Numerical Analysis of Thermal Stress and Shrinkage of Wax Patterns

III. Effects of Physical Properties and Cooling Conditions

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Thermal stresses in three brands of inlay waxes were numerically analyzed under different cooling conditions using the rheological and thermal properties of waxes. On the basis of this analysis, the shrinkage of the wax pattern was calculated. The larger the stress relaxation of the wax and the higher the ambient temperature, the smaller the thermal stress induced in the wax. The shrinkage of the wax pattern that was restrained by the die was extremely smaller than that of the male portion of the pattern. This indicates that the stress relaxation of the wax contributed greatly to the restriction of shrinkage of the wax pattern at the male portion of the die.

Key Words: Wax Pattern, Thermal Stress, Shrinkage

INTRODUCTION

The accuracy of the casting depends on the accuracy of the wax pattern. Various brands of dental inlay waxes with different properties have been formulated to minimize shrinkage and distortion of the wax pattern. This concept is not always right. For example, the male portion of the wax pattern requires sufficient shrinkage to save a layer of cement between the casting and the cavity. On the other hand, the shrinkage of the wax pattern at the male portion of the die should be minimized. These dimensional changes in the wax pattern are closely related to the induction and to the relaxation of the thermal stress.

When the thermal shrinkage of wax is restricted during cooling, thermal stress is induced in the wax pattern. Since wax is a viscoelastic material, the resultant thermal stress is released with time. Removal of the wax pattern from the die before the relaxation of thermal stress will result in shrinkage and distortion of the pattern. For this reason it is important to know the characteristics of the thermal stress and its relaxation process to obtain dimensional accuracy of the wax pattern and optimum adaptation of the final restoration.

As previously described, the generation and relaxation of the thermal stress take place simultaneously during fabrication of the pattern. The viscoelastic stress of the wax in this process can be theoretically analyzed on the assumption that there is a linear relationship between the stress and the strain at low extension states. The following equation was proposed2)
\[
\sigma(t) = \int_0^t \left[ \int_0^t \frac{1}{a_T(T(t'))} dt'' \right] \left\{ -\alpha \{T(t')\} \cdot \frac{dT(t')}{dt} \right\} dt'
\]  
\text{(1)}

where the functions \( E, a_T, \alpha \) and \( T \) are the relaxation modulus, the reducing factor, coefficient of thermal expansion and temperature, respectively; \( \text{variable } t \) represents time. Equation (1) indicates that the thermal stress \( \sigma(t) \) depends on the relaxation modulus \( E \), on coefficient of thermal expansion \( \alpha \) and on cooling rate \( dT/dt \).

There are number of brands of inlay waxes with different properties and each material claims to have different tendency for the induction and relaxation of the thermal stress. The purpose of this report is to estimate shrinkage of wax patterns under different cooling conditions using the numerical analysis of the thermal stress based on data acquired from commercially available inlay waxes.

**MATERIALS AND METHODS**

The three brands of commercially available dental inlay waxes used were GC Blue Wax, Shofu Blue Wax and Toyo Violet Wax. GC Blue Wax and Toyo Violet Wax are employed in the indirect technic. Toyo Violet Wax is classified as a soft wax with a low melting point. Shofu Blue Wax is designed for the direct technic and classified as a hard wax.

Stress relaxation experiments were carried out on specimens of 100 mm long (width; 10 mm, thickness; 1.8 mm) using the Shimadzu Autograph (DSS-2000 type). Whole specimens were dipped into a thermostatic water bath regulated within ±0.1°C of desired temperature. A tensile strain of about 0.2% was applied and the stress relaxation was measured at different temperatures ranging from 10°C to 37°C. By the application of the time-temperature superposition principle, the relaxation modulus and time relationships at each temperature were horizontally shifted to a curve at 20°C to construct a single master curve.

The method of the numerical analysis of the thermal stress induced in the wax pattern was described in detail in the previous paper of this series\(^ 2 \). As shown in equation (1), the integrand is the product of the master curve of the relaxation modulus, coefficient of thermal expansion and cooling rate.

Both the master curve of the relaxation modulus and the reducing factor-temperature relationship were reduced to second order expressions. The coefficient of thermal expansion was divided into several regression sections and was reduced to a polynomial expression. The cooling rate was reduced to a polynomial expression in the high temperature region and was approximated to an exponential function in the low temperature region of the cooling curve. The thermal stress was calculated using a computer (ACOS SERIES 77 NEAC SYSTEM 1000, NEC Corporation).

