Low-Complexity Differential Modulation for High Mobility MIMO-OFDM

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Abstract:
Differential Modulation is a non-coherent modulation technique which is used to solve the uncertainty channel problem. The most popular Differential Unitary Modulation technique uses a Space-Time Block Code (STBC) to improve the performance of Differential Modulation technique in multi-in multi-out (MIMO) system. However, using Differential Unitary Modulation technique in the multicarrier system become a big challenge because of the rising of the system complexity. In this paper, we introduce a low complexity differential modulation technique using the property of STBC. The proposed method is proven to provide high-quality performance with a low level of complexity.

Keywords: differential modulation, MIMO, OFDM, complexity

Classification: Wireless communication technologies

References

1 Introduction

High mobility communication until now still become an interesting research topic in the wireless communication. The most important problem which is occurred in high mobility condition is that the channel condition changes rapidly. This condition affects the accuracy of channel estimation while increasing the number of pilots is not a good option because it decreases the spectral efficiency. Non-coherent transmission system which does not employ the channel estimation is one of the solutions which is proposed to overcome this problem. Differential Modulation (DM) is the most interesting technique to be developed in non-coherent transmission system research. The differential modulation research aims to develop a good communication system which can be implemented even in the unpredicted channel condition.

In the era of the MIMO antenna system, the combination between DM technique and MIMO has produced some DM techniques [1, 2, 3, 4]. One of the most potential technique is called Differential Unitary Modulation (DUM) Technique which uses STBC Alamouti [5] as a Unitary Matrix [1, 2, 3]. In the papers [2, 3], Tran et al. introduce how to implement the DM technique in one of the multicarrier systems, Orthogonal Frequency Division Multiplexing (OFDM). This technique is called Unitary Differential Space-Time Frequency Modulation (UDSTFM). To arrange the symbols into Unitary Matrix, Tran et al. use diagonal matrix constellation. However, this diagonal constellation technique has a major problem, it has a very high system complexity and the worst part is the complexity increases significantly along with the increase of subcarriers. High complexity can be mean a lot, it may increase the power usage, the cost, and has a longer processing time.

In this paper, we propose a set of encoding and decoding of DM for MIMO OFDM. We use the property of STBC to create this technique, and we will show the significant difference between our method and DUM in term of complexity. We believe that our proposed method can be implemented widely in the DUM systems to solve the complexity problem of the other systems. This paper is arranged into the following section; 1. Introduction, 2. System Models, 3. Performance Evaluation, and 4. Conclusion.

Notations: In this paper, the superscript (.)* and (.)^T denotes the complex conjugate operation and transpose operation respectively. We denote the notation “∘” as the notation of element-wise (Hadamard) product multiplication.

2 System Models

In this paper, we propose a set of Differential Modulation encoding and decoding schemes which transmit and receive the message via 2x2 MIMO OFDM system. The concept of our proposed technique is to change the multiplication between two unitary matrices into element-wise multiplication. This idea is proposed for minimizing the system complexity of Differential Unitary Modulation technique.
The original concept of DUM is shown in the equation (1). The symbols $a, b, c, d$ are random symbols outputs of the modulator. The symbols are paired in two different STBC Alamouti matrices.

$$
\begin{bmatrix}
a & b \\
-b^* & a^*
\end{bmatrix}
\begin{bmatrix}
c & d \\
-d^* & c^*
\end{bmatrix} =
\begin{bmatrix}
ac - bd^* & ad + bc^* \\
-cb^* - a^*d^* & a^*c^* - db^*
\end{bmatrix}
\quad (1)
$$

The result of equation (1) is the same as the equation (2) below:

$$
\begin{bmatrix}
a & b \\
-b^* & a^*
\end{bmatrix} \circ \begin{bmatrix}
c & c^* \\
-d^* & d
\end{bmatrix} +
\begin{bmatrix}
b & a \\
-a^* & -b^*
\end{bmatrix} \circ \begin{bmatrix}
-d^* & d \\
-c^* & c
\end{bmatrix}
\quad (2)
$$

The element-wise calculation concept in equation (2) will be the basic concept of our proposed DM calculation process.

### 2.1 Encoding

As mentioned in the previous section, the proposed model uses the property of STBC Alamouti [5] to encode the symbols which come out from the PSK Modulator. The STBC Matrix can be written as the following equation:

$$
C_t = 1/\sqrt{2} \begin{bmatrix}
C_{t,1} & C_{t,2} \\
-C_{t,2}^* & C_{t,1}^*
\end{bmatrix}
\quad (3)
$$

$C_{t,m}$ is a transpose of column matrix with the size $(N_{fft} \times 1)$, where $N_{fft}$ is the total number of the subcarrier. The notation $t$ represents the time of transmission and $m$ is the number of antenna. The $C_{t,m}$ consists of $c_{t,m,k}$ which is the output of the PSK modulator. Notation $k$ represents the subcarrier’s number. Then, the $C_{t,m}$ can be written as: $C_{t,m} = [c_{t,m,1}, c_{t,m,2}, ..., c_{t,m,k}]^T$. Therefore, $C_t$ is a matrix size $(2N_{fft} \times 2)$. Equation (3) is similar to the concept with UDSTFM which is proposed by LC Tran et al. in [2, 3]. However, there is a major difference between UDSTFM and our proposed system, in UDSTFM, the $C_{t,m}$ is made into a diagonal matrix which made the matrix $C_t$ become a matrix with the size $(2N_{fft} \times 2N_{fft})$, while in our proposed method this diagonalization process is unnecessary.

