Analysis of the interference from GFDM to OFDM signals in same band

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Abstract: This paper focuses the interference from GFDM signal to already-existing OFDM signal when both signals coexist in same band, considering some migration scenario from 4G to 5G mobile system. After defining system model, the mathematical expression is derived theoretically and numerically verified by computer simulation. This mathematical expression indicates that the time difference does not affect the cross-correlation between GFDM and OFDM symbols, and that the interference decreases approximately by square of subcarrier interval between those symbols. Derived formula can be used not only for the interference evaluation in various configurations, but also for designing the waveform filter of GFDM.

Keywords: 5G, GFDM, OFDM, interference, cross-correlation

Classification: Wireless Communication Technologies

References


1 Introduction

Aiming at post 4G era, various multicarrier modulation schemes other than OFDM are being studied. FBMC (Filter Bank Multicarrier) [1], UFMC (Universal Filtered Multicarrier) [2], GFDM (Generalized Frequency Multicarrier Multiplexing) [3, 4] and so on are examples of these so called “New Waveforms”. Since these schemes have less OOB (out-of-band) emissions in general, more efficient spectrum usage is expected. On the other hand, inter-subcarrier interference occurs in such non-orthogonal schemes. When such non-orthogonal multicarrier schemes coexist with OFDM signal in the same band, this interference could be stronger than the OFDM signal because the OFDM assumes orthogonality between subcarriers. This case is, however, quite likely to occur in future, due to migration scenarios from 4G to 5G system. Although some numerical results have already been reported by computer simulation [5], an analytical approach is still essential to study or to design the coexisting multicarrier schemes.

This paper focuses on the interference from GFDM signal to already-existing OFDM signal when both signals coexist in the same band. After defining system model, the formula is derived theoretically in section 2, which is numerically verified in section 3. Through this work, some features are stated and concluded.

2 System model and interference analysis

2.1 Mixed transmission of multicarrier signals

Fig. 1(a) shows the concept of hybrid multicarrier system. At the transmitter side, a filtered multicarrier signal such as GFDM signal along with a conventional OFDM signal is generated. GFDM is provided for advanced receivers, while OFDM is for legacy receivers, i.e., Fourier transform reception. Note that the OFDM receiver can detect OFDM signals only. GFDM signal interferes with such legacy receiver’s detection. In Fig. 1(a), the $m$-th subsymbol $s_{G,m}$ at $k_G$-th subcarrier in GFDM signal appears at legacy OFDM receivers as interference.

Let the maximum number of OFDM subcarriers be $N_F$ which is equal to the effective length of OFDM symbol in samples, as shown in Fig. 1(b). In this figure, $k_F$ ($k_F = 0, 1, \ldots, N_F - 1$) indicates the OFDM subcarrier position.

As for GFDM parameters, let the maximum number of subcarriers be $K_G$, considering $k_G$-th GFDM subcarrier interfering $k_F$-th OFDM subcarrier, where $k_G = 0, 1, \ldots, K_G - 1$. The $K_G$ is the length of GFDM subsymbol as shown in Fig. 1(b). The integer $m$ indicates the GFDM subsymbol number ranging from 0 to $M - 1$. $M$ is the total number of subsymbols in one GFDM symbol.

In generating GFDM signal, a waveform filter is used, whose impulse response $h(n)$ is defined by:

$$h(n) = \sum_{k'=-k_0}^{k_0} H(k')e^{j2\pi k' n / N_G}$$

(1)

where $n$ is the sample number corresponding to time, taking the values $0, 1, \ldots, N_G - 1$. $N_G$ is the length of a GFDM symbol in samples, which is $M K_G$. $H(k)$ is a frequency transfer function of the waveform filter. The parameter $k_0$ corresponds to the stop frequency of the waveform filter. For instance, in case of the filter with roll-off factor $\alpha$: 

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\[ k_0 = \left[ \frac{1 + \alpha}{2} \right] M \]  

(2)

The GFDM symbol is composed of \( M K_G \) modulation symbols. Each modulation symbol is multiplied by the following complex coefficient at time \( n \).

\[ g(n) = h(n - mK_G)e^{j2\pi n k_0}/CM \]

(3)

where \( k_G \) and \( m \) indicate the subcarrier number and the subsymbol number of the modulation symbol, respectively. Note that, some references, e.g. [3], in the definitions use opposite sign for the exponent in Eq. (3), i.e. \( h(n - mK_G)e^{-j2\pi n k_0}/CM \). However in this paper, in order to reduce the interference to OFDM signal and to match the definition in generating OFDM signal, Eq. (3) is used as defined in [4].

\[ c(m, \Delta k_F) = \sum_{i=0}^{N_F-1} g(i + \Delta n)e^{-j2\pi n k_0}/CM = \sum_{i=0}^{N_F-1} h(i + \Delta n - mK_G)e^{j2\pi \left\{ \frac{i + \Delta n - mK_G}{N_F} \right\}} \\
= \sum_{i=0}^{N_F-1} \sum_{k'=k_0}^{k_0} H(k')e^{j2\pi \left\{ \frac{i + \Delta n - mK_G + \Delta k_F}{N_F} \right\}} \\
= e^{j2\pi \Delta k_F/n} \sum_{k'=k_0}^{k_0} H'(k')e^{j2\pi (\Delta n - mK_G + \Delta k_F)/N_F} \]

(4)

where:

\[ h(n) = h_0(n) + h_1(n), \quad h_0(n) = \sum_{m=0}^{M-1} \sum_{k=0}^{N_F-1} a_m e^{j2\pi n k}/CM \]

Fig. 1. System model and symbol configuration.

