Representation of an equivalent circuit for capacitive wireless power transfer using a distributed-constant circuit

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Abstract: Capacitive wireless power transfer is performed using large electrodes or using a medium having high relative permittivity such as water. The transfer efficiency demonstrates a pronounced change with frequency. A lumped-constant equivalent circuit with parasitic elements represents the change. However, a relationship between conditions for generating minimum/maximum values of the transfer efficiency and structural parameters of the electrodes in the coupler has not clarified. In this study, an equivalent circuit is proposed using a distributed-constant circuit. We converted the lumped-constant circuit into two open-ended directional couplers. From the result, we derived the conditions for generating the minimum and maximum values of the voltage input/output ratio. The efficiencies calculated by the distributed-constant circuit and electromagnetic simulation were in good agreement.

Keywords: wireless power transfer, capacitive wireless power transfer, directional coupler, equivalent circuit, maximum available efficiency

Classification: Energy in Electronics Communications

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1 Introduction

Many studies on capacitive wireless power transfer (WPT) use the lumped-constant equivalent circuit [1, 2, 3, 4, 5, 6]. The transfer efficiency shows significant change with frequency in capacitive WPT via water. The change in the transfer efficiency is represented by adding parasitic elements into a conventional equivalent circuit [4, 5, 6], and is predicted by a voltage input/output ratio derived from the equivalent circuit. However, a relationship between conditions for generating minimum/maximum values of the transfer efficiency and structural parameters of the electrodes in the coupler is not clarified. In this study, an equivalent circuit is proposed via fresh water using the distributed-constant circuit. It is elucidated from our circuit that the minimum and maximum values of the efficiency can be predicted depending on wavelength of the transfer frequency. First, the frequency characteristic of the maximum available efficiency was calculated by electromagnetic (EM) simulations. The lumped-constant equivalent circuit with the parasitic elements represents the characteristic. One study has used a general directional coupler for WPT [7]. We converted the equivalent circuit into the two open-ended directional couplers (i.e., a distributed-constant circuit). Using the couplers, the conditions for generating the minimum and maximum values of the voltage input/output ratio were derived. The efficiencies calculated by the distributed-constant circuit and EM simulation were in good agreement. Finally, it was shown that the minimum and maximum values of the transfer efficiency can be predicted from the structural parameters of the coupler electrodes.

2 Representation by lumped-constant circuit

Figures 1(a) and (b) show the coupler model and the equivalent circuit of the capacitive WPT, where #1–#3 and #2–#4 show the primary and secondary sides, respectively. The capacitances that occurred in #1–#4 and #2–#3 cannot be ignored, especially when there is misalignment between the primary and secondary sides [2]. The capacitances were ignored because this study assumes that there is perfect symmetry without misalignment. The capacitance C_s and loss R_s occurred at that adjacent electrodes (#1–#3, #2–#4). Furthermore, C_m and R_m occurred at the facing

Fig. 1. Representation by lumped-constant circuit.

electrodes (#1–#2, #3–#4). L_m and L_s are parasitic inductances in each capacitance. As shown in Fig. 1(c), the coupler model comprises four metal plates and four 50- Ω reference ports imitated subminiature connectors. Here, tap water is represented using the relative permittivity $\epsilon_r = 79$ and the conductivity $\sigma = 0.011$ S/m. We calculated the maximum available efficiency $\eta_{\max}$ [3], which indicates the achievable power transfer efficiency under conjugate matching conditions. First, we calculated the four-port single-ended S-parameters using the equivalent circuit (advanced design system) and EM simulation (CST microwave studio) and converted them into two-port differential S-parameters. Next, the results were converted into the two-port impedance matrix. Finally, $\eta_{\max}$ was calculated by substituting the impedance results into the following two equations:

$$\alpha = \frac{|Z_{21}|}{\sqrt{R_{11}R_{22} - R_{12}R_{21}}}. \quad (1)$$

$$\eta_{\max} = 1 - \frac{2}{1 + \sqrt{1 + \alpha^2}}. \quad (2)$$

Figure 1(d) shows the calculated $\eta_{\max}$ values. The element values are as follows: $C_s = 72$ pF, $R_s = 960$ Ω , $L_s = 95$ nH, $C_m = 840$ pF, $R_m = 74$ Ω , and $L_m = 8.5$ nH. The $\eta_{\max}$ values are calculated from the equivalent circuit in Fig. 1(b) and EM simulation, which are in good agreement up to around 60 MHz. However, the relationship between the conditions for generating the minimum/maximum values of the transfer efficiency and the structural parameters of the electrodes in the coupler is not clarified.

