Pattern synthesis algorithm with limited amplitude control range for active phased array antenna

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Abstract: An active phased array antenna is capable of controlling not only the phase of each element but also the amplitude, and can therefore form a highly accurate beam. When an amplitude distribution range is wide, there occurs the problem of the power efficiency of an amplifier connected to each element decreasing. To operate the amplifier with high efficiency, it is necessary to limit an amplitude control range. This paper proposes a new pattern synthesis algorithm adapted to optimize the excitation amplitudes and phases of an APAA within a predetermined amplitude control range. The proposed method enables designing within the predetermined amplitude control range by introducing a mapping function representing the amplitude of a driven element, although the values of independent variables to be optimized using a conjugate gradient method are not limited. In addition, the proposed method does not require iterative calculation, and makes it possible to obtain the optimum excitation amplitudes and phases by only performing optimization design once. The proposed algorithm will be described while taking null formation by a linear array antenna as an example, and effectiveness will be examined by simulation.

Keywords: active phased array antenna, restricted amplitude, power efficiency, null beam forming

Classification: Antennas and Propagation

References

1 Introduction

An active phased array antenna (APAA) is capable of performing high-speed beam scanning without mechanically moving the antenna itself by making a phase shifter connected to each element antenna control the phases of transmission and reception signals [1]. Also, the APAA suppresses interference waves such as clutter and interfering waves, and can therefore reduce a side lobe level in an interference direction, i.e. variously form a beam such as forming a null. Until now, there various pattern synthesis algorithms have been proposed based on phase control using nonlinear optimization or genetic algorithm (GA) [2, 3, 4, 5, 6, 7]. However, the pattern synthesis based on only phase control fails to form a null at a position, for example, symmetric with respect to a main beam or increases a reduction in gain when null formation is achieved over a wide range, and only the phase control restricts the beam formation.

By controlling the amplitude in addition to the phase, the degree of freedom of beam formation is increased [8]. The characteristics of a typical amplifier are designed to maximize power efficiency at the maximum output power of the amplifier. Accordingly, in the APAA, since the amplitude is controlled, the input power of an amplifier decreases, thereby making the output power smaller than the maximum value, and also decreasing the power efficiency [9]. In terms of the power efficiency of an amplifier, it is necessary to limit the amplitude control range.
Pattern synthesis algorithms taking account of an amplitude control range have been reported. In the method in Reference [10], amplitudes and phases allowing the formation of a desired null are first obtained under the condition of not imposing a limit on an amplitude control range. Then, the amplitudes are adjusted so as to fall within the amplitude control range by a projection method, and the amplitudes and phases allowing the formation of the desired null are again calculated. However, such a procedure should be iteratively performed until the amplitudes fall within the predetermined amplitude control range, and thus a calculation time is also problematic. Reference [11] proposes a method that sets amplitudes falling within a predetermined control range, and controls phases to form a null with the amplitudes as initial values. However, since the method does not simultaneously optimize the amplitudes and the phases, the optimum solution is not necessarily obtained.

This paper proposes a new pattern synthesis algorithm adapted to optimize the excitation amplitudes and phases of the APAA within the predetermined amplitude control range. The proposed method does not require iterative calculation, and makes it possible to obtain the optimum excitation amplitudes and phases by only performing optimization design once.

2 Issues of APAA

In the APAA, in order to control the amplitude of each element, power input to an amplifier differs among elements. When lowering side lobes over a wide range or forming a null in a high side lobe, a wide range amplitude width is required, and consequently an amplifier operating at a point where power efficiency is low exists. In particular, when mounting in a satellite, since power supply resources are limited, it is necessary to operate the amplifiers which the power efficiency is high. In order to operate the amplifiers within a certain range, it is only necessary to keep the excitation amplitudes of the APAA within a specified range. Fig. 1(a) illustrates a radiation pattern (solid line) obtained without taking account of an amplitude control range and a radiation pattern obtained when a null is formed at a first side lobe position (+6.8°) with excitation uniformed. Calculation specifications are the same as the simulation conditions in section 4, and Fig. 1(b) illustrates an excitation amplitude distribution. In order to obtain the radiation pattern using the pattern synthesis algorithm not taking account of the amplitude control range and then impose a limit on the amplitude control range as described above, a method adapted to limit excitation amplitudes equal to or less than a certain constant value to the same value is easily conceivable. Fig. 1 also illustrates a radiation pattern and excitation amplitude distribution obtained when limiting the amplitude control range to 2 dB. In actuality, the null formed at +6.8° is eliminated by the limitation of the excitation amplitudes and desired characteristics cannot be obtained. In order to resolve this problem, a pattern synthesis algorithm adapted to obtain a radiation pattern while taking proper account of the amplitude control range is required.

