Theoretical system capacity of multi-user MIMO-OFDM THP in the presence of terminal mobility

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Abstract: This letter investigates the theoretical system capacity of multi-user multiple-input multiple-output (MU-MIMO) orthogonal frequency division multiplexing (OFDM) Tomlinson-Harashima precoding (THP) in the presence of terminal mobility. Considering that MU-MIMO THP has been adopted for OFDM-based mobile broadband systems, analyzing the effects of time-selective fading and the modulo loss peculiar to THP on system capacity is essential. In this study, we theoretically derive both the multi-user interference (MUI) and inter-carrier interference (ICI) resulting from terminal mobility and incorporate these into the modulo loss analysis based on the mod-Λ channel. The theoretical results obtained by the proposed analysis are compared with those of linear precoding, which demonstrates the applicability of THP to OFDM-based mobile broadband systems.

Keywords: MU-MIMO-OFDM THP, multi-user interference (MUI), inter-carrier interference (ICI), system capacity, mod-Λ channel, terminal mobility

Classification: Wireless Communication Technologies

References

[3] 3GPP TS 36.211 v10.5.0, “Evolved universal terrestrial radio access (E-UTRA); Physical channels and modulation,” June 2012.
1 Introduction

In recent years, multi-user multiple-input multiple-output (MU-MIMO) has become a promising technique for high-speed and high-capacity wireless communication systems, as simultaneous transmission can be realized via a single antenna mounted on a mobile station (MS) [1]. Moreover, to further enhance the system capacity, MU-MIMO is normally applied to orthogonal frequency division multiplexing (OFDM), which has been adopted for IEEE 802.11ac [2] and LTE-Advanced [3].

To realize MU-MIMO, precoding techniques are essential and are categorized into two approaches: linear precoding (LP) and non-linear precoding (NLP). NLP provides the better system capacity than LP because it reduces noise enhancement and has thus emerged as a candidate technique to realize 5G systems [4, 5, 6]. Of the various NLP schemes, Tomlinson-Harashima precoding (THP) is considered a practical approach because the perturbation vector can be generated by a simple modulo operation [5, 7, 8].

In this letter, we investigate the theoretical system capacity of MU-MIMO-OFDM THP in the presence of terminal mobility. The primary objective of our investigation is to grasp the exact theoretical capacity of MU-MIMO-OFDM THP under time-selective fading channels caused by terminal mobility, which ought to be considered in mobile wireless communications. In a previous study, we successfully derived the exact system capacity of MU-MIMO THP under time-selective fading channels [9], and therefore this study extends our previous work to OFDM-based broadband wireless systems. More specifically, the effect of both the multi-user interference (MUI) and inter-carrier interference (ICI) caused by time-selective fading is analyzed considering the application of THP to OFDM-based systems, and its effect is included in the mod-Λ channel [9]. This makes it possible to provide an exact system capacity analysis based on the adoption of OFDM as well as the modulo loss peculiar to THP in the presence of terminal mobility. Moreover, our investigation enables a fair comparison in terms of the system capacity between THP and LP without time-consuming computer simulations, and it demonstrates the superiority of THP over LP even in the presence of terminal mobility.
2 System capacity analysis of MU-MIMO-OFDM THP

Figure 1 shows the system configuration of MU-MIMO-OFDM THP, where \( N_t, N_r, \) and \( N \) denote the number of transmit antennas, MSs with one received antenna element, and sub-carriers. In Fig. 1, the feedforward (FF) and feedback (FB) filters of THP can be implemented in each sub-carrier by an LQ decomposition [8, 9] so as to retain spatial orthogonality among multiple MSs. Especially in THP, the modulo operation is performed to limit the transmit power increased by the addition of an interference subtraction vector generated by the FB filter. Moreover, because the transmit power is changed by the FF filter, a power normalization factor is required. In the \( p \)-th sub-carrier, the power normalization factor is given by

\[
g_p = \sqrt{\text{tr}(F_p C_{V_p} F_p^H)/(N_r \sigma_z^2)},
\]

where \( F_p \in \mathbb{C}^{N_t \times N_t} \) is the FF filter, \( C_{V_p} \in \mathbb{C}^{N_r \times N_r} \) is the covariance matrix of the transmit signal after the modulo operation \( V_p \in \mathbb{C}^{N_r} \), and \( \sigma_z^2 \) denotes the modulated signal power. After the precoder output is converted into the time domain signal with the length of \( T_s \) by means of IFFT processing, these signals are transmitted from each transmit antenna.

In general, terminal mobility creates time-selective fading, which causes a mismatch between the channel state information (CSI) for precoding and actual channel condition in data transmission. This mismatch destroys the space-frequency orthogonality in precoding, which leads to both the MUI and ICI. In this letter, we derive both the MUI and ICI resulting from terminal mobility in MU-MIMO-OFDM and then incorporate its impact into the system capacity analysis based on the mod-A channel.

