A joint coding HARQ scheme for N-channel stop-and-wait protocol

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Abstract: N-channel stop-and-wait (SW) hybrid automatic repeat request (HARQ) protocol is widely used in wireless communication systems due to its simplicity and high efficiency. However, different HARQ processes are transmitted over sub-channels with individual encoding. We present a joint coding HARQ scheme in which erroneous codewords of N parallel channels are jointly encoded for retransmissions. Erroneous and retransmitted codewords form a linear concatenated block code which is jointly decoded with a low complexity iterative decoding algorithm. Simulation results show that the normalized throughput performance obtains a significant gain while complexity increasing is limited.

Keywords: hybrid automatic repeat request (HARQ), stop-and-wait (SW), joint coding

Classification: Wireless Communication Technologies

References


1 Introduction

Hybrid automatic repeat request (HARQ) is the combination of forward error correction (FEC) and automatic repeat request (ARQ). N-channel stop-and-wait (SW) protocol consists of N parallel HARQ processes, and is widely applied in wireless systems, such as high speed packet access (HSPA) and long term evolution (LTE) [1] since the design is simple and efficient.

The two basic HARQ variants of each HARQ process are chase combining (CC) [2] and incremental redundancy (IR) [3] where the sub-channels either repeat the erroneous codewords or send new parity bits. In the existing systems, the retransmission and decoding of each sub-channel work independently. It is known that linear combination of codewords is helpful to the performance of HARQ system. For example, in [4], the sender adopts data packet linear combinations based on feedbacks from several users and each user exploits its previously received information in decoding the linearly combined packets; In [5], it is shown that network coding HARQ can improve throughput performance significantly.

This paper proposes a joint coding HARQ scheme for N-channel SW protocol. The retransmitted codeword generated by the proposed joint HARQ scheme is neither the repetition of the initial erroneous codeword, nor the parity bits of a single sub-channel. The erroneous codewords and the retransmitted ones have constructed a concatenated codeword, in which low-density parity-check (LDPC) code is the inner code and the outer code is a linear block code. A low complexity decoding algorithm is also proposed to jointly decode the erroneous codewords in all sub-channels. Simulation results demonstrate that, compared with traditional HARQ, joint HARQ can improve the normalized throughput with limited complexity increasing.

2 System model

An example of the synchronous N-channel SW protocol of traditional HARQ is illustrated as Fig. 1. We assume that each frame contains N slots and each slot includes exactly one LDPC codeword which encodes k information bits into n code bits. The parity check matrix of the LPDC code is denoted by \( H \) and the code rate is denoted by \( r = k/n \). Based on decoding results, N bits of acknowledgement (ACK)/negative acknowledgement (NAK) are sent back to the transmitter over an error-free feedback channel. Upon the arrival of NAKs, the erroneous codewords will be retransmitted in the next frame.

Let \( c_1, \ldots, c_M \) denote M erroneous codewords (\( M \leq N \)). In the example of Fig. 1, \( N = 8, M = 2 \) and \( c_1, c_2 \) are the codewords transmitted at slot \#2 and \#7 of frame 1.

In order to explore the extra coding gain, we takes a linear coding to generate \( P ( P \leq M ) \) retransmitted codewords which are denoted by \( c_{M+1} \).
The conventional CC-HARQ is a special case where \( P = M \) and \( c_{M+1} = c_1, c_{M+2} = c_2, \ldots, c_{2M} = c_M \). In other words, the retransmission in conventional HARQ is the concatenation of inner LDPC code and outer repetition code. It is well known that repetition code has no code gain, hence there is space to optimize the outer code.

By the constraint that \( c_{M+1}, c_{M+2}, \ldots, c_{M+P} \) is the linear combination of \( c_1, c_2, \ldots, c_M \), we have defined a linear block code of code rate \( r_{II} = M / (M + P) \) and parity matrix \( H_{II} \). The detailed design of \( H_{II} \) will be presented in next section.

### 3 Joint encoding for parallel channels

The initial transmission and the retransmission has constructed a systematic linear code with its parity matrix \( H_{II} \) can be expressed as

\[
H_{II} = [H_{II_d} \ H_{II_p}]
\]

where \( H_{II_d} \) is a \( P \times M \) matrix corresponding to the systematic codewords, i.e. the initial erroneous codeword \( c_1, c_2, \ldots, c_M \), and \( H_{II_p} \) is a \( P \times P \) diagonal matrix corresponding to the parity codewords (the retransmitted codeword) \( c_{M+1}, c_{M+2}, \ldots, c_{M+P} \) which are the linear combinations of \( c_1, c_2, \ldots, c_M \).

Note that \( H_{II} \) is a small matrix since the number of sub-channels, \( N \), is generally small in practical N-channel HARQ systems. As a short code suitable for Belief Propagation (BP), \( H_{II} \) should have the following characteristics [6]: the number of ones in each row should be small while the number of ones in each column should be large; for all pairs of rows of the matrix, the number of columns that have a one in both rows (intersection sets) is small, ideally zero or one.

