Dynamic Characteristics of Rolling Agricultural Tire

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Abstract

When operating a tractor on a pavement, its vibration is greatly affected by the lug excitation force, which is caused by the interaction between the road and agricultural tire. Therefore, it is important to understand the characteristics of the lug excitation force to elucidate the mechanism of tractor vibration. However, it is difficult to directly measure the lug excitation force during rolling motion owing to the requirements of complicated measurement methods. Therefore, no studies have determined the lug excitation force. In this study, an agricultural tire is modeled as a 3-dimensional rigid ring model, which is useful for estimating the lug excitation force. Consequently, a new technique to determine the lug excitation force is presented. The shaft displacement and shaft force during rolling motion are measured in 3-dimensions through rolling test, and the lug excitation force is determined from the test data. The characteristics of the lug excitation and shaft forces are investigated, including the correlation between these and the tire vibration characteristics. Furthermore, the transfer characteristics of the lug excitation force with respect to the shaft force are analyzed, and the effect of the tire vibration characteristics on the transfer characteristics is examined. The developed technique is very useful in reducing tractor vibration caused by the lug excitation force.

[Keywords] agricultural tire, rigid ring model, identification, lug excitation force, vibration characteristic, transfer characteristics

I Introduction

Recently, farm management expansion scale has increased in Japan due to a decrease in the agricultural population, and tractors are increasingly running on paved roads. Accordingly, tractor speeds and weight reduction are advancing. Generally, agricultural tires have cross-stitch lugs. Therefore, when operating a tractor on the pavement, the vibration generated by the reciprocal action between the agricultural tire and the road surface is dominant (Kaneko and Saito, 2006; Takeda et al., 2009). It is understood that the lug excitation force is the main cause of tractor vibration. Furthermore, the characteristics of the agricultural tire vibration such as natural frequency and natural mode are considered to play significant roles in the lug excitation force. As a study on the dynamic characteristics of agricultural tire, the dynamic stiffness and the damping properties in longitudinal and lateral directions were experimentally derived from the dynamic test for dynamic analysis of farm tractors (Nguyen et al., 2009). The dynamic properties were compared to the static properties. However, the vibration characteristics of agricultural tire were not investigated. Regarding tractor vibration, the effects of agricultural tire on wheel load and rear axle vibrations at different tire forward speeds and inflation pressure have been investigated (Nguyen and Inaba, 2011). Further, the characteristics of the wheel load

fluctuation were examined, and it was shown that the wheel load variation was excited at the 1st- and the 2nd-order frequencies of tire lugs. However, only non-uniformity was considered as tire characteristic, and lug excitation force was not treated at all. Few studies have attempted to elucidate the mechanism of tractor vibration caused by lug excitation force. The purpose of this study is to elucidate phenomena and examine measures to reduce tractor vibration caused by lug excitation force. Tractors lack a suspension mechanism between the tire and the chassis due to problems of working on soft soil. For anti-vibration measures, cabin-type tractors use a structure that supports the cabin with vibration-proof rubber. Therefore, to examine vibration reduction measures such as the optimal placement of vibration-proof rubber and the rigidity characteristics of rubber, it is significant to understand the characteristics of the lug excitation force and the transmission characteristics through the tire.

In our previous study, it was confirmed that the agricultural tire vibration characteristics affect the dynamic behavior of rolling tires (Fujita et al., 2011), and only the rigid mode among the natural modes affects the rolling behavior of the tire (Fujita et al., 2014). Therefore, we modeled the tire with a 3-dimensional rigid ring referred to as Short Wavelength Intermediate Frequency Tire (SWIFT) model, which can represent rigid modes. This model is valid for estimating lug

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excitation forces. In this study, the shaft motion and the shaft force during rolling motion were measured in 3-dimensions by rolling test, and the lug excitation force in longitudinal, vertical, and lateral direction were determined from the measured data using the least square method with the SWIFT model. This new method of identifying lug excitation forces allows us to investigate the characteristics of lug excitation forces. The characteristics of the lug excitation force and shaft force were investigated, including the correlation between these characteristics and the agricultural tire vibration characteristics. Furthermore, the transfer characteristics relating the lug excitation force to the shaft force were analyzed, and the effect of the tire vibration characteristics on the transfer characteristics were examined.

II Materials and Methods

1. Rolling test experimental equipment and procedure

Fig. 1 shows the schematic of the rolling test. This test device was manufactured to measure shaft force and shaft displacement by rolling test. In the experiment, cultivator tire was used, and Table 1 shows the main specifications of the tire. The outer diameter of the drum was 600 mm. The tire was connected to the tire shaft, which was supported at a single end. The inflation pressure of the tire was 100 kPa and it was pressed to the drum with a preload of 200 N. When the drum was rotated by the drive motor, the tire rotated according to the rotation of the drum.