3 Proposed algorithm

The pattern synthesis algorithm adapted to limit the amplitude distribution of the driven elements of the APAA within a specific range will be described. In addition,
the conjugated gradient method employed in Reference [12] is used. The independent variables to be optimized take values within the range of $-\infty$ to $+\infty$, which can be achieved by introducing the mapping function resulting in a specific amplitude range. Here, using an arctangent function, the mapping function is defined as follows,

$$A_n = a \tan^{-1} x_n + \frac{\pi}{2},$$

where $x_n$ represents each independent variable to be optimized by the conjugated gradient method, and $A_n$ represents the excitation amplitude of each element. Coefficient $(a)$ is obtained from the amplitude control range $D$ [dB] as follows.

$$a = \frac{10^{D/20} - 1}{10^{D/20} + 1}.$$  \hspace{1cm} (2)

The Amplitude $A_n$ falling in the amplitude control range $D$ [dB] is obtained with Equation (1), and using Equation (2) the coefficient $(a)$ is found for determining the maximum and minimum of the amplitude $A_n$ with respect to the amplitude control range $D$ [dB]. As the excitation amplitude of an actual element, amplitude normalized as follows is used so as to satisfy the energy conservation law.
Next, an example of an objective function used when forming a null is defined as follows.

\[ F = \sum_{m=1}^{M} \exp\left(1 - \frac{g_m(\theta_m)}{G_m(\theta_m)}\right) + \sum_{i=1}^{I} \exp\left[-\left(1 - \frac{g_i(\theta_i)}{G_i(\theta_i)}\right)\right]. \]  

(4)

\[ g_{m,i}(\theta_{m,i}) = \left| \sum_{n=1}^{N} A'_n \exp(j\phi_n) \times e_n(\theta_{m,i}) \times \exp(jk\vec{r}_n \cdot \vec{R}(\theta_{m,i})) \right|^2 \]  

(5)

The first and second terms of Eq. (4) are objective functions related to gains in a desired beam direction and null formation direction, respectively. M represents the number of desired beam directions \( \theta_m \) (1 \( \leq \) m \( \leq \) M), and a gain in \( \theta_m \) and a value related to a target gain are respectively denoted by \( g_m(\theta_m) \) and \( G_m(\theta_m) \). \( G_m(\theta_m) \) is a constant, and adjusted so as to satisfy the target gain. Parameters \( N, \theta_i, g_i(\theta_i), \) and \( G_i(\theta_i) \) related to the null in the second term are also defined in a similar manner. Eq. (5) is a gain calculation expression, in which \( \phi_n \) represents the excitation phase of each element, \( e_n(\theta_{m,i}) \) the radiation pattern gain of that element, \( \vec{r}_n \) the position vector of the element, and \( \vec{R}(\theta_{m,i}) \) a direction vector in the desired direction or null formation direction. \( x_n \) and the phase \( \phi_n \) in Eq. (1) are set as the independent variables to obtain the gradient vectors of the objective functions in Eq. (4), and consequently the conjugated gradient method can be applied. By setting the target gain for the desired beam direction and null formation direction, and finding the solution where the objective function is minimized, a radiation pattern is obtained where the excitation distribution and null are formed in the specified amplitude control range.

4 Characteristics evaluation by simulation

In order to examine the proposed algorithm, effectiveness will be examined by the null formation simulation.

