Ship Routing Problem with Multi-Product Inventory Constraints in a Hub-and-Spokes Network

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1. INTRODUCTION

Distribution of multi type of products is a common business in product oil distribution to satisfy the need at various demand locations. Often the distributions are not initiated by orders from demand points but rather the company must schedule the shipment to assure the demand points do not run out of stock.

Many organizations ship their products to customers in a recurrent basis. This idea is also adopted into this study to solve ship routing with multi-product inventory constraints. This study focuses on determining a set of delivery routes for heterogeneous ships from a single distribution center to several demand locations as a distribution network is considered as a hub-and-spokes network. Moreover, this study addresses the question on how much to ship of each petroleum product to demand points within planning horizon. Due to its nature, the products must be shipped in separate compartments. There is a known limited storage capacity for each product at each demand point while distribution center is assumed never run out of stock availability.

2. SOLUTION APPROACH

Two phases of approach is employed to solve the problem. The main idea of the approach is to construct a set of delivery routes within shipment planning period and execute the routes in recurrent basis for longer planning horizon. In phase 1, by giving available ships with different compartment size and cost components, a set of delivery routes are generated. In phase 2, based on delivery routes obtained in phase 1, we decide how much of each product to ship at every shipment period.

Phase 1: Delivery Routes Generation

In this phase, the approach aims to generate a set of delivery routes for available ships to transporting products from a single distribution center to demand points in one shipment period.

To determine a set of delivery routes, first we define a cluster of ports to be a group of demand points that can be served cost effectively by a single ship. Shipping distance that corresponds to a cost serving a cluster depends on geographic locations, storage capacities, and usage rates for each product.

We estimate shipping distance to visit demand points in the cluster to be the length of an optimal traveling salesman problem (TSP) tour using Held and Karp dynamic programming. We employ a set partitioning in order to determine an optimal delivery routes from feasible routes using different ship sizes. The step of phase one can be seen in Fig.1.

Phase 2: Shipment Scheduling

This phase addresses the question on how much each product to ship from distribution center to demand points using delivery routes obtained from phase 1. The shipment period is extended to longer planning horizon. We model the problem into mixed integer linear programming (MILP) based on work by Campbell et al.2) by modified the model to cover multi-products and heterogeneous of ships. In the first step of this phase, we try to estimate how many routes should be made by the company within the planning horizon.

Let \( c_{v}^{s} \) be the cost of optimal route using ship \( v \) through a subset of demand points at cluster. \( SCA_{v}^{k} \) is the ship capacity for carrying product \( k \) and \( C_{v}^{k} \) is storage capacity of product \( k \) at demand point \( i \). Total volume of product \( k \) to deliver to demand point \( i \) on route \( r \) using ship \( v \) in the planning period and the route count \( x_{v}^{i} \), and consider following model:

\[
\text{Min } \sum_{r \in R} c_{v}^{s} x_{r}^{i} \tag{1}
\]
Subject to
\[
\sum_{v} \sum_{k \in \text{iter}} Q_{vr}^{v} \leq \min(\text{SCAP}_{k}^{v}, \sum_{k \in \text{iter}} C_{ik}) x_{vr}^{v} \quad \forall r
\]
\[
Q_{vr}^{v} \leq \min(\text{SCAP}_{k}^{v}, C_{ik}) x_{vr}^{v} \quad \forall v, k, i, r
\]
\[
\sum_{v} \sum_{k \in \text{iter}} Q_{vr}^{v} = T_{i}^{v}_{r} \quad \forall i, k
\]
\[
x_{vr}^{v} \text{ integer, } Q_{vr}^{v} \geq 0
\]

Constraints (2) are ship and storage tank capacities limitation based on number of times route \( r \) is executed. Constraints (3) ensure that we do not deliver more to customer than the minimum of the ship capacity and its storage capacity time route \( r \) was executed. Constraints (4) ensure that the total volume delivered in the planning period is equal to its total usage during planning period.

