Liner Shipping Routing and Scheduling Considering Empty Container Reposition

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Key Words: Empty Container Reposition, Liner Shipping, Genetic Algorithms, Linear Programming

1. INTRODUCTION

Empty container allocation problems arise due to imbalance on trades. Imbalanced trade is a common fact in the liner shipping, creating the necessity of repositioning empty containers from import-dominant ports to export-dominant ports. Empty containers movements represent approximately 20 per cent of the world total international container port throughputs, according to ESCAP1. Shipping companies face the challenge of repositioning empty containers between ports in an efficient and economic way, lease containers or reject cargo. Cargo rejection is very unlikely in practice due to the intensive market competition.

The present study configures a liner shipping network considering the allocation of empty containers for variable cargo demand, using genetic algorithms and linear programming methods. The study can be divided into two main parts, overall problem of routing and detailed scheduling problem. The routing problem deals with the choice of the best combination of routes for the problem as a whole; while the scheduling problem define in detail, the schedule of all ships on each route and their cargo movements during a defined time horizon.

A case study composed of 11 South and East Asian ports is used to configure a liner shipping network, considering the allocation of empty containers.

2. ROUTING PROGRAM

The routing program chooses the best combination of routes that maximizes the revenue of a liner shipping company. A set of predefined candidate routes is the input data of the program. The program, by using genetic algorithms will search for the best combination of candidate routes of the input data. For each possible combination of routes, linear programming methods are used to solve the cargo distribution between all chosen routes of loaded containers, empty containers reposition and determining the best ship capacity that minimizes the volume of cargo rejection.

2.1 Genetic algorithms I (GA-I)

Genetic algorithms (GAs) are search algorithms based on the mechanics of natural selection and natural genetic, according to Goldberg2. GAs efficiently exploit historical information to speculate on new search points with expected improved performance.

In the routing program, GAs are used to determine the best route combination of the predefined set of candidate routes. In this case, the individual structure is composed by a vector with length equal to the number of total predefined candidate routes, where each string represents one specific route, the vector is composed by binary variables, the string receives value 1 when the candidate route is chosen and 0 otherwise.

The fitness value of each individual is calculated after performing the optimization of the cargo distribution and defining the ship size for each valid route. The fitness value is the profit of the given network, obtained from the revenue of the transported cargo and the shipping costs. One time period is used in the GA-I program.

2.2 Linear Programming (LP-I)

For each individual of the population in GA-I, the cargo distribution and empty container reposition among all the candidate routes of the individual are done during this process.

For each o-d (origin-destination) pair, in each route, the legs (part of the route linking two consecutive calling ports) of the route that the cargo will remain in the ship can be predetermined. As ports can be called both at outbound and inbound parts of the route, there might be more than one alternative of transport. In this case, the minimum distance alternative is chosen. The following constant can be defined:

\[ Q_{ij} = \begin{cases} 1, & \text{if cargo from origin } i \text{ to destination } j \text{ travels on leg } g \text{ of route } r \\ 0, & \text{otherwise} \end{cases} \]

The linear programming will find the best distribution of cargo of all routes. The linear program formulation can be written as:

**Constants and Variables:**

\[ D_i \] demand cargo from port \( i \) to \( j \)
\[ E_{S_i} \] supply of empty container in surplus port \( i \)
\[ E_{D_i} \] demand of empty container in deficit ports \( i \)
\[ \text{ship}_\text{cap}_{\text{max}} \] maximum capacity in TEUs of the ship in route \( r \)

**Decision Variables:**

\[ X_{ij} \] amount of cargo (from \( i \) to \( j \)) transported in route \( r \)
\[ E_{S_i} \] amount of empty container from \( i \) to \( j \) transported in route \( r \)
\[ P_i \] amount of cargo not transported from \( i \) to \( j \)
\[ \text{ship}_\text{cap}_{r} \] ship capacity of route \( r \)
\[ \text{E}_{S} \] empty container on surplus port \( i \) not transported
\[ \text{E}_{D} \] empty container in deficit port \( i \) not received