**RESULTS**

Stress relaxation curves for Shofu Blue Wax and Toyo Violet Wax measured at different temperatures are shown in Figures 1 and 2. The relaxation modulus \( E(t) \) of Shofu Blue Wax decreased slowly with increasing temperature. On the other hand, the modulus of
Toyo Violet Wax depended greatly on temperature as the temperature of the solid-solid phase transition of this material was fairly low.

By the application of the time-temperature superposition principle, master curves shown in Figure 3 (curves 2 and 3) were constructed from the corresponding relaxation curves shown in Figures 1 and 2. These relaxation master curves overlapped each other well. In this paper 20°C was chosen as the reference temperature. The master curve of GC Blue Wax shown in Figure 3 was obtained by plotting the data published in the previous paper. The modulus of Shofu Blue Wax and Toyo Violet Wax changed slowly with time. The
modulus of GC Blue Wax in a long time region decreased rapidly with time. The reducing factor-temperature relationships are shown in Figure 4.

Both the cooling curves of various brands of waxes which were heated to 48°C initially and cooled to the room temperature of 20°C, and numerically determined thermal stresses are shown in Figure 5. The thermal stress in Toyo Violet Wax was not induced from 48°C to 41°C, as the modulus of this material in a high temperature region was small ($10^8$ dyne/cm²). A significant thermal stress was induced below 41°C, and a maximum value was observed at the vicinity of room temperature. Gradually this stress was released with time.

The thermal stress in GC Blue Wax in a high temperature region was slightly induced, as the stress was rapidly released in this region and also the modulus was fairly small ($10^8$ dyne/cm²).

![Figure 5](image1.png) Thermal stresses and cooling curves of waxes cooled from 48°C to the room temperature of 20°C.

![Figure 6](image2.png) Thermal stresses and cooling curves of waxes cooled from 48°C to the room temperature of 25°C.
On the other hand, the thermal stress in Shofu Blue Wax was induced simultaneously with the beginning of cooling and the value was the largest in all of the materials examined, since the stress of this material was slowly released even in a high temperature region and the modulus was fairly large (10^9 dyne/cm^2).

The cooling curves of various brands of waxes cooled to 25°C and the numerically analyzed thermal stresses are shown in Figure 6. The thermal stresses in GC Blue Wax and Toyo Violet Wax in a high temperature region were not induced, whereas the stress in Shofu Blue Wax was induced with the start of cooling. The thermal stresses in Shofu Blue Wax and Toyo Violet Wax were about 70% of the maximum value after 5 hours. While in the case of GC Blue Wax of which the stress relaxation was rapid above 23°C^3), the thermal stress decreased to 55% of the maximum value after 5 hours.

The cooling curves and the thermal stresses at room temperature of 30°C are shown in Figure 7. In this case the thermal stress of each material was small because of high room temperature and the slow cooling.

**DISCUSSION**

The thermal stress induced in the wax and its relaxation during fabrication of the wax pattern are important factors when the dimensional accuracy of the pattern is discussed. The thermal stresses induced in the simply shaped wax patterns were analyzed numerically^2,4^). If the rheological and thermal properties of any wax are known, the thermal stress can be calculated from equation (1). It is obvious from equation (1) that the magnitude of the thermal stress depends on the relaxation modulus, on coefficient of thermal expansion and on cooling conditions. The relaxation modulus and coefficient of thermal expansion are properties of the wax itself, but the former is a function of both time and temperature. The cooling condition is influenced by the temperature difference between the softened wax and the ambience.

During fabrication of wax patterns, the wax is subjected not only to thermal changes
but also to various mechanical deformation. It has been also reported that the preferred orientation occurs when the wax is softened and deformed plastically, and the anisotropic dimensional change caused by the recovery of deformation is observed\(^{5,6,7}\). Therefore, the magnitude of thermal stress and its distribution in the practical wax pattern are complicated. If it is intended to cover all of these factors, the analysis of thermal stress must be very much complicated. In this study, the thermal stresses induced in the plate-shaped wax patterns of three brands of inlay waxes were calculated numerically in various cooling conditions, assuming that the waxes have isotropy.

As can be noted from Figures 5, 6 and 7, each material has a different tendency for the induction of thermal stress and for its relaxation. The thermal stress induced in Shofu Blue Wax was the largest over the temperature range examined in all materials tested, since the stress relaxation of this material was slow and the modulus was large. Considerable amounts of some ingredients, that restrict stress relaxation, may be added in this inlay wax. It is obvious from Figure 3 that the stress in GC Blue Wax was rapidly released at the right side of graph (i.e., in a high temperature region). Therefore, the thermal stress in GC Blue Wax must be smaller than that in Shofu Blue Wax. The thermal stress in Toyo Violet Wax was the smallest because this material was a soft wax with a low transition temperature, and its modulus was small.

