The purpose of the present study was to establish a geometric design method for class I inlay cavities as a future method of computer-aided tooth preparation. Major occlusal fissures were used as a starting point of the cavity design and marked as multiple continuous line segments. An experimental cavity outline was defined by circular arcs and Bézier curves with three design parameters: minimum radius of curvature of the convex portion, taper angle of the convex portion of a cavity, and dovetail convexity angle. The experimental software was used to design class I inlay cavities for mandibular and maxillary right, first and second molars. Once the segments and the parameters were set, the outline was instantaneously drawn by the software. All design parameters worked as intended. Smooth class I inlay cavity outlines for molars with tool accessibility throughout the cavities could be obtained using the present design method.

Keywords: CAD/CAM, Cavity preparation, Class I inlay cavity, Geometric design

INTRODUCTION

Cavity preparation is one of the most fundamental dental treatments. For more than a hundred years, it has been performed on the basis of the principles proposed by G. V. Black. However, these principles are design guidelines and do not provide concrete dimensions for an individual cavity. There is almost no doubt that experienced dentists have an ideal outline for a finished cavity in their mind and make their abstract image concrete when preparing a real tooth. Nevertheless, a search for cavity preparation in the existing literature revealed that no clear and unified determination method is available for a well-proportioned outline. Little attention has been given to the geometric design of the outline. This is probably because tooth preparation today is performed manually using a dental handpiece, which makes it difficult to maintain design accuracy, even if an ideal outline was previously defined. Precise preparations are necessary not only for the traditional amalgam and cast metal restorations, but also for ceramic restorations made by rapidly advancing dental CAD/CAM systems. The geometric design of cavities will become an indispensable procedure when tooth preparation is computerized using the CAD/CAM technology in the future. The purpose of the present study is to establish a geometric design method for class I inlay cavities. A design method using simple geometric rules is proposed and applied to molars for evaluation as part of the study on computerization of tooth preparation.
distance \( r \) from each type-2 segment and auxiliary lines (T lines) in parallel with and at a distance \( r \) from each type-1 segment are drawn as shown in Figs. 1(d) and (e). Moving a cutting tool simply along the line segments minimizes the reduction of cusps and ridges. However, it can result in an outline with sharp corners, as shown in Fig. 1(f).

To assure the smoothness of the outline, a Bézier curve is introduced in the present study. A Bézier curve is often used in computer graphics to describe smooth curves\(^4\). Quadratic, cubic, and quartic Bézier curves are used, depending on the number of control points defined as mentioned later. For three points \((p_1, p_2, \text{ and } p_3)\), quadratic Bézier curve \(b_2\) described by the formula

\[
b_2(t) = (1-t)^2 p_1 + 2(1-t) t p_2 + t^2 p_3
\]

is used as the parameter \( t \) varied from 0 to 1. For four
points \((p_1, p_2, p_3, \text{ and } p_4)\) and five points \((p_1, p_2, p_3, p_4, \text{ and } p_5)\), cubic and quartic Bézier curves \(b_3\) and \(b_4\) are used, respectively, which are described by the following formulas.

\[
b_3(t) = (1-t)^2 p_1 + 3(1-t)t p_2 + 3t^2 p_3 + t^3 p_4
\]
\[
b_4(t) = (1-t)^3 p_1 + 3(1-t)^2 t p_2 + 3(1-t) t^2 p_3 + t^3 p_4
\]

The curve has the following important characteristics:

1) As the formula shows, the curve begins at \(p_1\) and ends at \(p_4\). 2) The start and the end of the curve are tangent to lines \(p_1 p_2\) and \(p_4 p_5\), respectively. 3) All points of the curve lie between the control points, where \(n\) is the number of control points. Owing to 1) and 2), the curve can be smoothly connected to arcs at \(p_1\) and \(p_4\).

For setting more concrete rules, three types of combinations of neighboring segments, namely, V, U, and W, are defined in the present study, as shown in Fig. 2. A V-shaped combination (type-V) is a combination of two neighboring type-1 segments. A U-shaped combination (type-U) is composed of one type-2 segment with two type-1 segments connected to a different end. The third combination (type-W) is composed of two type-2 segments with two type-1 segments connected inline like a four-unit folding ruler.