**Objective function**

\[ \text{minimize } \sum_{i,j} P_i + \sum_{i} \text{E}_{S_i} + \sum_{i} \text{E}_{D_i} \] (1)
Subject to:

\[ \sum_{i,j} Q_{g,i,j} \cdot X_{g,i,j} + \sum_{i,j} Q_{d,i,j} \cdot E_{g,i,j} \leq \text{ship}_{-}\text{cap,} \quad (2) \]

\[ \sum_{r} X_{g,i,j} + P_{g} = D_{g} \quad \text{for all } g, j \quad (3) \]

\[ \sum_{i} E_{g,i,j} + \text{P}_{s} = E_{s} \quad \text{for all } i \quad (4) \]

\[ \sum_{j} E_{g,i,j} + \text{P}_{d} = E_{d} \quad \text{for all } i \quad (5) \]

\[ \text{ship}_{-}\text{cap,} \leq \text{ship}_{-}\text{cap}_{\text{max}}, \text{for all } r \quad (6) \]

\[ X_{g,i,j} \cdot E_{g,i,j} \geq 0 \quad \text{for all } i, j, r \quad (7) \]

\[ P_{g} \geq 0 \quad \text{for all } i, j \quad (8) \]

\[ \text{P}_{s}, \text{P}_{d} \geq 0 \quad \text{for all } i \quad (9) \]

\[ \text{ship}_{-}\text{cap,} \geq 0 \quad \text{for all } r \quad (10) \]

The objective function (1) minimizes the amount of loaded cargo not transported, surplus empty containers on import-dominant ports not transported and empty container in deficit ports not received. Constraint (2) assures that the cargo loaded in the ship in each leg will not be larger than the ship capacity. Constraint (3) assures that the cargo demand requirements will be met. Constraints (4) and (5) ensure that the empty container supply and demand requirements will be satisfied. Constraint (6) limits the size of the ship to be used for a given route. Constraints (7)-(10) ensure the non-negativity of the decision variables.

3. SCHEDULING PROGRAM

The routing program chooses the best set of candidate routes, and the scheduling program will coordinate the cargo flow of loaded and empty containers and the best period to charter ships to handle peak season's cargo, in a time horizon with variable demand pattern.

The results of the routing program define the basic structure of the scheduling program. In this part, the distribution of cargo between all ships of all routes will be made. As the cargo demand is not constant, usually, during peak seasons, shipping companies charter ships to complement their fleet.

The scheduling program is divided into two main parts, a GA program (GA-II) is used to define the best time and the period to charter extra ships. The second part (LP-II) will configure the cargo distribution of all routes and ships according to the number and period of charter ships, using linear programming.

3.1 Genetic Algorithm II (GA-II)

The GA will search for the best time period to charter ships for different routes. The individual structure is a binary vector, where each string represents one time period, and receives value 1 if a ship will be chartered for the period, and 0, otherwise. The period of chartering ships should be the number of round trips that can be completed for the given route.

3.2 Linear Programming II (LP-II)

The basic structure of the scheduling program is to offer weekly service for all routes. To assure weekly service, the number of necessary ships per route should be equal to the turnaround period in weeks of the route. The basic structure is not enough to handle all cargo during peak seasons, creating the necessity to charter ships.

The LP-II will make the cargo distribution and empty container reposition between all fixed ships and charter ships. The calculation is done for each time step, and then the balance of containers (loaded and empty) for each port is done. If the balance of number of containers is negative for a given port, empty container should be leased. Leased containers are returned after the cargo delivery is completed. For the LP-II, the constant of cargo distribution is defined as:

\[ Q_{0,0,0,0} = \begin{cases} 1, & \text{if cargo from origin } i \text{ to destination } j \text{ of period } t \text{ travels on leg } g \text{ of ship } s \text{ of route } r \\ 0, & \text{otherwise} \end{cases} \]