2.2 Interference analysis

The interference from \( s_{kG,m} \) for the target OFDM symbol can be evaluated by calculating the cross-correlation between \( s_{kG,m} \) and the OFDM symbol. Let the correlation be noted as \( c(m, \Delta k_F) \), where \( \Delta k_F \) is the subcarrier interval defined as \( \Delta k_F = N_F k_G/N_G - k_F \). The metric \( c(m, \Delta k_F) \) is derived as follows.

where:

- \( s_{kG,m} \) is the GFDM symbol.
- \( K_G \) is the number of modulation symbols.
- \( m \) is the subsymbol number.
- \( N_F \) is the number of subcarriers in the OFDM symbol.
- \( \Delta n \) is the time delay in samples.
- \( H(k) \) is the frequency response of the channel.
- \( \alpha \) is a parameter related to the modulation scheme.

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= \sum_{i=0}^{N_F-1} \sum_{k'=k_0}^{k_0} H(k')e^{j2\pi \left\{ \frac{i + \Delta n - mK_G + \Delta k_F}{N_F} \right\}} \\
= e^{j2\pi \Delta k_F/n} \sum_{k'=k_0}^{k_0} H'(k')e^{j2\pi (\Delta n - mK_G + \Delta k_F)/N_F} \]

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where:

- \( h(n) = h_0(n) + h_1(n), \quad h_0(n) = \sum_{m=0}^{M-1} \sum_{k=0}^{N_F-1} a_m e^{j2\pi n k}/CM \)
\[ H'(k') = H(k') \] 
\[ \sin\left\{ \pi \left( \frac{N_F k'}{N_G} + \Delta k_F \right) \right\} \] 
\[ \sin\left\{ \pi \left( \frac{k'}{N_G} + \frac{\Delta k_F}{N_F} \right) \right\}. \] 

(5)

When \(|s_{kG,m}| = 1\), interference power \(I(\Delta k_F)\) from all \(M\) subsymbols of the GFDM subcarrier is as follows.

\[ I(\Delta k_F) = \sum_{m=0}^{M-1} |c(m, \Delta k_F)|^2 \]
\[ = \sum_{k'=-k_0}^{k_0} H'(k') \sum_{k''=-k_0}^{k_0} H''(k'') e^{j2\pi \left( \Delta n + \frac{N_F k'}{N_G} \right) k''} e^{-j2\pi m k''} \]
\[ = M \sum_{k'=-k_0}^{k_0} |H'(k')|^2 \] 

(6)

Therefore, the interference is independent of the time difference between symbols, \(\Delta n\).

If \(\Delta k_F\) is an integer:

\[ I(\Delta k_F) = M \sum_{k'=-k_0}^{k_0} |H(k')|^2 \sin^2\left\{ \pi \left( \frac{k'}{N_G} + \frac{\Delta k_F}{N_F} \right) \right\}. \]

(7)

Considering \(k_0\) as defined in Eq. (2), \(k_0/N_G \leq (1 + \alpha)/(2K_G) \ll 1\) usually holds true. Assuming that \(\Delta k_F/N_F < 1\), for \(\Delta k_F\) such that \(k_0/N_G \ll \Delta k_F/N_F \ll 1\), the following approximation is derived:

\[ I(\Delta k_F) \approx M \left( \frac{N_F}{\pi \Delta k_F} \right)^2 \sum_{k'=-k_0}^{k_0} |H(k')|^2 \sin^2\left( \pi \frac{N_F k'}{N_G} \right) \] 

(8)

This means that the interference decreases by square of subcarrier interval \(\Delta k_F^2\).

3 Numerical examples with simulation

Table I shows evaluation parameters in this section. By computer simulation, EVM (error vector magnitude) of detected OFDM symbol and the BER (bit error rate) are evaluated. In the theoretical analysis, the average square of EVM is considered as Eq. (7). If the interference is considered as Gaussian noise, the BER can be expressed as in Eq. (9) where \(E_b/N_0\) is the ratio of the received energy per bit to the density of AWGN power.

<table>
<thead>
<tr>
<th>Table I. Evaluation parameters</th>
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<tbody>
<tr>
<td>modulation</td>
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<tr>
<td>(N_F)</td>
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<td>(K_G)</td>
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<td>(M)</td>
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<td>(\Delta n)</td>
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Results are shown in Fig. 2, which indicate the computer simulation agrees the analysis in Section 2.2. Note that a slight difference in BER performance is seen between simulation and analysis results in Fig. 2(b). The reason can be that the interference from the GFDM signal with QPSK modulation cannot be regarded as Gaussian noise exactly.

\[ P_b = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{1}{2(\Delta k_F) + N_0/E_b}} \right) \]  

\( (9) \)

4 Conclusions

This paper focused on GFDM coexisting with OFDM signal for migration scenarios from 4G to 5G mobile system, and theoretically analyzed the interference from the GFDM signal to the standard OFDM receiver. It has been shown that the analytical interference formula agrees well to the simulation results. Through this
work, the following properties have been pointed out. (1) Although some references adopt DFT at the GFDM generator, IDFT should be used to suppress the interference. (2) The interference is independent of the time difference between GFDM and OFDM symbols, which means that the cross-correlation does not change in multipath channel. (3) Approximately, the interference decreases by square of the subcarrier interval between the GFDM and OFDM symbols.

The analytical expression that is derived in this paper can be used not only for the interference evaluation in various symbol configurations, but also for some filter design purposes such as to reduce the cross correlation.

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