Joint polar and run-length limited decoding scheme for visible light communication systems

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\textbf{Abstract:} In Visible Light Communication (VLC) systems, Run-length limited (RLL) codes are widely used to avoid the long runs of 1’s and 0’s. However, there are a small number of Forward Error Correction (FEC) codes that can well incorporate with the traditional RRL codes. In this paper, a serial joint of Polar code and Soft-Input Soft-Output (SISO) RLL decoding scheme is proposed to enhance the transmission efficiency while guaranteeing an equal number of 1’s and 0’s. The simulation results confirm that our proposed algorithm has consistently outperformed the existing schemes.

\textbf{Keywords:} Visible Light Communication (VLC), Polar Code, Dimming Control, RLL

\textbf{Classification:} Wireless communication technologies

\textbf{References}

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1 Introduction

Visible Light Communication (VLC), which provides data transmission as an additional functionality to illumination systems, has attracted increasing attention from academia and industry in recent years. In the simplest VLC system based on on-off keying (OOK) modulation technique, the long runs of 1’s and 0’s in the input data are the main cause of unstable brightness, clock and data recovery detection problems. As a result, Run-Length Limited (RLL) codes are applied to overcome these issues. RLL codes take in random input data and guarantee an equal number of 1’s and 0’s (DC balance) at the output. RLL codes generally show a low error correction ability; hence, Forward Error Correction (FEC) codes are essential to ensure a reliable data communication. Another drawback is that the traditional RLL codes only support hard decoding in the receiver’s side. This limits the performance of FEC codes. Three practical RLL codes (Manchester, 4B6B and 8B10B) and their well incorporate FEC code Reed-Solomon (RS) are employed in IEEE 801.15.7 standard [1].

Recently, many researchers have considered about FEC codes that not only archive better error correction performance but also keep the DC balance in VLC systems. Two FEC schemes based on modified Reed-Muller (RM) codes have been proposed for OOK-based VLC systems [2, 3]. These FEC codes can guarantee DC balance with short code length; however, their error control performances are also inferior to iterative decoding codes such as turbo or low-density parity-check (LDPC) codes. In the turbo codes-based coding scheme proposed by Lee and Kwon [4], acceptable number of 1’s and 0’s at the output are created by applying scrambling technique. This FEC algorithm requires long code length (1024 bits) to ensure the intensity of VLC system stable. In LDPC codes-based scheme proposed by Kim [5], the fixed code rate 1/2 LDPC code, code puncturing technique and Manchester RLL code are employed. Furthermore, the iterative FEC algorithms may increase complexity and decoding latency. Finally, the author in [6] developed a
SISO RLL decoding that combined with Bit-Level Soft (BLS) RS decoder. Although achieving better Bit Error Rate (BER) and Frame Error Rate (FER) performance than conventional schemes, the Soft RS decoder requires a large number of computation efforts.

This paper focuses on developing a joint FEC-RLL decoding mechanism for VLC systems. In this scheme, the state-of-the-art Polar code and SISO RLL decoding in [6] are embedded. Polar codes, introduced by Arkan [7], are the first probably capacity-achieving codes with low encoding and decoding complexity \(O(N\log N)\). Furthermore, the code rate can be adjusted freely to any rate between 0 and 1. The proposed decoding scheme has the following advantages: 1) It has equal probability of 1’s and 0’s in codewords, which is an important requirement to achieve dimming function in VLC system. 2) It supports various code rates while existing schemes can have limited code rate options. 3) It has higher coding efficiency than that of the other coding schemes.

2 Background

2.1 Polar codes

Polar codes, proposed by Arikan in 2009, provide a novel idea for the capacity-approaching code construction and create a new field of coding theory.

A polar code can be specified completely by \((N,K,I)\), where \(N\) is the length of a codeword in bits, \(K\) is the length of information bits vector \(u\) and \(I = |K|\) is the set of information bit indices. The remaining indicates subset \(\mathcal{F} = |N - K|\) is called as frozen bit indices \[8\]. For a \((N,K,I)\) polar code, \(R = K/N\) is the code rate and \(N\) is the power of 2. Let \(n = \log_2(N)\) and \(G = F \otimes n = F \otimes ... \otimes F(n \text{ copies})\) is the n-fold Kronecker product of Arikans [7] standard polarizing kernel \(F = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\). Then a codeword is generated as Eq. (1).

\[
x = u.G_I = d.F \otimes n
\]

where \(d\) is a vector of \(N\) bits length including information bits and frozen bits such that \(d_I = u\) and \(d_F = 0\), respectively. Polar codes follow the form in Eq. (1) are non-systematic codes.

On the other hand, a systematic polar code can be described as an equivalent to original polar code, except that the information vectors are mapped to codewords, such that the information bits are explicitly visible [9]. The most popular decoding algorithm for Polar code is the Successive Cancelation (SC) algorithm which is also introduced by Arikan. In fact, the same SC decoder can be used for both systematic and non-systematic codes.

2.2 SISO RLL decoder

The \((q,s)\) RLL encoder takes the \((m,q)\)-bits codeword of Polar encoder as its input \(c = [c_1, c_2, ..., c_i, ..., c_m]\), where \(c_i = [c_i^{(1)}, c_i^{(2)}, ..., c_i^{(q)}]\), to create \((m,s)\)-bits output \(\bar{c} = [\bar{c}_1, \bar{c}_2, ..., \bar{c}_i, ..., \bar{c}_m]\), where \(\bar{c}_i = [\bar{c}_i^{(1)}, \bar{c}_i^{(2)}, ..., \bar{c}_i^{(s)}]\).

