Geometric design method for occlusal outlines of complex class I and class II inlay cavities

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The purpose of this study was to establish a geometric design method for the occlusal outlines of complex inlay cavities as a continuation study of a previous design method for simple class I inlay cavity. A method for extending the occlusal outline to the buccal or lingual groove and to three preparation types of the proximal portions of class II inlay cavities—namely, straight line preparation, sweeping curve preparation, and reverse curve preparation—was investigated. To ensure the smoothness of the occlusal outline, a Bézier curve was introduced in the design. A minimal number of control points for the curve was applied to define each preparation type. The design method was experimentally applied to mandibular and maxillary first molars. Smooth outlines of the complex inlay cavities in the molars with tool accessibility throughout the cavities were achievable by using the present method.

Keywords: CAD/CAM, Cavity preparation, Complex cavity, Geometric design, Bézier curve

INTRODUCTION

Dental CAD/CAM is a rapidly advancing technology and is gradually becoming a popular method for fabricating dental restorations and prostheses. To achieve the ideal result with dental CAD/CAM, precise tooth preparation is necessary. However, tooth preparation is mostly performed freehand by using a dental handpiece, and the appropriateness of the prepared tooth completely depends on the dentist's technical skill. One way to solve this problem is to computerize the tooth preparation process by using CAD/CAM technology. However, there are many challenges before computerized tooth preparation can be realized, such as computer-aided design of inlay cavities.

Designing the entire cavity outline by freehand drawing is not only time consuming but also has drawbacks that cannot be ignored: smoothness and tool accessibility cannot be ensured. The most fundamental rule in cavity design is that the narrowest portion of the outline should be equal to or wider than the diameter of the cutting tool. Therefore, a simple method for designing a geometrically reasonable cavity outline must be developed. Such a method should be flexible to variations in tooth size and shape.

In a previous study, a geometric design method for simple class I inlay cavity was proposed; smooth class I inlay cavity outlines for molars with tool accessibility throughout the cavities could be obtained by this method. The method uses circular arcs and quadratic, cubic, and quartic Bézier curves to define the cavity outline. The purpose of the present study was to extend the design method to occlusal outlines of complex inlay cavities as a continuation study for the future application of CAD/CAM technology in tooth preparation.

MATERIALS AND METHODS

This paper deals with the outlines of typical complex class I and class II inlay cavities. To simplify the problem, we focus only on a two-dimensional (2D) occlusal outline of a cavity without a marginal bevel or proximal groove. The path of insertion of the inlay is assumed to be perpendicular to the design aspect. The concern here is not the optimal cavity design, because this major issue is beyond the scope of the present study. The cutting tool is assumed to have a single cutting radius (r). Most occlusal outlines of both complex class I and class II cavities can be designed by using the previously developed method for simple class I inlay cavities. Therefore, we limit this discussion to a method for extending the fundamental occlusal outline of a simple class I cavity along the buccal or lingual groove, or to the proximal portions. As in the previous study, a Bézier curve was used to ensure the smoothness of the outline.

Extending the cavity outline along the buccal or lingual groove

The convex portion of a cavity at the buccal or lingual groove is shown in Fig. 1. Tangents of the auxiliary circle with radius r that form angle θ with the type-1 segment are shown as T. The degree-n Bézier curve, b_n, is connected to the auxiliary circle at p_1. The ideal minimal width of the prepared groove equals the tool diameter (2r). However, as shown in Fig. 1(a), simply extending the open end (shown as an open circle) of a type-1 segment along the buccal or lingual groove outwardly toward the tooth contour (shown as the upward arrow) results in a prepared groove wider than 2r, even if the design parameter, θ, is zero. This causes unwanted removal of tooth structure. To avoid this problem, an additional type-1’ segment is introduced along the buccal or lingual groove and connected to the open end of the
type-1 segment, as shown in Fig. 1(b). The original type-1 segment thus becomes a type-2 segment. An auxiliary line parallel to and at distance \( r \) from the type-1 segment is drawn as \( G \). A portion of \( G \) becomes an extended part of the cavity outline. When \( G \) and \( T \) are collinear, the Bézier curve forming a part of the fundamental outline can be smoothly connected to \( G \) at point \( p_i \), because of the curve characteristics. When \( G \) is slanted away from the Bézier curve, as shown in Fig. 1(c), the curve can be smoothly connected to \( G \) through arc \( p_ip_2 \). On the other hand, when \( G \) is slanted toward the Bézier curve, as shown in Fig. 1(d), the curve cannot be smoothly connected as in the previous case. Here, \( p_1' \) (intersection point of \( T \) and \( G \)) is used instead of \( p_i \), and \( p_2 \) is added on segment \( p_1'p_n \), which becomes the new terminal point of the Bézier curve. The degree of the Bézier curve \( (n) \) increases by 1 accordingly. For example, if the original curve is a quartic Bézier curve, the modified curve will be quintic (one end is modified) or sextic (both ends are modified). Finally, the Bézier curve is smoothly connected to \( G \) at \( p_2 \). The \( p_1p_2 \) length must be greater than zero for this rule to take effect. However, unnecessary extension of the length increases the width of the prepared groove. In this paper, \( p_2 \) is positioned at distance \( r \) from the point of contact of the auxiliary circle and \( G \), as shown in Fig. 1(d). In any of the above-mentioned cases, the extended outline ends at or within the tooth contour, forming an outline of the groove with the narrowest width of the diameter of the auxiliary circle.