In time-selective Rayleigh fading channels, the channel matrix between the \( j \)-th transmit antenna and \( i \)-th MS \( H(t, f) \in \mathbb{C}^{N_t \times N_r} \) is correlated with the preceding channel condition \( H(t - \Delta t, f) \in \mathbb{C}^{N_t \times N_r} \), which is represented by [9]

\[
H(t, f) = K_{\Delta t} H(t - \Delta t, f) + M_{\Delta t},
\]

where \( K_{\Delta t} = \text{diag}(k_{1,\Delta t}, \cdots, k_{N_t,\Delta t}) \in \mathbb{R}^{N_t \times N_r} \) and \( M_{\Delta t} \in \mathbb{C}^{N_t \times N_r} \) denote the time correlation matrix and uncorrelated channel error matrix. Here, the diagonal element of \( K_{\Delta t} \) is given by \( k_{i,\Delta t} = J_0(2\pi f_{D_i} \Delta t) \), where \( J_0(\cdot) \) and \( f_{D_i} \) are Bessel function of the first kind of order 0 and the maximum Doppler frequency of the \( i \)-th MS, respectively. Moreover, each element of \( M_{\Delta t} \) follows the complex Gaussian distribution with mean 0 and variance \((1 - k_{i,\Delta t}^2)\sigma_z^2\).

Assuming that the time difference between the CSI for precoding and precoded data transmission is \( \Delta t \) as shown in Fig. 1, the received time domain signal vector \( r(t) \in \mathbb{C}^{N_r} \) is expressed as

\[
r(t) = \frac{1}{N} \sum_{k=0}^{N-1} g_k^{-1} e^{j2\pi k t} H(t, k/T_s) F_k V_k + z(t)
= \frac{1}{N} \sum_{k=0}^{N-1} g_k^{-1} e^{j2\pi k t} (K_{\Delta t} H(t - \Delta t, k/T_s) + M_{\Delta t}) F_k V_k + z(t),
\]

where \( z(t) \in \mathbb{C}^{N_r} \) is the noise vector. Moreover, it should be noted that \( H(t - \Delta t, k/T_s) \) denotes the CSI matrix which matches with the FF filter \( F_k \).
After conducting FFT processing, we can represent the received frequency domain signal vector of the \( p \)-th sub-carrier \( Y_p = [Y_{1,p}, \cdots, Y_{N_r,p}]^T \in \mathbb{C}^{N_r} \) by

\[
Y_p = g_p \sum_{m=0}^{N-1} e^{-j2\pi \frac{m}{N} T_s} \left( n \frac{T_s}{N} \right) \\
= \frac{1}{N} \sum_{n=0}^{N-1} \left( K_{\Delta t} H \left( n \frac{T_s}{N} - \Delta t, \frac{p}{T_s} \right) + M_{\Delta t} \right) F_p V_p \\
+ \frac{g_p}{N} \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} e^{-j2\pi \frac{k}{N} \frac{n}{N}} \left( K_{\Delta t} H \left( n \frac{T_s}{N} - \Delta t, \frac{k}{T_s} \right) + M_{\Delta t} \right) F_k V_k + g_p Z_p
\]

where \( Z_p = [Z_{1,p}, \cdots, Z_{N_r,p}]^T \in \mathbb{C}^{N_r} \) denotes the noise vector of the \( p \)-th sub-carrier.

In Eq. (3), since the FF filter \( F_p \) is originally determined by the CSI matrix which represents the preceding channel condition for \( \Delta t \) from actual data transmission \( H(nT_s/N - \Delta t, p/T_s) \), the first term of Eq. (3) corresponds to the desired signal component. Consequently, the received signal \( Y_p \) can be rewritten as [9]

\[
Y_p = K_{\Delta t} X_p + M_{\Delta t} F_p V_p \\
+ \frac{g_p}{N} \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} e^{-j2\pi \frac{k}{N} \frac{n}{N}} \left( K_{\Delta t} H \left( n \frac{T_s}{N} + \Delta t, \frac{k}{T_s} \right) + M_{\Delta t} \right) F_k V_k + g_p Z_p
\]