In the receiver, the inputs of RLL decoder are soft values that defined as
\( y = [y_1, y_2, \ldots, y_i, \ldots, y_m] \), where \( y_i = [y_i^{(1)}, y_i^{(2)}, \ldots, y_i^{(s)}] \); similarly, the outputs of the RLL decoder are represented as \( z = [z_1, z_2, \ldots, z_i, \ldots, z_m] \), where \( z_i = [z_i^{(1)}, z_i^{(2)}, \ldots, z_i^{(q)}] \). Based on this notation, the decoding algorithm is summarized as follow:

1. Each bit probabilities of ‘0’ and ‘1’ based on OOK in AWGN channel can be calculated as
   \[
   p(y_i^{(j)}|\bar{c}_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y_i^{(j)}^2}{2\sigma^2}}
   \]
   \[
   p(y_i^{(j)}|\bar{c}_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_i^{(j)}-I_p)^2}{2\sigma^2}}
   \]
   where \( j = 1, 2, \ldots, s \) and \( I_p \) is the peak photocurrent.

2. From each bit probabilities, the SISO RLL decoder calculates probabilities of \( s \)-bit codeword as
   \[
   p(y_i|\bar{c}_i) = \prod_{j=1}^{s} p(y_i^{(j)}|\bar{c}_i^{(j)})
   \]
   It should be noticed that there are \( 2^q \) possible codewords \( \bar{c}_i \) in total.

3. Each \( \bar{c}_i \) is the unique value that created by mapping rule \( M \) from the corresponding input \( c_i \). As a result,
   \[
   p(y_i|c_i) = p(y_i|\bar{c}_i)
   \]

4. The \( l \)-th bit probabilities in \( c_i \) are calculated by marginalizing the joint probability as
   \[
   p(y_i^{(l)}|c_i^{(l)}) = \sum_{c_i^{(l)} = \epsilon} p(y_i|c_i)
   \]
   where \( \epsilon \in \{0, 1\} \) and \( l = 1, 2, \ldots, q \).

5. Finally, the LLR value \( z_i^{(l)} \) is produced as
   \[
   z_i^{(l)} = \log \frac{p(y_i^{(l)}|c_i^{(l)}) = 0)}{p(y_i^{(l)}|c_i^{(l)}) = 1)}
   \]
   where this LLR value is used as the input of a soft-decision FEC decoder afterwards.

### 3 Proposed system model

In this paper, we primarily focus on the downlink transmission of VLC systems. The block diagram of system is shown in Fig. 1. Assuming \( n \)-bit codeword \( c \) is generated after passing information vector \( u \) with \( k \) bit length through Polar FEC encoder block. Then, the LUT-based \( (q, s) \) RLL encoder block is used to map every \( q \)-bit groups in \( c \) to \( s \)-bit groups in \( \bar{c} \), based on the mapping rule \( M \) that guarantees the DC balance in the system. Finally,
Fig. 1. Block diagram of the proposed VLC system.

$\bar{c}$ is converted into OOK signal $x(t)$ and emitted through the channel $h(t)$ by Light Emitted Diode (LED) front-end circuit. The photodiode (PD) receives the signal $r(t) = R.s(t) * h(t) + n(t)$, where $R$ is the receiver responsivity ($A/W$); $*$ denotes convolution; and $n(t)$ is Additive White Gaussian Noise (AWGN) which contains shot and thermal noises with the variance $\sigma^2$ [10]. Using match filter at the output of OOK demodulator, photo current values $y$ for the Soft-Input Soft-Output SISO $(q,s)$ RLL decoder are generated. From $s$ values $y_i$ in $y$, the SISO calculates 4 new Log-Likelihood Ratio (LLR) values $z_i$ for Polar decoder to estimates the transmitted information $\hat{u}$.

4 Simulation results

In this section, we present simulation results for the joints of SISO 4B6B RLL code ($q = 4; s = 6$) with Polar codes. In each Polar code, both Non-systematic (Polar-Nonsys) and Systematic (Polar-Sys) are considered to make comparisons with the combination of SISO 4B6B and BLS RS codes over GF($2^4$) which were investigated in [5]. Three code rates of Polar codes (64,44), (64,28), and (64,12) are used to compare with that of the reference BLS RS codes (15,11), (15,7), and (15,3), respectively.

As is highlighted in Fig. 2, the BER performances of proposed schemes are better than those of the reference BLS RS codes at all corresponding code rates. In the highest code rate case, the gains of proposed scheme nearly 0.3 dB (Polar-Nonsys) and 0.7 dB (Polar-Sys) than BLS RS (15,11). Furthermore, while the BER performances of BLS RS codes are reduced when the code rates decrease from the highest (15,11) to the lowest code rate (15,3), the BER performances of 4B6B-Polar codes show opposite trends. For the lowest code rate, the proposed scheme (Polar-Nonsys and Polar-Sys) can archive BER $= 10^{-5}$ when $E_b/N_0$ are 5.60 dB and 5.33 dB while BLS RS (15,3) requires $E_b/N_0$ up to 13.0 4dB. Moreover, the Systematic Polar code has higher BER performance compared with Non-systematic Polar at the same code rate. We next compared the FER performances in Fig. 3. To calculate the FERs, one codeword corresponds to one frame. It can be clearly seen that the proposed schemes outperform BLS RS codes in all cases.
5 Conclution

In this paper, we have presented a joint of Polar and Run-Length Limited scheme as a strong FEC candidate for VLC systems. With the proposed scheme, the number of 1’s and 0’s in codewords are exactly the same. Simulations proposed algorithm exhibit significantly better performance than the BLS RS codes. An interesting future direction would be to consider practical VLC systems with hardware implementation.

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