A maximum number of charter ships are allowed in each route. Each charter ship for each route charter_{r,s} has the following period of chartering tcharter_{r,s}=

The formulation of LP-II is as follows:

**Constants and Variables:**

\[ D_{g} \text{ demand cargo from port } i \to j \text{ at time } t \]

\[ E_{g,i,j} \text{ supply of empty container at surplus port } i \text{ at time } t \]

\[ E_{d} \text{ demand of empty container in deficit ports } i \text{ at time } t \]

\[ B_{i} \text{ balance of empty containers in port } i \text{ at time } t \]

\[ \text{ship}_{-}\text{cap,} \text{ capacity in TEUs of ship } r \text{ at time } t \]

\[ \text{ship}_{-}\text{space}, \text{ available capacity for cargo of ship } r \text{ at time } t \]

\[ \text{charter}, \text{ capacity in TEUs of ship in route } r \text{ and charter}_{r,s} \text{ capacity for ship } s \text{ in route } r \text{ at time } t \]

\[ TA_{r} \text{ turnaround period of route } r \]

**Decision Variables:**

\[ X_{g,i,j,t} \text{ amount of cargo (from } i \to j \text{ transported by ship } s \text{ of route } r \text{ at time } t \]

\[ C_{i,j,t} \text{ amount of cargo (from } i \to j \text{ transported by charter ship } s \text{ of route } r \text{ at time } t \]

\[ E_{g,i,j,t} \text{ amount of empty container from } i \to j \text{ transported by ship } s \text{ of route } r \text{ at time } t \]

\[ C_{i,j,t} \text{ amount of empty container from } i \to j \text{ transported by charter ship } s \text{ of route } r \text{ at time } t \]

\[ P_{g} \text{ amount of cargo not transported from } i \to j \text{ at time } t \]

\[ \text{P}_{s} \text{ empty container on surplus port } i \text{ at time } t \]

\[ \text{P}_{d} \text{ empty container in deficit port } i \text{ at time } t \]

**Objective function**

\[ \text{minimize } \sum_{j} \sum_{t} P_{g} + \sum_{i} \text{P}_{s} + \sum_{j} \text{P}_{d} \]

Subject to:

\[ \sum_{j} Q_{g,i,j,t} \cdot X_{g,i,j} + \sum_{j} Q_{d,i,j,t} \cdot E_{g,i,j,t} \leq \text{ship}_{-}\text{space}_{r,s,i,j} \quad \text{for all } g, s, t \quad (12) \]

\[ \sum_{j} Q_{g,i,j,t} \cdot X_{g,i,j} + \sum_{j} Q_{d,i,j,t} \cdot E_{g,i,j,t} \leq \text{charter}_{-}\text{space}_{r,s,i,j} \quad \text{for all } g, s, t, \text{if } t \in t_{\text{charter}} \quad (13) \]

\[ \sum_{j} \left( X_{g,i,j} \cdot E_{g,i,j,t} + P_{g} \right) = D_{g} \quad \text{for all } i, j \quad (14) \]

\[ \sum_{j} \left( E_{g,i,j,t} \cdot E_{g,i,j,t} + \text{P}_{s} \right) = E_{s} \quad \text{for all } i \quad (15) \]

\[ \sum_{j} \left( E_{g,i,j,t} \cdot E_{g,i,j,t} + \text{P}_{d} \right) = E_{d} \quad \text{for all } i \quad (16) \]
\[ X_{ij,r,s}^C, X_{ij,r,s}^C, E_{ij,r,s}, E_{ij,r,s}^C, E_{ij,r,s}^C \geq 0 \text{ for all } i, j, r, s \]  
\[ p_{ij} \geq 0 \text{ for all } i, j \]  
\[ p_{es,ij}, p_{ed,ij} \geq 0 \text{ for all } i \]  

The objective function (11) minimizes, for every step time, the amount of cargo not transported, surplus empty containers on import-dominant ports not transported and empty containers in export-dominant ports not received. Constraints (12) and (13) ensure that the loaded cargo and empty containers shipped is not larger than the available space for cargo in the ship. Constraint (14) assures that the cargo demand is met. Constraints (15) and (16) ensure that the empty container supply and demand are met. Constraints (17)-(19) assure the non-negativity of the decision variables.

