Flexible fractional K-best sphere decoding for uncoded MIMO channels

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Abstract: In K-best sphere decoding algorithm (KB), the number of survivor paths K, can be integer values in all tree levels. A certain complexity is provided for each value of K. In this letter, a variant of the K-best sphere decoding algorithm for uncoded MIMO channels is proposed, namely, flexible fractional K-best algorithm (FFKB). The FFKB provides a degree of freedom in complexity in-between the discreet complexities of the traditional KB. Through the offline calculations, the FFKB determines the best operation mode for KB relative to the hardware capability from the complexity point of view.

Keywords: MIMO channels, K-best (KB) sphere decoder

Classification: Wireless Communication Technologies

References

1 Introduction

In multi-input multi-output (MIMO) spatial division multiplexing (SDM) systems, the traditional K-best sphere decoder (KB) keeps the best K-nodes at each level of the search tree [1]. The complexity is usually measured by the average visited nodes (VN)s, consumed by solver during the running of algorithm and it depends on the value of K [1]. In real time detection devices, the hardware specification can treat limited number of nodes to fully estimate the received symbol [2, 3]. The complexity of KB algorithm supports limited number of complexity which may not be suitable for all implementation purposes. The limiting in complexity came from, the big gap between successive complexities of traditional KB [2, 3]. Authors in [4] provide a basic solution to this problem by finding only the intermediate complexity level in-between two successive complexity levels, corresponding to two successive values of Ks, based on pruned radius specified in their paper. Beside limitation in complexity problem, many algorithms are presented in [1, 2, 3, 5] to reduce the complexity but without taking the hardware capability into consideration.

In this letter, a flexible fractional K-best algorithm (FFKB) for uncoded MIMO channels is presented1. FFKB provides a solution for the complexity limitation problem by finding all complexity sub-levels in-between any two successive complexity levels. Then, FFKB determines the best operation mode for K-best algorithm, valid for capability of working hardware, through offline calculations. This enables the proposed FFKB algorithm to be combined with any complexity reduction technique in the desired operation mode or to be considered as a pre-KB algorithm.

2 Background

In additive white Gaussian noise environment, the maximum likelihood (ML) decoder is the optimum one where the ML solution finds the symbol estimate \( \hat{x}_{ML} \) that minimizes the 2-norm of the residual error. In particular

\[
\hat{x}_{ML} = \arg \min_{x \in \mathbb{C}^M} \| y - Hx \|^2,
\]  

(1)

where \( x \) is the \( M \times 1 \) transmitted vector whose elements are drawn from the \( q \)-QAM constellation, \( y \) is the \( N \times 1 \) received vector, \( H \) is the \( N \times M \) channel matrix.

1The letter is an extended version of the studies presented in part at an international conference, JEC-ECC 2013 [4].
whose elements, $h_{nm}$, represent the Rayleigh flat-fading gains from transmitter $m$ to receiver $n$, and $\Lambda$ is the lattice whose points represent all possible codewords at the transmitter.

The sphere decoder (SD) reduces the computational complexity by limiting the search space inside a sphere of radius $\rho$ centered at the received signal vector $y$. The estimated signal vector, $\hat{x}$, should satisfy the radius constraint $\|y - H\hat{x}\| < \rho$ [5]. The SD transforms the closest-point search problem into a tree-search problem by factorizing the channel matrix, $H = QR$, where $Q$ is a $N \times N$ unitary matrix and $R$ is an upper triangular matrix of size $N \times M$. Thus, (1) can be rewritten as

$$\hat{x}_{SD} = \arg \min_{x \in \hat{\Lambda}} \|\tilde{y} - Rx\|,$$  

(2)

where $\tilde{y} = Q^H y$ and $\hat{\Lambda}$ is the subset of the lattice that lies inside the sphere of radius $\rho$ centered at the received signal vector $y$. The distance metrics $d(x) = \|\tilde{y} - Rx\|$ can be computed recursively with partial Euclidean distances (PED) [5]. The upper triangular nature of $R$ allows the optimization problem to be structured as a tree search\(^2\), with each transmitter representing one level of the tree, and the branches representing a choice of one of the constellation points available for each transmitter [4].