We consider $X_t$ as the symbol of the transmitted matrix, which is the results of the DM calculation. For the first transmission, the transmitted matrix is $X_t = C_t$ ($X_1 = C_1$). For the next transmissions (from $t = 2$), we consider $Ca_t$ and $Cb_t$ as the encoding matrices of the proposed DM.

$$
Ca_t = 1/\sqrt{2} \begin{bmatrix}
C_{t,1} & C_{t,1}^* \\
C_{t,1}^* & C_{t,1}
\end{bmatrix}
\quad (4)
$$

$$
Cb_t = 1/\sqrt{2} \begin{bmatrix}
-C_{t,2}^* & C_{t,2} \\
-C_{t,2} & C_{t,2}
\end{bmatrix}
$$

The multiplication between two Unitary STBC Alamouti results in another Unitary Matrix. Therefore, for our proposed system, this property also can be applied. Thus, the transmitted matrix $X_t$ can be written as:

$$
X_t = \begin{bmatrix}
X_{t,1} & X_{t,2} \\
-X_{t,2}^* & X_{t,1}^*
\end{bmatrix}
\quad (5)
$$

To encode the symbols, we need a reverse matrix of $X_t$ which swaps the symbols between column 1 and column 2. We call this matrix as $Xr_t$.
\( X_r_t = \begin{bmatrix} X_{t,2} & X_{t,1} \\ X_{t,1}^* & -X_{t,2} \end{bmatrix} \) \hspace{1cm} (6)

And finally, the differential calculation of the proposed technique can be written as:

\[ X_t = X_{t-1} \circ C_a_t + X_{r_t} \circ C_b_t \] \hspace{1cm} (7)

The elements of \( X_t \) then will be transmitted by two antennas after Inverse Fast Fourier Transform (IFFT) process. The transmission model is written as:

\[ Y_t = X_t \circ H_t + N = \begin{bmatrix} X_{t,2} & X_{t,1} \\ -X_{t,1}^* & H_{t,2} \end{bmatrix} \circ \begin{bmatrix} H_{t,1} & H_{t,2} \\ H_{t,3} & H_{t,4} \end{bmatrix} + \begin{bmatrix} N & N \\ N & N \end{bmatrix} \] \hspace{1cm} (8)

Where, \( Y_t \) is the received matrix after Fast Fourier Transform (FFT), \( H_t \) is the channel coefficient and \( N \) is the Noise coefficient. Identical with the transmitter, in the receiver two antennas also employed.

### 2.2 Decoding

To decode the received matrix \( Y_t \), we consider \( Y_{a_t} \) and \( Y_{b_t} \) to be used as decoding matrices. We also consider \( \hat{C}_t \) as the result of the decoding process. For the first received transmission \( Y_t = \hat{C}_t (Y_{1} = \hat{C}_1) \).

\[ Y_t = \begin{bmatrix} X_{t,1} \circ H_{t,1} + N & X_{t,2} \circ H_{t,2} + N \\ -X_{t,2}^* \circ H_{t,3} + N & X_{t,1}^* \circ H_{t,4} + N \end{bmatrix} \] \hspace{1cm} (9)

For a better understanding of our proposed decoding, \( Y_t \) also can be written as the following:

\[ Y_t = \begin{bmatrix} Y_{t(1,1)} & Y_{t(1,2)} \\ Y_{t(2,1)} & Y_{t(2,2)} \end{bmatrix} \] \hspace{1cm} (10)

Where, in the \( Y_{t(p,q)} \), \( p \) and \( q \) represent the number of row and column of the matrix \( Y_t \), respectively. As has been mentioned in the previous subsection, one row in this matrix represent a set of matrix size \((N_{fft} \times 1)\).

\[ Y_{a_t} = \begin{bmatrix} Y_{t(1,1)} & Y_{t(1,1)}^* \\ Y_{t(2,1)} & Y_{t(2,1)}^* \end{bmatrix} \] \hspace{1cm} (11)

\[ Y_{b_t} = \begin{bmatrix} Y_{t(2,1)} & Y_{t(2,1)}^* \\ Y_{t(1,2)} & Y_{t(1,2)}^* \end{bmatrix} \]

We also need the reverse matrix of \( Y_t \) which swaps the symbols between rows of \( Y_t \) we call it \( Y_{r_t} \):

\[ Y_{r_t} = \begin{bmatrix} Y_{t(2,1)} & Y_{t(2,2)} \\ Y_{t(1,1)} & Y_{t(1,2)} \end{bmatrix} \] \hspace{1cm} (12)