In order to taper the convex portion of a cavity, the second design value \(\theta\) which is a taper angle, is introduced, as shown in Fig. 2(a). Here, T lines are redefined as tangential lines of the circular arc that form angle \(\theta\) with the type-1 segment. \(\theta\) must be equal to or greater than 0 to ensure complete accessibility of the tool to the convex portion when the tool radius is \(r\). The auxiliary T lines are drawn as shown in Fig. 1(g).

The two circular arcs of type-V segments become an occlusal dovetail portion of the preparation. To provide convexity to the dovetail, the third design parameter \(\phi\), which is a dovetail convexity angle, is introduced, as shown in Fig. 2(b). When \(\phi\) is equal to or greater than 0, complete accessibility of the tool with radius \(r\) within the dovetail is guaranteed. \(\phi\) can be a negative number to give concavity to the dovetail; however, the lower limit depends on the proportion of the type-V combination. The auxiliary tangential lines (D lines) of circular arcs of type-V segments that form angle \(\phi\) with a common tangent line \(p_1 p_2\) are drawn as shown in Fig. 1(g). The two circular arcs are connected by a quadratic Bézier curve defined by three points \((p_1, p_2, \text{ and } p_3)\), an intersection point \(p_4\), and \(p_5\).

The two circular arcs of type-U segments are connected by a cubic Bézier curve defined by four points \((p_1, p_2, p_3, \text{ and } p_4)\), as shown in Fig. 2(c). In case lines \(p_1 p_2\) and \(p_2 p_3\) in Fig. 2(c) intersect before reaching a P line, a quadratic Bézier curve defined by three points \((p_1, p_2, \text{ and } p_3)\) is used instead as shown in Fig. 2(d). With regard to type-W segments, a quadratic Bézier curve is applied. \(p_2\) and \(p_4\) are intersections of T lines and the nearest P lines. \(p_3\) is an intersection of a bisector (B line) of the two T lines and a P line. A lower-degree Bézier curve is applied as necessary as in the case of type-U segments.

### Preparation of tooth image
Photographs of occlusal surfaces of right first and second molars of an anatomical tooth model (B3-305, Nissin Dental Products Inc., Kyoto, Japan) are taken using a digital camera. They are resized to 40 pixels/mm.

### Prototyping of computer-aided cavity design software
The above-described design rules are implemented using a visual programming language (LabVIEW 8.6,
National Instruments, Austin, USA). When the software is started, a selected image of an occlusal surface of the target tooth is loaded. The operator marks the major occlusal fissures as continuous line segments on a computer screen by clicking the ends of each segment with a mouse. After the segments and design parameters are set, the software instantaneously draws the outline. The software is applied to mandibular and maxillary right first and second molars; and their class I inlay cavities are experimentally designed.

RESULTS

Effects of the minimum radius of curvature of the convex portion $r$, the taper angle of the convex portion $\theta$, and the angle of the recess portion $\phi$ on the cavity design. (a) $r=0.5$, $0.6$, $0.7$, and $0.8$ mm ($\theta=5^\circ$, $\phi=10^\circ$), (b) $\theta=0$, $10$, $20$, and $30^\circ$ ($r=0.5$ mm, $\phi=10^\circ$), (c) $\phi=-20$, 0, and $20^\circ$ ($r=0.5$ mm, $\theta=5^\circ$).

Fig. 3 Effect of $r$, $\theta$, and $\phi$ on the cavity design. (a) $r=0.5$, $0.6$, $0.7$, and $0.8$ mm ($\theta=5^\circ$, $\phi=10^\circ$), (b) $\theta=0$, $10$, $20$, and $30^\circ$ ($r=0.5$ mm, $\phi=10^\circ$), (c) $\phi=-20$, 0, and $20^\circ$ ($r=0.5$ mm, $\theta=5^\circ$).