### 3 Flexible fractional K-best sphere decoder

In FFKB, we can obtain some regular steps of the complexity by increasing the value of VNs in some tree levels into $K + 1$ nodes, whereas the other tree levels still $K$ nodes. These steps, between two successive $K$’s, act as if the value of $K$ is fraction (not integer). Fig. 1-a illustrates the procedure of increasing one node in each tree level ($2M$ levels, in real form representation, for $M \times N$ MIMO and $M = 3$) in the available tree levels, where the available tree levels are defined as number of available tree levels between two successive integer values of $K$’s, valid for increasing one node per level without reaching to the next value of integer $K$. In this figure, we have 6 levels ($i = \{6 5 4 3 2 1\}$), where there are 4 available tree levels ($i = \{6 5 4 3\}$) valid to increase one node in each of them to get the fractions.

\(^2\)This letter uses the real form representation which deals with the two components (real & imaginary parts) of the estimated symbols separately (the upper half of the tree represents imaginary parts and the lower half of it represents real parts as shown in Fig. 1-b). So, the number of tree levels is doubled [7].
of K. For example in Fig. 1-a, for $K = 2$ and $K = 3$, number of best nodes taken from each tree level are, $K_2 = [2 2 2 2 2 1]$ at $i = [6 5 4 3 2 1]$ and $K_3 = [3 3 3 3 3 1]$ at $i = [6 5 4 3 2 1]$, respectively. The best nodes at a specific tree level are defined as the nodes that have the smallest PED at this tree level. Hence, by increasing only one node in the highest tree level, we can obtain $K = 2.2$, while it is denoted as $K_{2.2}$. Number of best nodes in $K_{2.2}$ is $K_{2.2} = [3 2 2 2 2 1]$ at $i = [6 5 4 3 2 1]$. Similarly, $K_{2.4}$ can be obtained by increasing one node in each of the first two highest tree levels ($i = 6$ and $5$) and so on till reaching the last fractional value $K_{2.8}$. Thus, the available tree levels for all fractional values of $K = 2$ ($K_{2.2}, K_{2.4}, K_{2.6}, K_{2.8}$) are given as $i = [6, 5, 4, 3]$. Note that the lowest tree level ($i = 1$) is extracted, since the available tree levels are increased from the level 2 ($i = 2$).

4 Complexity analysis

In what follows, the complexity of KB and FFKB are presented. The complexity in this letter is defined as the average number of VNs to find solution.

4.1 Complexity of the K-best

To determine the complexity of the K-best algorithm, tree levels are divided into two groups. The first group contains tree levels where number of available nodes in each tree level, $N_i^{KB-} \leq K$, whereas the second group contains tree levels where number of available nodes per level $> K$ (see Fig. 1-b for $K = 3$). Note that, each survived node is expanded into $\sqrt{q}$ child nodes in the next tree level. Then, number of VNs (the same as available nodes in this group), $N_i^{KB-} = \sqrt{q} \sum_{j=0}^{P_K} N_j^{KB-}$, for the first group is

$$N_i^{KB-} = \sqrt{q} \sum_{j=0}^{P_K-1} (\sqrt{q})^j, \quad (\sqrt{q})^{P_K} \leq K, \quad (3)$$

where $P_K$ is number of tree levels in first group for specific $K$. Given that $(\sqrt{q})^{P_K} \leq K < (\sqrt{q})^{P_K+1}$ and knowing $q$ and $K$, we can determine $P_K$ [6].

$$P_K = \left[ \frac{\ln(K)}{\ln(\sqrt{q})} \right], \quad (4)$$

where $[\cdot]$ is the floor operation. For example, consider 4-QAM for $2 \times 2$ MIMO shown in Fig. 1-b. In case of $K = 3$, from Eqs. (3) and (4), the number of tree levels for the first group $P_K = [1.58] = 1$ and the total nodes of the first group $N_i^{KB-} = 2$.