And finally, the decoding process can be written as:

\[ \hat{C}_t = Y_t \circ Y_{a_{t-1}} + Y_{r_t} \circ Y_{b_{t-1}} \] \hspace{1cm} (13)

Because \( \hat{C}_t \) is the received \( C_t \). We can write it into this following form:

\[ \hat{C}_t = \begin{bmatrix} \hat{C}_{t,1} & \hat{C}_{t,2} \\ -\hat{C}_{t,2}^* & \hat{C}_{t,1}^* \end{bmatrix} \] \hspace{1cm} (14)
In order to take the advantage of diversity, $\hat{C}_{t,m}$ which is the received form of $C_{t,m}$ can be calculated with the following equations:

$$\hat{C}_{t,1} = \frac{1}{2} (\hat{C}_{t(1,1)} + \hat{C}_{t(2,2)})$$

$$\hat{C}_{t,2} = \frac{1}{2} (\hat{C}_{t(1,2)} - \hat{C}_{t(2,1)})$$

(15)

The received PSK symbol $\hat{c}_{t,m,k}$, which is the element of $\hat{C}_{t,m}$ can be calculated with the following maximum likelihood equation:

$$\hat{c}_{t,m,k} = \arg \min_{c_{t,m,k} \in \mathcal{C}} \left\{ (\Re \left| \hat{c}_{t,m,k} - c_{t,m,k} \right|)^2 + (\Im \left| \hat{c}_{t,m,k} - c_{t,m,k} \right|)^2 \right\}$$

(16)

### 3 Performance Evaluation

#### 3.1 Complexity Analysis

As mentioned in the two previous sections, the goal of this research is to solve the complexity problems of DUM technique in the MIMO OFDM. In this section, we show how much the improvement of the proposed system compared to the UDSTFM [2, 3].

If we use Big O notation for calculating the system complexity, the complexity of calculation between square matrices is $O(n^3)$, where $n$ is the number of row (or column). Several new ways to multiply two matrices have been proposed, with the lowest complexity is $O(n^{373})$ [6]. However, in this paper we choose to not to use the Big O, instead, we calculate the real complexity of our proposed model compared to DUM model.

As a comparison, we calculate the complexity of UDSTFM in the paper [2, 3]. Using conventional matrices multiplication, we can find that the number of multiplications is $(2N_{fft})^3$ and the number of additions is $(2N_{fft})^3 - (2N_{fft})^2$. If we include the total number of differential calculations which have to be done until all data decoded, then the number of multiplications is $r(2N_{fft})^3$ and the number additions is $r((2N_{fft})^3 - (2N_{fft})^2)$, which $r$ represents the number of differential calculation processes.

In the element-wise multiplication, the number of steps is the same with the total number of the elements of the matrix. In this proposed system we have two element-wise multiplication processes in one differential calculation process, in which each matrix size is $(2N_{fft} \times 2)$. Therefore, the number of multiplications can be written as $2r(4N_{fft})$ and the number of additions is $r(4N_{fft})$. The complexity decreases significantly because it removes the unnecessary multiplications between zeros which happens in the UDSTFM because of the diagonalization process.

Based on the explanations above, we can understand that the complexity of DUM will increase exponentially along with the increase of subcarrier, while in the proposed scheme, it increases arithmetically. Moreover, another advantage of using our proposed scheme is if we transmit the same number of symbols using DUM, a system which employs a higher number of subcarriers has higher complexity. However, with our proposed scheme the complexity is decreased. This
condition happens because when the system doubles the number of subcarriers, the number of $r$ becomes $\frac{r-1}{2}$ which results in lower complexity.

### 3.2 Experimental Result

We simulate our proposed scheme and compare it with UDSTFM scheme which is proposed by Tran et al in [2, 3]. The parameters including; the carrier (2.6 GHz), subcarrier spacing (15 kHz), the number of the subcarrier (128), the velocities (200 k/h and 300 k/h), the modulator (QPSK), and the channel model (Rayleigh Fading Channel). Both schemes are simulated in the exactly same channel condition and added by the same additive noise.

![UDSTFM vs. Proposed model](image)

**Fig. 1. UDSTFM vs. Proposed model**

As shown in **Fig. 1**, there is no single difference in term of performance between UDSTFM [2, 3] model and our proposed model. Beside the performance comparison, we also calculate the running time of both systems from the encoding process until finish the decoding process when using 128 and 256 subcarriers. By using 128 subcarriers, the average running time of UDSTFM [2, 3] is 5.5 times longer than our proposed model, while 256 subcarriers results in the average running time of UDSTFM [2, 3] 15.6 times longer than our proposed model.

### 4 Conclusion

In this paper, a low complexity differential modulation scheme is proposed. The experimental result shows the proposed low complexity differential modulation technique successfully matches the performance of UDSTFM technique, but with lower complexity. The complexity reduction can be achieved by removing the diagonalization process in UDSTFM and change the original matrix multiplication into the proposed element-wise multiplication. We believe that this concept also can be implemented into the other DUM techniques for multicarrier system in order to minimize their system complexity.