Based on these rules, the construction of \( H_{II_d} \) is given as follows. First, the variable nodes degrees (column weights) of \( H_{II_d} \) are fixed to 2; Then, we use Progressive Edge-Growth (PEG) algorithm [7] to construct \( H_{II_d} \) in order to minimize the girth; The fixed degree distribution and short code length may still cause the existence of 4-cycle, hence the final step is added which choose lest ones in intersection sets and change them to zeros, thus all intersection sets have only one elements. An example of \( H_{II} \) for \( M = 5 \), \( P = 4 \) is

\[
H_{II} = \begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
4 Iterative decoding with complexity reducing

This section presents the iterative decoding of constructed concatenated codes. As depicted in Fig. 2, \( c_m = (c_{m,1}, \ldots, c_{m,n}) \), \( m = 1, 2, \ldots, M + P \), is the \( m \)-th inner LPDC code. For each bit \( i = 1, 2, \ldots, n \), \( (c_1,i, c_2,i, \ldots, c_{M+P},i) \) form the outer code. Define \( \mathcal{V}^l \) and \( \mathcal{C}^l \) as the set of variable nodes and check nodes in each inner code, \( \mathcal{V}^{ll} \) and \( \mathcal{C}^{ll} \) as the set of variable nodes and check nodes in each outer code respectively.

The soft information propagating in the joint HARQ decoder can be further categorized into the following types:

- the full soft information of all variable nodes: \( \lambda(m,i), i \in \mathcal{V}^l, m \in \mathcal{V}^{ll} \);
- the soft information input to the decoder: \( \lambda_{ch}(m,i), i \in \mathcal{V}^l, m \in \mathcal{V}^{ll} \);
- the soft information from \( j \)-th check node to \( i \)-th variable node in \( m \)-th inner code: \( \lambda_{C^{2V}}^{l,j,i}(m,j,i), j \in \mathcal{C}^l, i \in \mathcal{V}^l, m \in \mathcal{V}^{ll} \);
- the soft information from \( i \)-th variable node to \( j \)-th check node in \( m \)-th inner code: \( \lambda_{C^{2V}}^{l,j,i}(m,j,i), j \in \mathcal{C}^l, i \in \mathcal{V}^l, m \in \mathcal{V}^{ll} \);
- the soft information from \( m \)-th variable node to \( p \)-th check node in \( i \)-th outer code: \( \lambda_{C^{2V}}^{l,p,i}(m,p,i), m \in \mathcal{V}^{ll}, p \in \mathcal{C}^{ll}, i \in \mathcal{V}^l \);
- the soft information from \( m \)-th variable node to \( p \)-th check node in \( i \)-th outer code: \( \lambda_{C^{2V}}^{l,p,i}(m,p,i), m \in \mathcal{V}^{ll}, p \in \mathcal{C}^{ll}, i \in \mathcal{V}^l \).

and have the following relationships:

\[
\lambda(m,i) = \lambda_{ch}(m,i) + \sum_{j:H^l(j,i)=1} \lambda_{C^{2V}}^{l,j,i}(m,j,i) + \sum_{p:H^{ll}(p,m)=1} \lambda_{C^{2V}}^{l,p,i}(m,p,i), \quad (3)
\]

\[
\lambda_{C^{2V}}^{l,j,i}(m,j,i) = \lambda(m,i) - \lambda_{C^{2V}}^{l}(m,j,i), \quad (4)
\]

\[
\lambda_{C^{2V}}^{l,j,i}(m,j,i) = f(\lambda_{C^{2V}}^{l}(m,q,i)); q : H^l(q,i) = 1, q \neq j, \quad (5)
\]

\[
\lambda_{C^{2V}}^{l,p,i}(m,p,i) = \lambda(m,i) - \lambda_{C^{2V}}^{l}(m,p,i), \quad (6)
\]

\[
\lambda_{C^{2V}}^{l,p,i}(m,p,i) = \omega \cdot f(\lambda_{C^{2V}}^{l}(m,l,i)); l : H^{ll}(l,m) = 1, l \neq p, \quad (7)
\]