As a rolling test condition, the tire revolution was changed from 50 to 500 rpm by 10 rpm steps. Shaft forces and shaft motion during the rolling motion were measured in 3-dimensions in each condition.

Fig. 1 shows the supporting structure of the tire. The tire shaft was supported only by a bearing holder that incorporates 2 sets of bearings, and it was joined to the 6-component force transducer (LFM-A-3KN, Kyowa Electronic Instruments Co. Ltd., Japan). By this structure, the force acting on the tire shaft was transferred only to the bearing holder, thus the shaft forces (longitudinal, lateral, and vertical shaft force: F_x , F_y , and F_z) could be measured by the 6-component force transducer. For the shaft motion, longitudinal and vertical shaft displacements x_a and z_a were measured by a laser displacement sensor (LB-1000QS, KEYENCE Corp., Japan), and lateral shaft displacement y_a was measured by eddy current displacement sensor (VS-041, Kyowa Electronic Instruments Co. Ltd.). The rotation speed of the tire Ω was measured by a rotary encoder joined to the tire shaft via flexible coupling.

2. Identification of lug excitation force

(1) Three-dimensional rigid ring tire model

The rigid ring model is based on the study of Zegelaar (1998). Later, it was expanded to a 3-dimensional model by

The tire is fixed to the tire shaft and pressed to the drum with preload. When the drum is rotated by a drive motor, the tire rotates according to the rotation of the drum. Tire shaft is supported by bearing holder, which is joined to 6-component force transducers. Shaft forces F_x , F_y and F_z are measured by 6-component force transducers; shaft displacements x_a and z_a by a laser displacement sensor; y_a by an eddy current displacement sensor; and rotation speed of the tire by a rotary encoder.

Fig. 1 Schematic of rolling test

Table 1 Main specifications of tire

Characteristic	Dimension
Outer diameter	308 mm
Width	93 mm
Number of lugs	10
Lug height	12 mm
Weight	3.7 kg

Maurice (2000) and finally integrated as the SWIFT model by Pacejka (2006). Fig. 2 shows the SWIFT model, which is an inflated tire wheel system, consisting of the tire ring, the sidewalls, and the wheel. This model can take into account the shaft forces F_x , F_y , F_z (N) and the lug excitation forces

F_{lugx} , F_{lugy} , and F_{lugz} (N). The tire ring modeled as a rigid ring and the wheel modeled as a rigid body are connected via sidewalls in 3-dimensions.

Table 2 shows the parameters of the model; m_a and m_b (kg) are the masses of the wheel and tire ring; c_{bx} , c_{by} , c_{bz} (N s / m) are translational sidewall damping; and k_{bx} , k_{by} , k_{bz} (N / m) are translational sidewall stiffnesses.

The wheel has 4 degrees of freedom: 3 translational movements (longitudinal, lateral, and vertical direction) and 1 rotational movement (around an axis vertical to the wheel surface). The equation of motion of the wheel can be derived as follows by limiting it to translational motion.

$$\left\{ \begin{array}{l} F_x(t) = m_a \ddot{x}_a(t) + c_{bx} \{\dot{x}_a(t) - \dot{x}_b(t)\} \\ \quad + k_{bx} \{x_a(t) - x_b(t)\} - c_{bz} \Omega \{z_a(t) - z_b(t)\} \\ F_y(t) = m_a \ddot{y}_a(t) + c_{by} \{\dot{y}_a(t) - \dot{y}_b(t)\} \\ \quad + k_{by} \{y_a(t) - y_b(t)\} \\ F_z(t) = m_a \ddot{z}_a(t) + c_{bz} \{\dot{z}_a(t) - \dot{z}_b(t)\} \\ \quad + k_{bz} \{z_a(t) - z_b(t)\} - c_{bx} \Omega \{x_a(t) - x_b(t)\} \\ \quad + F_0 + F_d \end{array} \right. \quad (1)$$