For the second group, each tree level has a fixed number of VNs, $N_i^{KB+} = K \sqrt{q}$ nodes. Then, total number of VNs in the second group

$$N_i^{KB+} = (2M - P_K - 1)K\sqrt{q}. \quad (5)$$

Using $P_K$ from (4), the complexity of the KB, $C_K^{KB}$, is total number of VNs

$$C_K^{KB} = \sqrt{q} \left[ \frac{1 - (\sqrt{q})^{P_K+1}}{1 - \sqrt{q}} + (2M - P_K - 1)K \right]. \quad (6)$$
4.2 Complexity of the FFKB

To analyze the complexity of FFKB, we will describe number of available nodes between two successive $K$’s ($K$ and $K + 1$), $C^K_{bet}$, which is given from

\[
C^K_{bet} = C^{KB}_{K+1} - C^K_{K} = \sqrt{q} \left( (\sqrt{q})^{P_K + 1} \times \frac{1 - (\sqrt{q})^\alpha}{1 - \sqrt{q}} + 2M - (aK + P_{K+1} + 1) \right),
\]

(7)

where $P_K$ and $P_{K+1}$ given from (4), and $\alpha = P_{K+1} - P_K$.

Based on the fact that, each visited node is expanded into $\sqrt{q}$ child nodes in the next tree level, increasing one node in any tree level will increase the resulted complexity by $\sqrt{q}$ nodes. Hence, we can define a complexity resolution, $C_{ros}$, as the smallest step in complexity:

\[
C_{ros} = \sqrt{q}.
\]

(8)

Number of available tree levels between two successive values of $K$’s, is valid for increasing nodes without reaching to the next value of $K$, which can be denoted as $L_{sub}$ and given by:

\[
L_{sub} = 2M - P_K - 1.
\]

(9)

The fraction resolution of $K$ can be defined as, $f_{ros}$

\[
f_{ros} = \frac{1}{L_{sub} + 1}.
\]

(10)

The fraction value of $K$ can be obtained, $K_f$

\[
K_f = K + \beta f_{ros} \quad \text{for} \quad \beta = 1, 2, \cdots L_{sub}.
\]

(11)

Finally, the complexity corresponding to value of $K_f$ is given as

\[
C_{K_f} = C^K_{K} + C^K_{bet}(\beta f_{ros}) \quad \text{for} \quad \beta = 1, 2, \cdots L_{sub}
\]

(12)
5 Simulation results and discussion

5.1 Simulation environments and results

In this section, Bit error rate (BER) performance and complexity of the KB and FFKB are assessed. We assume that the transmitted power is independent of number of transmit antennas, $M$, and equals the average energy per symbol in Rayleigh fading channels. Figs. 2 and 3 illustrate BER performance and complexity of the traditional KB and FFKB for 4-QAM & $2 \times 2$ MIMO and 16-QAM & $3 \times 3$ MIMO systems, respectively. Note that, the FFKB supports all sub-complexities (dashed lines) between two successive complexities of the traditional KB (solid lines). These sub-complexities provide sort of degree of freedom in hardware designing.

5.2 Discussion

Suppose we have a hardware specification that can treat 82 nodes per symbol estimation for 16-QAM and $3 \times 3$ MIMO channels. Using offline calculations to determine the maximum complexity which can fit the hardware capability, we will work on $C^K_B = 64$ ($K = 3$) and $P_3 = 1$. We can calculate $L_{sub} = 4$ and $f_{ros} = 0.2$ using (9) and (10), respectively. By using (12), we can find the value of $\beta$ by solving the inequality; $C^K_B + C^K_{f_{ros}}(\beta f_{ros}) \leq 82 \Rightarrow \beta \leq 4.5 \Rightarrow \beta = 4$ ($\beta$ should be integer). By using (11) and (12), the hardware can work on $K_{3,8}$ with 80 VNs per symbol estimation. From Fig. 3, the performance of $K_{3,8}$ is very close to $K_4$.

6 Conclusion

The proposed algorithm, namely, flexible fractional K-best (FFKB) has two benefits. First, solves the complexity limitation problem by increasing number of survivor paths into $K + 1$ regularly in some tree levels and staying $K$ paths in other levels. Second, chooses the best operation mode for KB algorithm through the offline calculations without utilizing any prior knowledge of current channel state relative to the hardware capability.