Once the linear programming calculations are done, the balance of the number of containers is done for each port. The balance on each time step considers the previous balance, all the cargo and empty containers received on the previous period minus all the cargo and empty containers sent to other ports, the number of leasing containers. For each period, the balance of each port should be equal or greater than zero, \( B_{ij} \geq 0 \). The available ship space for new cargo for the next period is also calculated in equations (21) and (22).

\[ B_{ij,t} = B_{ij,t-1} + \sum_{j} \sum_{r} (X_{ij,r,s,t}^C + X_{ij,r,s,t}^C + E_{ij,r,s,t} + E_{ij,r,s,t}^C) \]  
\[ - \sum_{j} \sum_{r} (X_{ij,r,s,t} + X_{ij,r,s,t}^C + E_{ij,r,s,t} + E_{ij,r,s,t}^C) + L_{ij,t} \]  
\[ \text{ship}_{-} \text{space}_{r,s,t} = \text{ship}_{-} \text{cap}_{r,s} \]  
\[ - \sum_{r} \sum_{t} Q_{ij,r,s,t} \text{ (X}_{ij,r,s,t}^C + E_{ij,r,s,t}^C) \text{ for all } r, s \]  
\[ \text{charter}_{-} \text{space}_{r,s,t} = \text{charter}_{-} \text{cap}_{r,s} \]  
\[ - \sum_{t} \sum_{i} Q_{ij,r,s,t} \text{ (X}_{ij,r,s,t}^C + E_{ij,r,s,t}^C) \text{ for all } r, s \]  

4. CASE STUDY

The forecast containerized trade among South and East Asian countries for 2005, published by UNCTAD is used in this case study. The ports considered in the study represent the flow of their respective country and they are Shanghai, Jakarta, Hong Kong, Yokohama, Port Klang, Manila, Singapore, Busan, Kaohsiung, Bangkok and Ho Chi Minh. Fixing the port of Yokohama as the head-end port and Singapore as the tail-end port, and using GA based on TSP, the primary sequence can be set as shown in Table 1. Table 1 also shows the annual cargo flow on each port between the ports in the network.

4.1 Routing program results

The input data of 30 candidate routes was randomly generated and for each candidate route, 3 possible ship sizes were previously defined, varying from 1500 to 5000 TEU. As the demand of cargo transport is not constant, a demand pattern as shown in Figure 1 is considered. For the routing program, the average share of flow demand is considered in the analysis.

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Table 1: Ports of case study and annual cargo flow of the network (in TEUs)

<table>
<thead>
<tr>
<th>Port</th>
<th>Imports</th>
<th>Exports</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>262,202</td>
<td>209,038</td>
<td>53,164</td>
</tr>
<tr>
<td>2</td>
<td>105,592</td>
<td>135,764</td>
<td>-30,172</td>
</tr>
<tr>
<td>3</td>
<td>294,947</td>
<td>407,179</td>
<td>-112,232</td>
</tr>
<tr>
<td>4</td>
<td>91,250</td>
<td>151,421</td>
<td>-60,171</td>
</tr>
<tr>
<td>5</td>
<td>205,171</td>
<td>13,298</td>
<td>191,873</td>
</tr>
<tr>
<td>6</td>
<td>56,226</td>
<td>31,277</td>
<td>24,949</td>
</tr>
<tr>
<td>7</td>
<td>30,287</td>
<td>14,809</td>
<td>15,478</td>
</tr>
<tr>
<td>8</td>
<td>47,198</td>
<td>73,876</td>
<td>-26,678</td>
</tr>
<tr>
<td>9</td>
<td>36,303</td>
<td>95,964</td>
<td>-59,661</td>
</tr>
<tr>
<td>10</td>
<td>68,267</td>
<td>68,665</td>
<td>-398</td>
</tr>
<tr>
<td>11</td>
<td>71,019</td>
<td>67,171</td>
<td>3,848</td>
</tr>
</tbody>
</table>

Fig. 1 Demand pattern

The total profit of the network is calculated as the difference of the revenue and the shipping costs. Revenues are based on the value of Ocean Freight published by OOCL. Shipping costs are calculated using Cullinane & Khanna's work.