It is also noted from Figures 5, 6 and 7 that the higher the ambient temperature, the smaller the thermal stress in each material. This is because the increase of temperature tended to promote stress relaxation and the temperature difference between the softened wax and the ambience was small, and also the cooling rate of the pattern was slow.

From the clinical point of view, various types of cavity are encountered. Figure 8 shows a MOD cavity (a) and a complex occlusal cavity (b). Portion A has a slope that diverges toward the root. On the other hand, portion B has a slope that converges toward the root. Shrinkage of the wax pattern facing portion A is restrained by the die, whereas the free shrinkage takes place at the pattern facing portion B.
Based on the numerical analysis of thermal stress, shrinkage of the wax pattern facing portion A (i.e., shrinkage of the wax pattern restrained by the die) was simply calculated from the residual stress after 5 hours and the modulus at the ambient temperature.

Table 1 shows the restricted shrinkage of portion A and free shrinkage of portion B of the wax pattern. One third of observed cubical shrinkage was adopted as the linear free shrinkage of portion B of the pattern. The measurement of shrinkage was performed between the temperature at which 50% flow was observed under 2.55 kgf/cm² and ambient temperature.

The shrinkage of portion A tended to become smaller with increasing ambient temperature, because the increase of temperature promoted the stress relaxation and depressed the induction of thermal stress, and also the residual thermal stress became small. When the ambient temperature increased, the temperature difference between the softened wax and the ambience became small. Thus shrinkage of portion B was small.

A comparison of shrinkage between portions A and B in Table 1, indicated that the value of the former was extremely smaller than that of the latter in all materials and over the ranges of temperature examined. This finding shows that the stress relaxation of the wax contributed greatly to the restriction of shrinkage of portion A.

There are many types of wax patterns that have portions A and B such as a MOD pattern. Those wax patterns require sufficiently large shrinkage at portion B and at the same time small shrinkage at portion A, in other words, they require a large shrinkage difference between the two portions, in order to ensure a proper layer of cement between the casting and the cavity.

As previously stated, the shrinkage of portion A of the wax pattern can be depressed by stress relaxation. The shrinkage of portion B is governed by factors such as temperature, pressure, and the adhesional condition of wax to the die. Therefore, suitable inlay wax should be chosen to obtain a casting with good fitting, considering widths of both portions A and B of the wax pattern. For example, Shofu Blue Wax which had the largest shrinkage difference between portions A and B in all materials tested, may be suitable for

<table>
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<th>Waxes</th>
<th>Temperature (°C)</th>
<th>20</th>
<th>25</th>
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<tr>
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<td></td>
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<tr>
<td></td>
<td>difference (%)</td>
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Restricted means shrinkage of the wax pattern facing portion A (shown in Figure 8). Free means free shrinkage of the pattern facing portion B.
the casting that has a narrow B portion.

The results of this study suggest a direction in selecting and manufacturing appropriate inlay waxes for fabrication of complex wax patterns such as a MOD type.

CONCLUSIONS

The thermal stresses of commercially available waxes at different temperatures were analyzed numerically using the data obtained by rheological and thermal experiments. On the basis of this analysis, shrinkage of portions A and B of the wax pattern was calculated. By the application of the time-temperature superposition principle for each material, the relaxation curves at different temperatures were superimposed to a single master curve with good overlapping.

The magnitude of the thermal stress and its relaxation were influenced by the characteristics of waxes and by the cooling conditions. The larger the stress relaxation of the wax and the higher the ambient temperature, the smaller the thermal stress induced in wax.

Shrinkage of portion A of the wax pattern was smaller than that of portion B. This indicated that the stress relaxation of the wax contributed greatly to the restriction of the shrinkage of portion A.

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REFERENCES

ワックスバタンの熱応力と収縮に関する数値解析

III. ワックスの物理的性質と冷却条件の影響
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ワックスバタンの外側性部分に発生する熱応力とその緩和は、バタンの寸法精度を論ずる際に、きわめて重要な因子である。著者らはこれまでに粘弾性応力の理論解析を行ない、冷却過程におけるワックスの熱応力を、ほぼ定量的に算出できることを明らかにした。今回は物性値の異なる3種類のワックスを用いて、冷却条件を変えた時の熱応力を数値解析によって求め、バタンの収縮について検討した。

その結果によれば、ワックスの応力緩和が大きい程、また軟化したワックスと外界との温度差が小さい程、バタンに発生する熱応力は小さくなった。解析された残留応力値から求めたバタンの外側性部分の収縮値は、内側性部分の自由収縮値よりもきわめて小さいことが分かった。この事実は、ワックスの応力緩和が、バタンの外側性部分の収縮抑制に大きく寄与していることを示すものである。