Extending the cavity outline to the proximal portions

Several preparation types for joining the occlusal portion to the proximal portions of class II cavities are clinically used\(^3\). The three basic designs discussed here are straight line preparation, sweeping curve preparation (flare preparation), and reverse curve (s-shaped curve) preparation.

1. **Straight line preparation**

The proximal portion of the outline of this type is defined by two outward-opening tangents \( (l_i \text{ and } l'_i) \) to the isthmus of the fundamental occlusal outline connected at \( q_1 \) and \( q'_1 \), as shown in Fig. 2(a). They can be moved along the fundamental outline, although the width of the narrowest portion between the two tangents must be at least the tool diameter. Points \( q_2 \) and \( q'_2 \) are the intersection points of the tangents and the tooth contour. In reality, the control points should be positioned according to various factors, such as the extent of tooth decay, tooth structure, clean area of the tooth, and the restorative material properties. Line segments \( q_1q_2 \) and \( q'_1q'_2 \) override the fundamental design. Thus, this fundamental design can differ from that of the simple class I cavity.

A proximal box (axial wall) can be displayed on the design as parallel curve \( q_0q_n \) offset from the tooth contour or simply as a straight line. Both ends of the curve or the line are rounded by inscribed arcs of radius \( r_{p_2} \), which is equal to or greater than the diameter of the tool used for the box preparation.

2. **Sweeping curve and reverse curve preparations**

The difference between these types and the preceding

![Types of ends of line segments](https://example.com/types_ends_line_segments)

**Fig. 1** Extended outline of a simple class I inlay cavity to the buccal or lingual groove. Only the left side of the groove is shown. The dotted circles represent auxiliary circles with radius \( r \). The \( T \) lines are the tangents of the auxiliary circles that form angle \( \theta \) with the type-1 segments. \( b_n \) represents the Bézier curve of degree \( n \) that form a part of the cavity outline. (a) Simply extending the open end (open circle) of the type-1 segment along the buccal or lingual groove outwardly results in a prepared groove wider than \( 2r \), even if \( \theta \) is zero. (b) An additional segment along the buccal or lingual groove is connected to the originally open end of the type-1 segment (open double circle). Auxiliary lines parallel to and at distance \( r \) from the additional segment are shown as \( G \). When \( G \) and \( T \) are collinear, \( b_n \) can be smoothly connected to \( G \) at \( p_1 \). The extended outline ends at the intersection points of \( G \) and the tooth contour (not shown). (c) When \( G \) is slanted away from \( b_n \), \( b_n \) can be smoothly connected to \( G \) via arc \( p_ip_2 \). (d) When \( G \) is slanted toward \( b_n \), \( b_n \) cannot be smoothly connected at \( p_1 \) as in (b) or (c). In this case, control points \( p_1' \) (intersection point of \( T \) and \( G \)) and \( p_2 \) on segment \( p_1'p_n \) (the new terminal) are drawn and the degree of the Bézier curve increases by 1.
Intersection points are shown as $q_2$ and $q_2'$. Points $q_3$ and $q_3'$ are the intersection points of the auxiliary lines and the tooth contour. Finally, two quadratic Bézier curves, $b_2$ and $b_2'$, are drawn by using the combination of $q_3$, and $q_3'$, $q_2$, and $q_2'$ as three control points. For the additional control points, this method enables a wider variety in design than the preceding method. The control points should be positioned properly as in the case of the straight line preparation. For the reverse curve preparation, $q_2$ and $q_2'$ are placed outside the cavity outline, as shown in Fig. 2(d). The proximal box can be added in the same way as for the straight line preparation.

