elements of $\mathbf{\Psi}_t$ are the eigenvalues of $\mathbf{\Theta}_t = \mathbf{T}^{-1} \mathbf{\Psi}_t \mathbf{T}$ which satisfy $\mathbf{E}_{s2} = \mathbf{E}_{s1} \mathbf{\Theta}_t$. The DOD can be estimated as $\hat{\varphi}_p = \arcsin(2 \arctan(\lambda_p) / \pi)$, and the estimated transmit steering vector for the p th target can be written as

$$\mathbf{a}(\hat{\varphi}_p) = [1, \exp(j\pi \sin \hat{\varphi}_p), \dots, \exp(j\pi(M-1) \sin \hat{\varphi}_p)]^T \quad (7)$$

According to each estimated transmit steering vector, by substituting the expression (7) in (5), a Unitary Root-MUSIC case is obtained. The receive steering vector $\mathbf{a}(\theta)$ can be rewritten as

$$\mathbf{a}(z_r) = [1, z_r, \dots, z_r^{N-1}]^T \quad (8)$$

where $z_r = \exp(j\pi \sin \theta)$. For the estimated transmitter steering vector $\mathbf{a}(\hat{\varphi}_p)$ of the p th target, the DOA of the p th target is equivalent to find the root $\hat{z}_p$ inside and closest to the unitary circle of the following polynomial

$$\mathbf{a}(z_r^{-1})^T \mathbf{B}^{(p)} \mathbf{a}(z_r) = 0 \quad (9)$$

where $\mathbf{B}^{(p)} = [\mathbf{a}(\hat{\varphi}_p) \otimes \mathbf{I}_N]^H \mathbf{U}_{MN} \mathbf{E}_n \mathbf{E}_n^H \mathbf{U}_{MN}^H [\mathbf{a}(\hat{\varphi}_p) \otimes \mathbf{I}_N]$. Then the DOA of the p th target can be derived as $\hat{\theta}_p = \arcsin(\angle(\hat{z}_p) / \pi)$, where $\angle(r)$ denotes the phase of r . Due to the p th DOA derived from the p th DOD, the DODs and DOAs are paired automatically. Up to now, we have proposed the Unitary ESPRIT-MUSIC algorithm for angle estimation in bistatic MIMO radar, and the same as to ESPRIT and ESPRIT-MUSIC method, the proposed method only can be used for the MIMO radar with uniform linear array. The maximum number of targets that can be identified by the proposed method is depend on the real-valued covariance matrix $\mathbf{Y}_{real}$ and Eq. (6). Owing to all targets are uncorrelated, the rank of $\mathbf{R}_{real}$ is MN . On the other hand, from the Eq. (6), it is indicated that the subarray corresponding to $N(M-1)$ virtual subarray elements is used to estimate DOD. Thus, the maximum number of targets that can be identified by the proposed method is $N(M-1)$.

4 Simulation results

Assume that the number of targets is known, and there exist $P = 3$ noncoherent targets located at angles $(\varphi_1, \theta_1) = (20^\circ, 10^\circ)$, $(\varphi_2, \theta_2) = (30^\circ, -10^\circ)$, $(\varphi_3, \theta_3) = (50^\circ, -20^\circ)$. The bistatic MIMO radar with $M = 6$ and $N = 8$ is considered.

Fig. 1 shows the paired results of three targets with 200 Monte Carlo trials for $SNR = 5 \text{ dB}$. From Fig. 1, we can easy to see that the DODs and

Fig. 1. Paired results for three targets with 200 Monte Carlo trials

Fig. 2. RMSE of angle estimation against SNR for $P = 3$ targets

Fig. 3. CPU time of both Unitary ESPRIT-MUSIC and ESPRIT-MUSIC algorithm against $M = N$ for $P = 3$ targets

DOAs of all three targets are paired correctly by our method. Fig. 2 shows the RMSE of angle estimation against SNR, where 200 Monte Carlo trials are used. It is showed that the proposed method has lower RMSE than ESPRIT and ESPRIT-MUSIC when the snapshots is $L = 50$. It also can be seen that the angle estimation performance of all methods are remarkably improved

when the snapshots increases, and the proposed method also outperforms both ESPRIT and ESPRIT-MUSIC. We use the CPUTIME instruction in MATLAB to evaluate the computational complexity of the proposed algorithm and ESPRIT-MUSIC method. Fig. 4 shows the CPU time against the number of sensors $M = N$ in MIMO radar, compared with ESPRIT-MUSIC. From the Fig. 4, we can observe that the CPU time of the proposed method is much smaller than the ESPRIT-MUSIC algorithm.

5 Conclusion

A new combined Unitary ESPRIT-MUSIC algorithm with real-valued processing for angle estimation in bistatic MIMO radar is presented. This technique uses Unitary ESPRIT to estimate DODs and Unitary Root-MUSIC for DOA estimation. Simulation results verify that the proposed method has an automatic pairing and better angle estimation performance than ESPRIT-MUSIC and ESPRIT. Furthermore, the proposed method is more computationally efficient than ESPRIT-MUSIC.

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