where \( X_p = [X_{1,p}, \cdots, X_{N_r,p}]^T \in \mathbb{C}^{N_r} \) denotes the original modulated signal vector of the \( p \)-th sub-carrier.
With a focus on the $i$-th MS, the received signal $Y_{i,p}$ is represented by
\[
Y_{i,p} = k_i \Delta t X_{i,p} + m_i \Delta t F_p V_p + \frac{g_p}{N} \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} g_k e^{j2\pi k - p n \frac{2}{N}} \left( k_i \Delta t h \left( n \frac{T_s}{N} - \Delta t \frac{k}{T_s} \right) + m_i \Delta t \right) F_k V_k + g_p Z_{i,p},
\]
where $m_i \Delta t$ and $h(\cdot, \cdot)$ are the $i$-th row vectors of $M_i \Delta t$ and $H(\cdot, \cdot)$, respectively. From Eq. (5), the received signal $Y_{i,p}$ contains the desired signal, MUI, ICI, and noise components, and in consequence, the powers of these terms are calculated as
\[
P_D = \mathbb{E} \left[ |k_i \Delta t X_{i,p}|^2 \right] = k_i^2 \Delta t \sigma_x^2,
\]
\[
P_{\text{MUI}} = \mathbb{E} \left[ |m_i \Delta t F_p V_p|^2 \right] = \text{tr} \left( F_p F_p^\dagger \right) (1 - k_i^2 \Delta t) \sigma_y^2,
\]
\[
P_{\text{ICI}} = \mathbb{E} \left[ \left| \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} g_k e^{j2\pi k - p n \frac{2}{N}} \left( k_i \Delta t h \left( n \frac{T_s}{N} - \Delta t \frac{k}{T_s} \right) + m_i \Delta t \right) F_k V_k \right|^2 \right]
\]
\[
= \frac{g_p^2 k_i^2 \Delta t^2 N \sigma_y^2}{N^2} \left[ N(N-1) - 2 \sum_{n=1}^{N-1} (N-n) J_0 \left( 2\pi f_D \frac{T_s}{N} n \right) \right],
\]
\[
P_N = \mathbb{E} \left[ |g_p Z_{i,p}|^2 \right] = g_p^2 \sigma_n^2,
\]
where $\sigma_y^2$ and $\sigma_n^2$ are the transmit signal power after the modulo operation and noise power, respectively.

The system capacity as well as the effect of the modulo loss peculiar to THP can be derived by the mod-$\Lambda$ channel [9], which is represented by
\[
C_{\text{sum}} = \frac{1}{N} \sum_{p=0}^{N-1} \sum_{i=1}^{N_i} 2 \left[ \log_2 \tau + \int_0^\tau p(z_{\text{mod}}) \log_2 p(z_{\text{mod}}) dz_{\text{mod}} \right] \quad \text{[bps/Hz]},
\]
where $\tau$ denotes the modulo width. Moreover, $p(z_{\text{mod}})$ ($-\tau/2 < z_{\text{mod}} < \tau/2$) is the probability distribution function of the white Gaussian noise after the modulo operation $z_{\text{mod}}$, which is given by
\[
p(z_{\text{mod}}) = \sum_{l=-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{(z_{\text{mod}} + l\tau)^2}{2(P_{\text{MUI}} + P_{\text{ICI}} + P_N)} \right).
\]

3 Numerical results

We demonstrate the theoretical results of MU-MIMO-OFDM THP in terms of both the signal-to-interference-plus-noise ratio (SINR) and system capacity to clarify the impact of terminal mobility and then compared it to MU-MIMO-OFDM LP. In our performance evaluation, spatially uncorrelated Rayleigh fading is assumed for the MIMO channel, where each channel follows a 16-ray exponentially decaying multi-path channel. Here, the delay spread normalized by the sampling rate $T_{\text{sam}}$ ($= T_s/N$) is set to be $\tau_{\text{rms}} = 1.0 T_{\text{sam}}$. Moreover, the perfect CSI feedback is assumed and its feedback error and delay are negligible. To enhance the transmission performance of THP, the ordering process [8, 9] is adopted.

Figure 2 shows the cumulative distribution function (CDF) of the SINR with parameters of the normalized maximum Doppler frequency $f_D T_{\text{sam}}$ and number of
sub-carriers $N$, where the MIMO antenna configuration and average CNR are set to be $8 \times 8$ and 25 dB, respectively. From Fig. 2, we can see that because THP effectively suppressed the effect of the MUI and ICI as well as the noise enhancement, THP achieves better SINR than LP even in the presence of terminal mobility. Moreover, it is observed that the SINR is degraded with an increase in the number of sub-carriers $N$ regardless of the precoding scheme because the effect of ICI becomes critical.

Figure 3 shows a performance comparison between THP and LP in terms of the sum-rate versus the normalized maximum Doppler frequency $f_{D}T_{sam}$ with a parameter of the MIMO antenna configuration, where the number of sub-carriers $N = 256$ and the average CNR is set to be 25 dB. From Fig. 3, it can be seen that the performance gap between THP and LP decreases in the range of $f_{D}T_{sam} > 10^{-4}$ regardless of the MIMO antenna configuration. This is because terminal mobility escalates the effect of the modulo loss with the MUI and ICI. Moreover, the superiority of THP over LP is enlarged with an increase in the MIMO antenna configuration as a result of the space diversity effect.

4 Conclusion

In this letter, we theoretically analyzed the exact system capacity of MU-MIMO-OFDM THP in the presence of terminal mobility. Considering the application of THP to OFDM transmission, we derived the effect of both the MUI and ICI due to terminal mobility and this effect was incorporated into the system capacity analysis based on the mod-$\Lambda$ channel. Numerical results showed that THP still achieved the higher system capacity than LP even in the presence of terminal mobility. The study showed that the proposed approach provides a comprehensive performance evaluation of MU-MIMO-OFDM THP when considering possible effects such as the MUI, ICI, and modulo loss without any time-consuming computer simulations. In general, the proposed analysis can be used to verify the applicability of THP to OFDM-based mobile broadband systems.

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