Cavity design using the proposed method

The extended design rules were additionally implemented in the original software by using a visual programming language (LabVIEW 2009, National Instruments, Austin, TX, USA), a C++ programming language (Microsoft Visual C++ 2010 Express Edition, Microsoft Corporation, Redmond, WA, USA), and an image processing library (OpenCV 2.1, Willow Garage, Menlo Park, CA, USA). The root-finding algorithm was used for calculating the coordinates of the points of tangency to the Bézier curves. A rolling ball algorithm that virtually rolls a ball with radius $o_{PB}$ along the inside of the tooth contour was used to determine the contour-parallel curve.

To check the feasibility of geometric modeling of typical complex cavities by the prototype software visually, the software was applied to prepare mandibular and maxillary right first molars; complex class I metal inlay cavities and mesio-occlusal (MO) class II metal and ceramic inlay cavities were experimentally designed. Photographs of the occlusal surfaces of the right first molars of an anatomic tooth model (B3-305, Nissin Dental Products, Inc., Kyoto, Japan) were used as in the previous study. For the complex class I metal inlay cavities, design parameters of $r=0.6$ mm, $θ=5°$, and $ϕ=10°$ were applied. For the class II metal inlay cavities, the same design parameters were applied to the fundamental cavity and the sweeping curve preparation with $o_{PB}=1.5$ mm and $r_{PB}=0.5$ mm was applied to the proximal portion. For the class II ceramic inlay cavities, the reverse curve preparation and straight line preparation were selected for the mandibular and maxillary first molars, respectively. The design parameters of $r=0.8$ mm, $θ=20°$, $ϕ=10°$, and the distance of $P$ lines from the type-2 segments of $d=1.1$ mm (in the previous study, the value equaled $r$) were applied to the fundamental cavity, and those of $o_{PB}=1.8$ mm and $r_{PB}=0.5$ mm were applied to the proximal portion. The positions of the line segments of the ceramic inlay cavities were not changed from those of the metal inlay cavities on purpose for easier comparison of the effects of the design parameters.

RESULTS

After the tooth image was loaded and the line segments, control points, and design parameters were set by the operator, the software instantaneously drew the outline.

**Fig. 2** Extended outline of a simple class I inlay cavity to the proximal portion. (a) Straight line preparation with a proximal box. Tangents $l_1$ and $l_1'$ to the fundamental outline form the extended outline. The proximal box is displayed as parallel curve $o_{PB}$ offset from the tooth contour by two arcs with radius $r_{PB}$ connected to both ends. (b) Sweeping curve preparation 1. The outline is extended by outwardly moving the dovetail portion of the fundamental design. The outline beyond the tooth contour is neglected. (c) Sweeping curve preparation 2. Auxiliary lines $l_2$ and $l_2'$ in addition to tangents $l_1$ and $l_1'$ are drawn. Quadratic Bézier curves $b_2$ and $b_2'$ form the extended outline. (d) Reverse curve preparation. Quadratic Bézier curves $b_3$ and $b_3'$ form the extended outline. Note that intersection points $q_2$ and $q_2'$ are placed outside the outline, not inside as in (c).
The appropriate degree of the Bézier curve was automatically selected as the type-1′ segment was added or repositioned. As for the tangents to the Bézier curves of the fundamental design, the software automatically set the point of tangency on the curve and drew the tangent by directing one point equivalent to \( q_2 \) or \( q_2' \) in Fig. 2 and another point nearby the curve on the computer screen by a mouse. The software also automatically set the points of tangency between the arc with radius \( r_{PB} \) and the contour-parallel curve as well as between the arc and the straight lines or quadratic Bézier curves to form the occlusal outlines of the proximal box.

Design examples of complex class I metal inlay cavities of mandibular and maxillary first molars are shown in Fig. 3, and those of class II metal and ceramic inlay cavities are shown in Figs. 4 and 5, respectively. The ceramic inlay cavities have a wider isthmus and larger radius of curvature than those of the metal inlay cavities. To avoid formation of both unsupported enamel and sharp corners on ceramic, the angle formed by a tangent of the tooth contour and a tangent of the ceramic inlay cavity outline at the proximal end [equivalent to \( q_2 \) and \( q_2' \) in Fig. 2(a) and \( q_3 \) and \( q_3' \) in Fig. 2(d)] is designed to be a near right angle (90°). In any case, the narrowest portions of the outline are equal to or wider than the tool diameter, and a smooth outline is obtained. The design can be readily modified by moving the line segments or control points, or adjusting the design parameters.
DISCUSSION