Fig. 4 Design examples of class I inlay cavities. (a), (a'): Mandibular right second molar; (b), (b'): Maxillary right first molar; (c), (c'): Maxillary right second molar. (a), (b), and (c): Occlusal fissures to be included in the preparation are marked as line segments; (a'), (b'), and (c'): Designed cavity outline ($r=0.5$ mm, $\theta=5^\circ$, $\phi=10^\circ$).
of a cavity \( \theta \), and the dovetail convexity angle \( \phi \) are shown in Fig. 3. The curvature radius of convex portions of the outline increases with an increase in \( r \), while that of the concave portions decreases, as shown in Fig. 3(a). As a result, the cavity becomes larger. As \( \theta \) increases, the narrow parts of the outline, including dovetails, become wider and result in a larger cavity as shown in Fig. 3(b). When \( \phi \) is positive or negative, the dovetails become convex or concave, respectively, as shown in Fig. 3(c).

Design examples of mandibular and maxillary right first and second molars are shown in Figs. 1(i) and 4. Design parameters of \( r=0.5 \) mm, \( \theta=5^\circ \), and \( \phi=10^\circ \) are applied. In any case, the narrowest portions of the outline are larger than the tool diameter, and a smooth outline is obtained. The design can be easily modified by moving the line segments and changing the design parameters.

**DISCUSSION**

Although the present design method targets a minimum-sized occlusal cavity that includes major fissures, they are in no way intended to deny different designs. Once the segments and the parameters are set, the smooth outline can be obtained. All design parameters work as intended. Accessibility of the tool throughout a cavity follows the bare-bones design rule. The designed outline is successful with respect to at least this point. Smoothness of the outline is also promised. The present method can be readily applied to teeth and tools of different sizes. These are the advantages of the present method as compared to a time-consuming freehand drawing of the entire outline.

When \( r \) is the tool radius and \( \theta \) and \( \phi \) are zero, the reduction of the tooth structure is suppressed. Because of the third characteristics of the Bézier curve mentioned above, even if \( r \) is the tool radius, the width of the narrowest portion along the type-2 segments defined by the Bézier curve is always greater than the tool diameter, which is against the minimizing of the removal of the tooth structure. To solve this problem, P lines can be placed closer (\( <r \)) to the type-2 segment, resulting in a narrower outline. However, care must be taken so as not to design a cavity whose outline’s narrowest part has a width smaller than a tool diameter. When two T lines intersect before reaching a P line as shown in Fig. 2(d), moving the P line closer to the type-2 segment is ineffective. The design parameters \( r \) and \( \theta \) should be increased as necessary to provide adequate width for less reliable restorations in terms of strength, such as ceramic inlays, although it increases the removal of the tooth structure.

The minimum radius of curvature of the convex portions of the present outline is constrained by the parameter \( r \). On the other hand, that of the concave portions depends on the Bézier curve, and its minimum value is not prescribed. Therefore, depending on the positions of the control points of the Bézier curves, an outline with sharp concave portions may be designed. Such a cavity can be cut from the viewpoint of tool path. However, it should be avoided to prevent the formation of sharp edges on the tooth structure and notch-like portions on a restoration. Two ways are available for such a case. One way is to move P lines farther away (\( >r \)) from the type-2 segment. The other way is to round the outline, that is, to replace the sharp corners by inscribed circular arcs that have the desired radius of curvature.

It is obvious that a wide variety of cavities exist in practice. The present method can be utilized to some extent by adjusting positions of line segments and design parameters. In exchange for some effort, design parameters can be changed from portion to portion. Caries lesion or occlusal contact points marked using an articulating paper can be recognized from an image of a tooth and thus can be reflected in the outline design. As for exceptional cases that the present method cannot cope with, the outline has to be partially modified manually by an operator.

There may be no point in designing a cavity for clinical purpose as long as the cavity is prepared manually. Nevertheless, the author believes the unified geometrical design of the cavity outline in some way is still necessary not only to discuss the optimal design but also to realize computer-aided tooth preparation. Appropriateness of the designed cavities in the present study remains to be proved. However, a comparative study among the different cavity designs is difficult unless the cavities can be prepared just as designed. When computer-aided preparation becomes a reality and exact cavity preparation becomes possible, comparative study becomes easier, and thus facilitates the optimization of cavity design.

**REFERENCES**