Following integer programming model is to minimize cost for delivering product \( k \) by determining volume to deliver in a manner that each demand point does not run out of stock. Let
\[
d_{ik}^{v} = \max(0, -l_{ik}^{v+1} + t_{ik})
\]
be the minimum amount to deliver and
\[
D_{ik}^{v} = C_{ik} - l_{ik}^{v-1} + t_{ik}
\]
be the maximum amount to be delivered to the demand points. The inventory balance is defined by
\[
l_{ik}^{v} = l_{ik}^{v-1} + \sum_{v, k \in \text{iter}} Q_{vk}^{v} - t_{ik}
\]
\[
\min \sum_{v} \sum_{k \in \text{iter}} c_{ik} x_{ik}^{v}
\]
\[
d_{ik}^{v} \leq \sum_{v} \sum_{k \in \text{iter}} \sum_{\text{iter}} Q_{vk}^{v} \leq D_{ik}^{v} \quad \forall i, k, v, t
\]
\[
\sum_{v} \sum_{k \in \text{iter}} Q_{vk}^{v} \leq \text{SCAP}_{k}^{v} x_{vk}^{v} \quad \forall v, k, r, t
\]
\[
\sum_{v} x_{vr}^{v} \leq m \quad \forall t
\]
\[
x_{vr}^{v} \text{ binary, } 0 \leq Q_{vr}^{v} \leq \min(\text{SCAP}_{k}^{v}, C_{ik})
\]

Constraints (6) ensure that the demand points do not run out of stock. Constraints (7) ensure that we do not deliver more than ship capacity. Constraints (8) ensure that we only use \( m \) number of ships in each shipment period where \( m \) also means number of delivery routes.

3. NUMERICAL EXAMPLE

The same problem discussed by Suprayogip is on distribution of fuel products in Eastern Part of Indonesia will be carried as a numerical example. There is a single distribution center and eight demand points among network. Three different product types (gasoline, kerosene, and Diesel oil) need to deliver to demand points using small tanker ships. It is assumed that two types of small tanker ships are available. Small tanker type 1 has dedicated compartment for gasoline, kerosene, and Diesel oil respectively 500:700:700 cubic meter while small tanker type 2 has 700:1000:1000. All ships have the same 12.5 knots of service speed.

Solution in Phase 1

In this phase, feasible delivery routes for small tanker type 1 and 2 are generated within a single shipment planning (one week). Delivery routes are obtained by performing TSP over demand ports clustering and 24 hours is added into route time when the ship visit a port for unloading cargos and other port activities. To select the minimal cost routes over all feasible routes, Set partitioning problem formulation is performed. Fig. 2 shows delivery routes obtained from phase 1. To deliver cargos over one shipment period, three delivery routes using three small tankers type 1 are required and one delivery route using one small tanker type 2 is required.

Solution in phase 2

Numbers of routes to execute within planning horizon of one month are four times except for route of Kupang-Ende-Waingapu-Kupang is executed three times. Following tables are example of results for the route Kupang-Dili-Kupang using small tanker type 2 with compartment capacity 700:1000:1000 m³ and initial inventory 164.85: 44.84:420.63 m³ for gasoline, kerosene, and Diesel oil respectively.

Table 1 Amount of products to deliver

<table>
<thead>
<tr>
<th>Prod. type</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gasoline</td>
<td>700.00</td>
<td>615.15</td>
<td>236.00</td>
<td>236.00</td>
</tr>
<tr>
<td>Kerosene</td>
<td>236.00</td>
<td>236.00</td>
<td>236.00</td>
<td>24.16</td>
</tr>
<tr>
<td>Diesel Oil</td>
<td>1000.00</td>
<td>1000.00</td>
<td>1000.00</td>
<td>409.61</td>
</tr>
</tbody>
</table>

Table 2 Inventory level at the end of period

<table>
<thead>
<tr>
<th>Prod. type</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gasoline</td>
<td>514.85</td>
<td>164.85</td>
<td>430.00</td>
<td>80.00</td>
</tr>
<tr>
<td>Kerosene</td>
<td>97.09</td>
<td>148.34</td>
<td>201.59</td>
<td>42.00</td>
</tr>
<tr>
<td>Diesel Oil</td>
<td>514.83</td>
<td>609.03</td>
<td>703.23</td>
<td>207.04</td>
</tr>
</tbody>
</table>

4. CONCLUSION

In this paper, we have presented a method for solving ship routing with multi-products inventory constraints. Based on the idea of cyclic distribution, we proposed two phases of optimization approach in order to combined ship routing and distribution planning. The proposed approach worked well on a given numerical example.

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