Results for the routing program are the profit of the network, the set of candidate routes and their ship capacity, using the average cargo flow of one week. The total profit of the network obtained is USD 6,942,506, the set of candidate routes and their shipping capacity are shown in Table 2.

Table 2: Results of the Routing Program

<table>
<thead>
<tr>
<th>Route</th>
<th>Ship capacity</th>
<th>Port calling sequence</th>
<th>Turnaround (weeks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>5000</td>
<td>3-5-4-5-4</td>
<td>1</td>
</tr>
<tr>
<td>II</td>
<td>3000</td>
<td>1-2-3-9-10-9-6-5-3-2-1</td>
<td>4</td>
</tr>
<tr>
<td>III</td>
<td>1300</td>
<td>1-3-4-5-8-9-10-11-9-7-1</td>
<td>4</td>
</tr>
<tr>
<td>IV</td>
<td>2000</td>
<td>1-2-4-5-6-8-11-10-9-7-4-3-2-1</td>
<td>5</td>
</tr>
</tbody>
</table>

4.2 Scheduling program results

Based on the results obtained in the previous part, it is possible to define the ships' schedule for each route. The scheduling program will make the distribution of the cargo between all ships in all routes and also the charter ships.

A time horizon of 52 weeks is considered. The maximum allowable number of charter ships per route assumed is 5 ships, and their capacity is the same as the ships of the routes they are allocated to. The scheduling results obtained for charter ships are shown in Table 3.

The total profit of the network obtained was USD 249,301,780. Empty container reposition accounted for 41 per cent of the total transported flow in the network, 6 per cent of the cargo was transported using leasing containers and less than 1 per cent of the cargo could not be transported. A
summary for each route is shown in Figures 3-5, where the total share of loaded cargo and empty containers are shown for each leg of the route.

<table>
<thead>
<tr>
<th>route</th>
<th>charter start period (week)</th>
<th>duration (weeks)</th>
<th>route</th>
<th>charter start period (week)</th>
<th>duration (weeks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
<td></td>
<td>III</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>#1 19</td>
<td>32</td>
<td>#2</td>
<td>12</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>#2 26</td>
<td>16</td>
<td>#3</td>
<td>17</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>#3 28</td>
<td>20</td>
<td>#4</td>
<td>20</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>#4 29</td>
<td>20</td>
<td>#5</td>
<td>34</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>#5 38</td>
<td>12</td>
<td>IV</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>#1 12</td>
<td>35</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>#2 23</td>
<td>25</td>
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<td>#3 25</td>
<td>20</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>#4 26</td>
<td>20</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

This work addressed the problem of routing and scheduling of a container liner shipping network by considering the empty container reposition. Genetic algorithms and linear programming calculations are combined to better deal with the problem; genetic algorithms to find best combination of routes and charter periods, while linear programming makes the cargo distribution.

Taking into account the empty container reposition in the routing and scheduling program, results showed, that doing small modifications in the route configurations can be effective for the reposition problem. For example, when considering sailing on legs which will not generate revenues, but strategic for empty container reposition; or increasing the ship capacity to accommodate extra flow of empty containers.

Further research will focus on the integration of routing and scheduling program as a whole. Additionally, transshipment points at some specific points should be considered and added in further studies, to increase the coverage of the network.

REFERENCES

1) ESCAP (Economic and Social Commission for Asia and the Pacific), Regional Shipping and Port Development Strategies (Container Traffic Forecast), United Nations, 2005