Characteristics of multi-dimensional modulation with MIMO signal processing

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Abstract: We numerically evaluate optical transmission characteristics of four- and eight-dimensional modulation formats, and find that they are effectively improved by wavelength-diversity or cross-talk (XT) compensation with 4 × 4 adaptive multiple-input multiple-output (MIMO) equalization. Further, we quantitatively verify that the four-dimensional formats with the diversity configuration is superior to the eight-dimensional formats using dual carriers in terms of the robustness against unbalanced OSNR of the two carriers. We also quantitatively demonstrate the XT compensation effectiveness for the eight-dimensional modulation formats in case that the dual carrier spacing is less than the symbol rate.

Keywords: adaptive MIMO equalization, multi-dimensional modulation, wavelength-diversity, cross-talk compensation

Classification: Fiber-Optic Transmission for Communications

References

1 Introduction

Digital coherent technologies have successfully realized high-speed optical transmission systems of 100 Gbps and beyond in commercial use, adopting dual polarization multiplexed optical signals in one wavelength, each of which has two-dimensional (2-D) in-phase and quadrature (IQ) modulation components. Four-dimensional (4-D) modulation format has recently attracted attention due to its better power efficiency, namely, higher sensitivity taking into account the data rate, than the 2-D format above, where all components of polarization modes as well as IQ modes are comprehensively treated as a single symbol [1, 2]. Moreover, eight-dimensional (8-D) or more dimensional formats have been intensively investigated to achieve further efficiency improvement, utilizing time-slot mode in single-mode fiber [3], frequency (wavelength) mode in single-mode fiber [4], or space mode in multi-core fiber [5] in addition to the IQ and polarization modes.

On the other hand, adaptive $4 \times 4$ or more MIMO (multiple-input and multiple-output) equalization processing techniques have been examined to handle four or more polarized optical signals in plural wavelengths or plural space modes, expanding conventional $2 \times 2$ MIMO for two polarized signals in one wavelength. One application of $4 \times 4$ or more MIMO is to configure the wavelength diversity system which has higher sensitivity by realizing maximum ratio combining (MRC) of two or more wavelength signals having the same data sequence which are separately transmitted on different optical grids in one fiber, or are transmitted along different routes [6]. Transmission experiments with the diversity configuration in multi-core fiber have also been reported [7]. Another application of the $4 \times 4$ or more MIMO is crosstalk (XT) compensation between neighboring wavelength channels in super-dense wavelength division multiplexed systems in a single mode fiber [8]. The XT compensation between the signals in different spatial modes in multi-mode fiber have also been studied [9].

The multi-dimensional modulations and the MIMO techniques could be applied at the same time, however, to our best knowledge, few reports have been confirmed. In this paper, we numerically evaluate transmission characteristics of 4-D and 8-D modulation formats with $4 \times 4$ adaptive MIMO equalization. The obtained results quantitatively show that the diversity configuration realizes the MRC to improve the Q-value of 4-D formats, and the XT compensation also improves the 8-D format in effective manner.
2 Calculation model

2.1 Modulation formats

Five formats are evaluated as shown in Table I, all of which have the same symbol rate of 32 Gbps. The format (A) is a conventional 2-D format called DP-QPSK (dual-polarization quadrature phase shift keying) having bit rate of 128 Gbps using its 16 (= \(2^4\)) symbols, each of which corresponds to the complex electric field’s x and y polarization components of \((E_{x,I}, E_{x,Q}, E_{y,I}, E_{y,Q}) = (\pm 1, \pm 1, \pm 1, \pm 1)\). In this format, in-phase and quadrature-phase of two polarized signals are individually modulated with Nyquist pulse-shaping (roll-off factor of 1.0), in the same manner as the other four formats are modulated.

The format (C) is 4D-PS-QPSK (PS: polarization switched) having 8 (= \(2^3\)) symbols, which is a subset of the 16 symbols in the format (A), specifically described as \((E_{x,I}, E_{x,Q}, E_{y,I}, E_{y,Q}) = (+1,+1,+1,+1), (+1,+1,-1,-1), (+1,-1, -1, -1), (-1,-1, +1, +1), (-1,-1,-1,-1), (-1, +1, +1, -1), (-1, +1, -1, +1)\) [1, 2]. Each of the electrical field components has at least two different signs from each other, which leads to the enlargement of the minimum Euclidean distance to improve sensitivity. Eight codes composed of three digits making its bit rate 96 Gbps, are respectively mapped to corresponding symbols as (000), (001), (010), (011), (110), (111), (100), (101). This mapping cannot be a perfect Gray code, since each symbol has 6 nearest neighbors. Thus, the pairs furthest away from each other is set to be inverted binary codes to minimize bit error. To be noted, the format (C) is one of three 4D-PS-QPSK formats characterized by Stokes parameter of S1 [1].