3 Representation by distributed-constant circuit

3.1 Formulation of the voltage input/output ratio

Figure 2 shows the procedure of the conversion from the capacitive coupler into the directional coupler. To simplify the figures, L_s , L_m , R_s and R_m are not listed. Figure 2(a) is represented as the balanced circuit topology by including the power

Fig. 2. Procedure of conversion into directional coupler.

source and load in Fig. 1(b). Here, there are three points to be noted. First, as mentioned in Section 2, C_m and C_s are the broadside coupling and edge coupling, respectively. Second, the four electrodes will be electrically floating metal plates (not four striplines structures). Third, the power supply position opposite the electrode is an open condition. Figure 2(b) is converted from Fig. 2(a) using an imaginary ground. Furthermore, Fig. 2(b) decomposes into Figs. 2(c) and (d), which are unbalanced circuits. Next, as shown in Fig. 2(e), Figs. 2(c) and (d) are converted into a directional coupler. A general directional coupler is the terminated Z_0 at all the ports [8]. In this study, half of all the ports are the terminated Z_{open} .

Next, we derive the voltage input/output ratio using Figs. 2(e) and (f). C_{in-GND} and $C_{out-GND}$ denote the capacitances between the two transmission lines to the ground. Furthermore, $2C_{in-out}$ denotes the capacitance between the two transmission

lines for the odd mode in Fig. 2(g). The capacitances of the two modes are expressed by Eqs. (3) and (4) [8]. Equations (5) and (6) show the characteristic impedances for the even and odd modes. Note that R_e and R_o show the loss generated at the inductances L_e and L_o , respectively, in each mode. Furthermore, G_e and G_o show the loss generated in Eqs. (3) and (4). Based on the results of Section 2, the conditions $R_s > 1/(\omega C_s)$ and $R_m > 1/(\omega C_m)$ are satisfied at 3 MHz. Therefore, we apply the lossless transmission line theory for conversion into the directional coupler as follows:

$$C_e = C_{\text{in-GND}} = C_{\text{out-GND}}. \quad (3)$$

$$C_o = C_{\text{in-GND}} + 2C_{\text{in-out}} = C_{\text{out-GND}} + 2C_{\text{in-out}}. \quad (4)$$

$$Z_{0e} = \sqrt{\frac{R_e + j\omega L_e}{G_e + j\omega C_e}} \approx \sqrt{\frac{L_e}{C_e}}. \quad (5)$$

$$Z_{0o} = \sqrt{\frac{R_o + j\omega L_o}{G_o + j\omega C_o}} \approx \sqrt{\frac{L_o}{C_o}}. \quad (6)$$

The input impedance for each mode is expressed by Eqs. (7) and (8). These equations can be simplified because of the open condition of Z_{open} .

$$Z_{\text{in}}^e = Z_{0e} \frac{Z_{\text{open}} + jZ_{0e} \tan \theta}{Z_{0e} + jZ_{\text{open}} \tan \theta} \approx \frac{Z_{0e}}{j \tan \theta}. \quad (7)$$

$$Z_{\text{in}}^o = Z_{0o} \frac{Z_{\text{open}} + jZ_{0o} \tan \theta}{Z_{0o} + jZ_{\text{open}} \tan \theta} \approx \frac{Z_{0o}}{j \tan \theta}. \quad (8)$$

The voltages at #1 for each mode are listed below:

$$V_{\text{in}}^{e'} = V_{\text{in}}' \frac{Z_{\text{in}}^e}{Z_{\text{in}}^e + Z_0}, \quad V_{\text{in}}^{o'} = V_{\text{in}}' \frac{Z_{\text{in}}^o}{Z_{\text{in}}^o + Z_0}. \quad (9)$$

Here, as long as $Z_{\text{in}} = Z_0$ is satisfied, we have $V_{\text{in}}' = V_{\text{in}}$ by voltage division in Fig. 2(e) because there was no voltage reflection. Moreover, by the preceding voltage definition in Fig. 2(f), V_{out} is expressed as follows:

$$V_{\text{out}} = V_{\text{out}}^e + V_{\text{out}}^o = V_{\text{in}}^{e'} + (-V_{\text{in}}^{o'}) = V_{\text{in}} \left[\frac{Z_{\text{in}}^e}{Z_{\text{in}}^e + Z_0} - \frac{Z_{\text{in}}^o}{Z_{\text{in}}^o + Z_0} \right]. \quad (10)$$

Equation (10) reduces to Eq. (11) using Eqs. (7) and (8). The voltage ratio of $V_{\text{out}}/V_{\text{in}}$ is given by Eq. (12).