Ease and the degree of freedom of design are contradictory. Just because a design method has a high degree of freedom, it does not necessarily produce a good result; freehand drawing is the perfect example. As already mentioned, the narrowest portion of a cavity designed in any way must be equal to or wider than the tool diameter for the design to be feasible, but it should not be unnecessarily large. The cavity outline should be smooth to minimize the interface between the tooth and the restoration, by which the risk of marginal leakage would be reduced. A smooth outline would also facilitate conventional or digital impression processes and consequently improve the fit of the restoration\(^3\). The present method succeeded in modeling smooth outlines of complex cavities similar to those of the typical ones by using a small number of design elements. The proposed method is in no way intended to deny different designs and the experimentally designed cavities are not necessarily optimized. Needless to say, the number of line segments or control points can be increased and other geometric curves can be applied for a higher degree of design freedom, or the design can be partially modified manually, in exchange for simplicity of the present method.

In the present design method, the cavity margins are not beveled. This is not a problem in ceramic restorations, because marginal beveling that leads to thin edges of material is harmful for the brittle material\(^3,4\). Instead, for the same reason, rounding of internal line angles may be necessary. For metal restorations, proximal grooves should be prepared at the

**Fig. 4** Design examples of sweeping curve-type class II metal inlay cavities in the maxillary right first molar (a, a’) and mandibular right first molar (b, b’). The line segments and control points are shown in (a) and (b), and the resulting cavity outlines are shown in (a’) and (b’) (r=0.6 mm, \(\theta=5^\circ\), \(\phi=10^\circ\), \(d=r\), \(o_{PB}=1.5\) mm, and \(r_{PB}=0.5\) mm).
proximal portion as necessary for enhanced retention and the margins should be finally beveled\(^{3}\).

Although the present design method is 2D and uses an occlusal image, it can be replaced with the occlusal view of 3D shape data of the tooth. Unlike direct restorations, there must be no undercuts in the preparation that prevent insertion of an inlay\(^{3}\). Ideally, no undercut will be formed in the prepared tooth if the preparation is done by translational motion of the tool axis, which is usually oriented parallel to the long axis of the tooth. The motion can be realized by 3D machining or, more simply and easily, by 2.5D machining, using a three-axis milling machine. The latter cuts only in two of the three axes at a time. For the assumption that the path of insertion of the inlay is perpendicular to the design aspect (i.e., to the sheet), the designed class II cavities including proximal boxes can be theoretically prepared by 2.5D machining using a cutting tool with a single cutting diameter.

In practice, the cavity walls parallel to the path of insertion are generally tapered by using a tapered tool to ensure undercut-free preparation by freehand and enhance the ease of luting the restoration to the tooth. The diameter of the tapered tool increases from the tip to the shank. When the tip diameter is used as a design parameter of the present design method and the occlusal portion of the cavity is prepared by 2.5D machining to a final depth of cut, it is the outline of the cavity floor that matches the design. In other words, the external outline of the prepared cavity becomes slightly larger than the designed outline, depending on the taper angle of the tool and the depth of the cavity. For example, if the taper

**Fig. 5** Design examples of reverse curve-type and straight line-type class II ceramic inlay cavities in the maxillary right first molar (a, a’) and mandibular right first molar (b, b’). The line segments, control points, and auxiliary figures are shown in (a) and (b), and the resulting cavity outlines are shown in (a’) and (b’) (\(r=0.8\) mm, \(\theta=20^\circ\), \(\varphi=10^\circ\), \(d=1.1\) mm, \(oPB=1.8\) mm, and \(r_{PB}=0.5\) mm).
(a half of the cone angle) is 4° and the maximum depth of the cavity is 2 mm, the discrepancy from the original design will be about 0.14 mm.

Buccal and lingual grooves can also be prepared by 2.5D machining when allowing the buccal or lingual view of the outline to match the profile of the tool. When the required degree of freedom of the tool path for the cavity preparation exceeds the limits of 2.5D machining, 3D machining or multi-axial machining that supports rotary motion in addition to translational motion is necessary. However, if the necessity of the higher-dimension machining is limited, it can be substituted by manual finishing of a 2.5D-machined cavity. The present method is limited to the occlusal view design of complex inlay cavities and full 3D designing remains a future topic of research.

REFERENCES