Signals of the formats (B) and (D) are respectively modulated by DP-QPSK and 4D-PS-QPSK, as same as the formats (A) and (C), and each of the formats (B) and (D) has the diversity configuration with doubled bandwidth. The format (E) is 8D-DC-PS-QPSK (DC: dual-carrier) composed of 16-symbols which is a subset of two wavelength 4D-PS-QPSK formats. The square of the minimum Euclidean distance of this format is four times larger than that of the format (A). The code mapping of four digits cannot be a perfect Gray code, either, since each symbol has 14 nearest neighbors.

2.2 Transmission line conditions and adaptive MIMO equalization

Two polarized optical signals in one wavelength are assumed to suffer some polarization rotations and differential group delay (DGD) of about 0.5 bit through 2.5 bit between them. Each of two wavelength signals of the format (B), (D) and (E) in the diversity configuration is transmitted at different wavelength from each other in a single-mode fiber, suffering different value of DGD from each other. Each optical signal is amplified by an optical amplifier with white optical noise as ASE (amplified spontaneous emission), the amount of which determines the OSNR condition. The transmitted signal is detected by optical hybrid receivers following the 3rd-order Butterworth bandpass filter whose bandwidth is 22 GHz. Local-spontaneous beat noise is assumed to be dominant for simplicity, accordingly local-shot noise and thermal noise are ignored. The signals of the formats (A) and (C) are respectively equalized by 2 × 2 MIMO whereas the signals of the formats
(B) and (D) are applied to $4 \times 4$ MIMO to make the MRC at the diversity configuration. Both of the MIMOs are based on least-mean-square (LMS) algorithm. The formats (E) is applied to $4 \times 4$ or $2 \times 2$ MIMO subject to performing the XT compensation or not. Finally, the symbols are made threshold decision by maximum likelihood estimation where minimum Euclidean distance from the received signal point is examined.

### Table I. Modulation and configuration formats

<table>
<thead>
<tr>
<th>Format</th>
<th>Dimension</th>
<th>Bit rate [Gbps]</th>
<th>Bandwidth [GHz]</th>
<th>MIMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) DP-QPSK</td>
<td>2</td>
<td>128</td>
<td>$32 \times 1$</td>
<td>$2 \times 2$</td>
</tr>
<tr>
<td>(B) DP-QPSK w/Diversity</td>
<td>2</td>
<td>128</td>
<td>$32 \times 2$</td>
<td>$4 \times 2$</td>
</tr>
<tr>
<td>(C) 4D-PS-QPSK</td>
<td>4</td>
<td>96</td>
<td>$32 \times 1$</td>
<td>$2 \times 4$</td>
</tr>
<tr>
<td>(D) 4D-PS-QPSK w/Diversity</td>
<td>4</td>
<td>96</td>
<td>$32 \times 2$</td>
<td>$4 \times 4$</td>
</tr>
<tr>
<td>(E) 8D-DC-PS-QPSK</td>
<td>8</td>
<td>128</td>
<td>$32 \times 2$</td>
<td>$2 \times 2$ or $4 \times 4$</td>
</tr>
</tbody>
</table>

### 3 Calculation result

#### 3.1 Q-value performance

Fig. 1 shows the calculated Q-value performance as a function of OSNR for the five formats. The XT between dual-carriers is not considered here at the formats (E), so $2 \times 2$ MIMO is applied for equalization. The five dotted lines are approximated theoretical Q-values corresponding to the five formats, which are derived from complementary error function of OSNR in accordance with respective Euclid distance taking into account the imperfect Gray code mappings for the formats (C), (D) and (E). The imperfect Gray code mappings induce Q-values deterioration at low Q-value conditions according to bit error rate increase in proportion to the imperfectness. In addition, exact theoretical Q-values of the format (C) are a little worse than the approximated ones at low Q-value conditions due to its 8-ary biorthogonal constellation characteristic [1], which shall cause the calculated Q-values deviated from the dotted lines.

The calculated Q-value performance of 2-D format (B) is exactly 3 dB better than that of 2-D formats (A), thanks to the diversity configurations with $4 \times 4$ MIMO. The differences from theoretical curves are caused by inter-symbol interferences, since the Nyquist pulse-shaped waveform is deformed by the filter at the receiver. And Q-value performance of 4-D format (C) is also about 3 dB better than that of formats (A), thanks to the enlarged Euclid distance, though it is slightly degraded from that of 2-D format (B) at low Q-value due to the imperfect Gray code and the constellation characteristic.