$$V_{\text{out}} = V_{\text{in}} \left[\frac{Z_{0e}}{Z_{0e} + jZ_0 \tan \theta} - \frac{Z_{0o}}{Z_{0o} + jZ_0 \tan \theta} \right]. \quad (11)$$

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o} + j[Z_0 \tan \theta - Z_{0e}Z_{0o}/(Z_0 \tan \theta)]}. \quad (12)$$

From Eq. (12), we confirmed that the minimum and maximum values of the voltage ratio occurred when the imaginary parts were equal to $\pm\infty$ and 0, respectively. Furthermore, θ_{min} in Eq. (13) shows the phase difference when the imaginary part was equal to $\pm\infty$; the minimum values occurred when the phase difference was $(n-1)\pi/2$ (with the wavelengths 0, $\lambda/4$, and so on). From Eq. (14), $\arctan \pm 1$ (the longest wavelength is $\lambda/8$) holds when $\sqrt{Z_{0e}Z_{0o}} = Z_0$ is satisfied. If all the

ports were terminated at Z_0 in Fig. 2(e), we would have $\sqrt{Z_{0e}Z_{0o}} = Z_0$ [8]. However, $\arctan \pm 1 (\lambda/8)$ does not hold because $Z_{\text{open}} \neq Z_0$. Therefore, the wavelength having the maximum values are determined by the $\sqrt{Z_{0e}Z_{0o}}/Z_0$ ratio.

$$\theta_{\min} = \arctan 0, \arctan (\pm \infty) = \frac{(n-1)\pi}{2} \quad (n : 1, 2, 3, \dots). \quad (13)$$

$$\theta_{\max} = \arctan \left(\pm \frac{\sqrt{Z_{0e}Z_{0o}}}{Z_0} \right). \quad (14)$$

3.2 Verification

Figure 3(a) shows the equivalent circuit represented by two open-ended directional couplers. For verification, we assign concrete values to ϵ_r , σ , S , L , W , and B in Fig. 2(g). Here, L is the transmission length across which the phase difference occurs. The phase difference is governed by the length of the electrode, which is affected by the wavelength shorting derived from ϵ_r . Therefore, L is the diagonal length of the electrode (141 mm) in Fig. 1(c). W is $100 \times 100/141 = 71$ mm, which ensures that the area size is constant. Based on the information in Section 2, we specify $\epsilon_r = 79$, $\sigma = 0.011$ S/m, and the distance between the facing electrodes $S = 10$ mm. C_s corresponds to $C_{\text{in-GND}}$ and $C_{\text{out-GND}}$, as shown in Fig. 2(g). C_s is the edge-coupled capacitance, and the distance is 50 mm. However, $C_{\text{in-GND}}$ and $C_{\text{out-GND}}$ are the broadside-coupled capacitances and the distance is $(B - S)/2$. We compared the capacitances in the broadside-coupled and edge-coupled states, and specified that the broadside > edge. Therefore, B was more than 50 mm in length, and the fitting result was 360 mm.

Figure 3(b) shows the calculated $\eta_{\max}$ values; the minimum value was 59.8 MHz, and maximum values was 35.1 MHz. When we considered the effect of the wavelength shorting, we reported that 59.8 MHz was $\lambda/4$, and 29.9 MHz was $\lambda/8$ of $L = 141$ mm. The minimum values were in good agreement. The calculated results of Eq. (12) with $Z_0 = 1$, $Z_{0e} = 4$ and the various Z_{0o} values of 0.125, 0.25, and 0.5 are shown in Fig. 3(c). $\sqrt{Z_{0e}Z_{0o}} = Z_0$ holds at $Z_{0o} = 0.25$, and the maximum value occurred in $\lambda/8$ (45°). Moreover, we confirmed that the center around $\lambda/8$ was affected by the change in the $\sqrt{Z_{0e}Z_{0o}}/Z_0$ ratio. From the above, we observe that the difference between 29.9 MHz and 35.1 MHz occurred because $\sqrt{Z_{0e}Z_{0o}} > Z_0$.

Fig. 3. Representation of distributed-constant circuits.

4 Conclusion

In this study, we represented the equivalent circuit of the capacitive WPT using a distributed-constant circuit. The lumped-constant equivalent circuit was converted into the two open-ended directional couplers. Consequently, we determined the condition for the generation of minimum and maximum value of the voltage ratio. The $\eta_{\max}$ values were calculated using the distributed-constant equivalent circuit and EM simulation, which were in good agreement. Finally, we clarified that the minimum and maximum $\eta_{\max}$ values occurred at 0 and $\lambda/4$, respectively, and centered around $\lambda/8$. The changes in $\lambda/8$ depended on the $\sqrt{Z_{0e}Z_{0o}}/Z_0$ ratio.

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