where the multi-dimensional function \( f(\cdot) \) is the check node update function such as sum-product algorithm (SPA) and min-sum algorithm (MSA), the weighting factor \( \omega \) is to change the ratio of external information coming from inner and outer codes and \( H^l(j,i) \) is the \((j,i)\)-th entry of \( H^l \).
When retransmission happens, decoding of joint HARQ starts. Firstly, the soft information of all variable nodes both in inner and outer codes, $\lambda(m, i) (m \in \mathcal{V}^\Pi, i \in \mathcal{I}^\Pi)$, is initialized by channel input $\lambda_{ch}(m, i)$. Then the iterative decoding of inner codes updates propagated soft information from variable nodes to check nodes $\lambda_{C^2V}(m, j, i) (j \in \mathcal{C}^\Pi, i \in \mathcal{V}^\Pi, m \in \mathcal{V}^\Pi)$ and from check nodes to variable nodes $\lambda_{C^2V}(m, j, i)$ for $niter^I$ times, as Eq. (4) and Eq. (5). After that outer codes update their propagated soft information from variable nodes to check nodes $\lambda_{C^2V}(m, p, i) (m \in \mathcal{V}^\Pi, p \in \mathcal{C}^\Pi, i \in \mathcal{V}^I)$ and from check nodes to variable nodes $\lambda_{C^2V}(m, p, i)$ for 1 time, as Eq. (6) and Eq. (7). Finally, the information of all variable nodes is updated by Eq. (3). The whole iteration will continue for $niter^\Pi$ times unless all erroneous codewords are decoded successfully. The parameters $niter^I$, $niter^\Pi$ and $\omega$ are set to be 5, 8 and 0.7, which come from simulations and can be further optimized by Density Evolution [8].

To further reduce the decoding complexity, $H^\Pi$ can be shrunk whenever one or more codewords are decoded successfully in the iteration course. Consider an example where $M = 5$ erroneous codewords $c_1, \ldots, c_5$ generate $P = 4$ retransmitting codewords $c_6, \ldots, c_9$ with $H^\Pi$ given by Eq. (2). Suppose that $c_2$ and $c_8$ have been correctly decoded, then the check equation $H^\Pi(c_1, c_2, \ldots, c_9)^T = 0$ can be rearranged as follows:

$$
\begin{bmatrix}
1 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\
\lambda_3 \\
\lambda_4 \\
\lambda_5 \\
\lambda_6 \\
\lambda_7 \\
\lambda_9
\end{bmatrix}
= \begin{bmatrix}
0 \\
c_2 + c_8 \\
c_2 \\
0
\end{bmatrix},
$$

which has reduced the $H^\Pi$ to a smaller matrix $\tilde{H}^\Pi$ of size $4 \times 7$. Additionally, the third row of $\tilde{H}^\Pi$ in Eq. (8) indicates that $c_3 = c_2 + c_8$. Thus, Eq. (8) further becomes

$$
\begin{bmatrix}
1 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\
\lambda_4 \\
\lambda_5 \\
\lambda_6 \\
\lambda_7 \\
\lambda_9
\end{bmatrix}
= \begin{bmatrix}
\lambda_3 \\
\lambda_2 \\
0
\end{bmatrix}.
$$

5 Simulation results

This section compares the performance of traditional HARQ and proposed joint HARQ with different code rate $r^\Pi$ when 1 retransmission is allowed. The number of sub-channel is $N = 8$. The inner code is the $n = 768, k = 576$ LDPC code defined in 802.16e. Coded bits are modulated by BPSK and...
transmitted over an AWGN channel. For the proposed scheme, $H^\Pi$ is constructed using the method presented in Sect. 3 where $M$ is determined by the number of erroneous codewords in frame 1 and $P = \lfloor M(1 - r^\Pi)/r^\Pi \rfloor$. We use MSA as check node update function and maximum iteration number in traditional HARQ is 50 while joint HARQ use the iteration number stated previously.

![Fig. 3. Performance comparisons of traditional HARQ and joint HARQ (a) normalized throughput (b) average accumulative iteration number](image)

The normalized throughput performance of all schemes is shown in Fig. 3(a). Joint HARQ of $r^\Pi = 1/2$ performs best in low SNR regime, while $r^\Pi = 4/7, 2/3$ have best normalized throughput performance when SNR increases. As a result, the optimal joint HARQ scheme is to apply $r^\Pi = 1/2, 4/7, 2/3$ in $[-1 \text{dB}, -0.2 \text{dB}], (-0.2 \text{dB}, 0.2 \text{dB}], (0.2 \text{dB}, 3 \text{dB}]$, while the thresholds are $-0.2 \text{dB}$ and $0.2 \text{dB}$. Joint HARQ improves normalized throughput by 10% to more than 100% compared with traditional HARQ.

Additionally, we compare average accumulative iteration number of inner codes in joint HARQ and traditional HARQ to verify complexity as Fig. 3(b), since outer codes include few check equations. By obtained thresholds above, iteration number of optimal joint HARQ is reduced by about 5% at $-0.5 \text{dB}(r^\Pi = 1/2)$, and is increased by about 18% and 9% at 0 dB and 1.5 dB($r^\Pi = 4/7, 2/3$). The increasing of complexity is proved to be limited.

### 6 Conclusion

A joint coding HARQ scheme for N-channel SW protocol and the corresponding low complexity decoding algorithm have been proposed. Erroneous codewords in sub-channels are jointly encoded to enhance the performance. Simulation results show that proposed scheme can improve the normalized throughput with limited increasing of complexity.