Moreover, Q-value performance of 4-D format (D) is also exactly 3 dB better than that of 4-D formats (C), which shows that the diversity configuration with $4 \times 4$ MIMO effectively improves the Q-value of the 4-D formats. And Q-values of 8-D formats (E) is almost about 3 dB better than that of 4-D format (C) in accordance with larger Euclid distance. The degradation of 8-D formats (E) from
that of 4-D format (D) is also observed at low Q-value due to larger impact of the imperfect Gray code in higher dimensions.

Two constellation maps of the formats (A) and (E) are inset in Fig. 1, which are obtained at the conditions of OSNR = 17[dB/0.1 nm] and 12[dB/0.1 nm]. Although the map of the format (E) does not have clear constellation anymore in 2-D in contrast to the map of (A), it can be detected at Q-value of about 13 dB.

![Constellation Maps](image)

**Fig. 1.** Q-value as a function of OSNR. (A) through (E) are the formats described in Table I. The dotted lines are respective theoretical Q-values. The two inset constellation maps are respectively obtained at the conditions indicated in the figure.

### 3.2 MRC characteristics and XT compensation

MRC characteristics are further examined in detail for the format (D). Fig. 2(a) shows Q-value performance of the format (D) as a function of OSNR of one wavelength signal while the OSNR of another wavelength is fixed to be 10[dB/0.1 nm]. At the same time, Q-values of the two wavelength signals, each of which is composed of the format (C) having the same data sequence from each other, are individually calculated. A dashed line is derived from weighed averages of these two individually calculated Q-values of the format (C) following MRC theory [7]. The obtained result of the format (D) is in good agreement with the dashed lines, which indicates that the diversity configuration with 4 × 4 MIMO achieves the MRC at unbalanced OSNR conditions.

Q-value of the format (E) is also evaluated at the same unbalanced OSNR conditions for two carriers, shown in Fig. 2(a) as well. The format (E) receives a greater influence from the worse OSNR carrier than the format (D), though the Q-value of the format (E) is almost equivalent to that of the format (D) at OSNR = 10[dB/0.1 nm] of CH1 and CH2. The format (D) of 4D-PS-QPSK with the diversity configuration is superior to the 8D-DC-PS-QPSK format (E) in terms of the robustness against unbalanced OSNR of the two carriers. The OSNR degradation of one channel by more than 4[dB/0.1 nm] out of two carriers could be observed due to erroneous operation. The format (D) offers a practical option to configure robust systems in view of such a case.

The XT compensation with 4 × 4 MIMO is examined for the format (E) with dual-carrier. Fig. 2(b) shows Q-values as a function of dual-carrier frequency spacing at OSNR = 10[dB/0.1 nm] and 12[dB/0.1 nm] with and without the XT compensation. The optical spectrum is presented as an inset in Fig. 2(b) at 32 GHz.
for reference. Although the obtained result shows that the Q-value drastically deteriorates as the spacing narrows, Q-value is around 0.4 and 0.9 dB improved by way of the XT compensation. The amount of the improvement is less at \( \text{OSNR} = 10 \text{[dB/0.1 nm]} \) than the amount at \( 12 \text{[dB/0.1 nm]} \). Qualitatively speaking, optical noise is assumed to interfere with the MIMO operation. However, quantitative evaluation is further required.

\[ \text{OSNR} = 10 \text{[dB/0.1 nm]} \]

Fig. 2. Q-value performances with \( 4 \times 4 \) MIMO.
(a) Q-value of the format (D) with MRC as a function of OSNR of one wavelength channel. A dashed line calculated from the two dotted lines for reference. The Q-value of the format (E) are also presented for comparison.
(b) 8D-DC-PS-QPSK with and without the XT compensation as a function of dual-carrier spacing in frequency at \( \text{OSNR} = 10 \text{[dB/0.1 nm]} \) and \( 12 \text{[dB/0.1 nm]} \).

4 Conclusion

We numerically evaluated transmission characteristics of multi-dimensional modulation formats with \( 4 \times 4 \) adaptive MIMO equalization which realizes wavelength-diversity or XT compensation. The obtained results quantitatively demonstrated not only that the multi-dimensional formatting improves Q-value according to the enlarged minimum Euclidean distance with slight degradation caused by the imperfect Gray code mapping, but also that the diversity configuration effectively improves Q-value of the 4-D formats with the MRC function of the \( 4 \times 4 \) MIMO. Further, we detailed the MRC characteristics and showed that the four-dimensional formats with the diversity configuration is superior to the 8-D formats using dual-carriers in terms of the robustness against unbalanced OSNR of the two carriers. Moreover, we qualitatively demonstrated the XT compensation effectively works for the 8-D modulation format in case the dual-carrier frequency spacing is less than the symbol rate.

Acknowledgments

This work was supported by JSPS KAKENHI Grant Number 16K06333.

The authors would like to thank Dr. Shigeru Saito at Ritsumeikan University for useful discussion.