4.1 Simulation conditions

Table I lists the simulation conditions. The number of element antennas is 16, the elements are arrayed linearly, the interval between elements is 0.76\( \lambda \), and an element pattern is set to have cosine electric field directivity (p = 1.0). The simulation is performed under the conditions that the initial excitation distribution is based on equal amplitude and equal phase, the desired direction is a front direction (0°), and the first side lobe position (+6.8°) is in the null formation direction. The target gain in the desired direction is set to 12 dBi, and the target gain in the null formation direction is set to −18 dBi. Here, the gain in the desired direction and the gain in the null formation direction are values calculated with Equation (5), and in this simulation the gain in the desired direction and the gain in the null formation direction were set so the gain in the null formation direction is −30 dB or less. As a simulation procedure, the predetermined amplitude control
range is first set. Then, the desired direction and the null formation direction are determined. Subsequently, excitation amplitudes and phases allowing the formation of the null are obtained using the pattern synthesis algorithm proposed in the previous section, and finally, the radiation pattern is calculated.

### Table 1. Simulation conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of element antennas</td>
<td>16</td>
</tr>
<tr>
<td>Array type of elements</td>
<td>Linear array</td>
</tr>
<tr>
<td>Interval between elements</td>
<td>0.76 λ</td>
</tr>
<tr>
<td>Element antenna pattern</td>
<td>( \cos^p \theta ) (( p ) = 1.0)</td>
</tr>
<tr>
<td>Desired direction ( \theta_d ) (( M ) = 1)</td>
<td>0°</td>
</tr>
<tr>
<td>( G_m(\theta_d) ) (( m ) = 1)</td>
<td>12 dBi</td>
</tr>
<tr>
<td>Null direction ( \theta_i ) (( I ) = 1)</td>
<td>+6.8°</td>
</tr>
<tr>
<td>( G_i(\theta_i) ) (( i ) = 1)</td>
<td>−18 dBi</td>
</tr>
<tr>
<td>Initial excitation distribution</td>
<td>Equal amplitude and equal phase</td>
</tr>
</tbody>
</table>

#### 4.2 Radiation pattern at null formation

Fig. 2(a) illustrates radiation patterns obtained when changing the amplitude control range. In the diagram, the radiation patterns are normalized to the maximum gain. Also, Fig. 2(b) and 2(c) respectively illustrate the amplitude distributions and phase distributions of the elements. In Fig. 2(a) to 2(c), the solid line, dotted line, dashed line, and dashed-dotted line represent characteristics obtained when the amplitude control range is unlimited, limited to 3 dB, limited to 2 dB, and limited to 1 dB, respectively. In Fig. 1(a), when the amplitude control range is limited to 2 dB, the null is eliminated; however, it turns out that when using the proposed algorithm, excitation amplitudes within the amplitude control range allow the formation of the null. Gain in the desired direction is 11.83 dBi when the amplitude constraint range is 3 dB, 11.80 dBi when it is 2 dB, and 11.74 dBi when it is 1 dB, and a gain was obtained within 0.3 dB with respect to the target gain. A rise in a side lobe level associated with the null formation increases as the amplitude control range is narrowed. In Fig. 2(b), the amplitude distributions respectively satisfy corresponding predetermined amplitude control ranges, which differs from the amplitude distribution illustrated in Fig. 1(b) where the amplitude control range is limited to 2 dB. From Fig. 2(c), we can see that as the amplitude control range is narrowed, a phase difference for forming the null increases.
5 Conclusion

We propose a pattern synthesis algorithm taking into consideration the control range of the excitation amplitude of each element in the active phased array antenna. In the algorithm, a mapping function adapted to limit the excitation amplitude range of each element is introduced, and the independent variables are

![Radiation pattern](image1)

![Amplitude distribution](image2)

![Phase distribution](image3)

Fig. 2. Null formation when changing amplitude control range.
optimized by the conjugated gradient method. In addition, an array antenna having 16 elements has been numerically simulated and it has been determined that a null is formed within the predetermined amplitude control range.