The tire ring has 6 degrees of freedom: 3 translational movements and 3 rotational movements. The equation of motion of the tire ring can be derived as follows by limiting it to translational motion in the same way as the wheel:

$$\left\{ \begin{array}{l} F_{lugx}(t) = m_b \ddot{x}_b(t) + c_{bx} \{\dot{x}_b(t) - \dot{x}_a(t)\} \\ \quad + k_{bx} \{x_b(t) - x_a(t)\} - c_{bz} \Omega \{z_b(t) - z_a(t)\} \\ F_{lugy}(t) = m_b \ddot{y}_b(t) + c_{by} \{\dot{y}_b(t) - \dot{y}_a(t)\} \\ \quad + k_{by} \{y_b(t) - y_a(t)\} \\ F_{lugz}(t) = m_b \ddot{z}_b(t) + c_{bz} \{\dot{z}_b(t) - \dot{z}_a(t)\} \\ \quad + k_{bz} \{z_b(t) - z_a(t)\} - c_{bx} \Omega \{x_b(t) - x_a(t)\} \\ \quad + F_0 + F_d \end{array} \right. \quad (2)$$

where x_a , y_a , z_a (m) are shaft displacements; x_b , y_b , z_b (m) are tire ring displacements; Ω (rad / s) is steady speed of rotation.

(2) Parameter identification

In Eq. (1), x_a , y_a , z_a , F_x , F_y , F_z , Ω can be measured by rolling test. Further, $\dot{x}_a$, $\ddot{x}_a$, $\dot{y}_a$, $\ddot{y}_a$, $\dot{z}_a$, $\ddot{z}_a$ can be calculated from the measured data x_a , y_a , z_a using the 2nd-order accurate central difference method; for example,

$$\dot{x}_a(t_i) = \frac{x_a(t_{i+1}) - x_a(t_{i-1}))}{2\Delta t}$$

Tire ring modeled as a rigid ring and wheel modeled as a rigid body are connected via sidewalls in 3 dimensions.

Fig. 2 Three-dimensional rigid ring model (SWIFT model)

Table 2 Parameters of rigid ring model

Parameter	Dimension
m_a	2.52 kg
m_b	2.39 kg
c_{bx}	1.38×10^3 N s / m
c_{by}	1.21×10^3 N s / m
c_{bz}	1.38×10^3 N s / m
k_{bx}	3.84×10^5 N / m
k_{by}	2.81×10^5 N / m
k_{bz}	3.84×10^5 N / m

The tire ring displacement x_b , y_b , z_b is assumed to be the vibration in which lug excitation frequency is dominant. Therefore, the displacement of the tire ring can be expressed as:

$$\left\{ \begin{array}{l} x_b(t) = X_1 \cos \Omega_1 t + X_2 \sin \Omega_1 t + \dots \\ \quad + X_7 \cos \Omega_4 t + X_8 \sin \Omega_4 t \\ y_b(t) = Y_1 \cos \Omega_1 t + Y_2 \sin \Omega_1 t + \dots \\ \quad + Y_7 \cos \Omega_4 t + Y_8 \sin \Omega_4 t \\ z_b(t) = Z_1 \cos \Omega_1 t + Z_2 \sin \Omega_1 t + \dots \\ \quad + Z_7 \cos \Omega_4 t + Z_8 \sin \Omega_4 t \end{array} \right. \quad (3)$$

where Ω_i is the i th lug excitation frequency. Here, the lug excitation frequency from 1st to 4th is considered. Substituting Eq. (3) into Eq. (1) gives

$$\begin{aligned}
\begin{Bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \end{Bmatrix} &= X_1 \begin{Bmatrix} u_{11}(t) \\ 0 \\ u_{12}(t) \end{Bmatrix} + \dots + X_8 \begin{Bmatrix} u_{81}(t) \\ 0 \\ u_{82}(t) \end{Bmatrix} \\
&+ Y_1 \begin{Bmatrix} 0 \\ v_1(t) \\ 0 \end{Bmatrix} + \dots + Y_8 \begin{Bmatrix} 0 \\ v_8(t) \\ 0 \end{Bmatrix} \\
&+ Z_1 \begin{Bmatrix} w_{11}(t) \\ 0 \\ w_{12}(t) \end{Bmatrix} + \dots + Z_8 \begin{Bmatrix} w_{81}(t) \\ 0 \\ w_{82}(t) \end{Bmatrix} + F_d \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}
\end{aligned} \quad (4)$$

where

$$\begin{cases} F_1(t) = F_x(t) - m_a \ddot{x}_a(t) - c_{bx} \dot{x}_a(t) \\ \quad - k_{bx} x_a(t) + c_{bz} \Omega z_a(t) \\ F_2(t) = F_y(t) - m_a \ddot{y}_a(t) - c_{by} \dot{y}_a(t) \\ \quad - k_{by} y_a(t) \\ F_3(t) = F_z(t) - m_a \ddot{z}_a(t) - c_{bz} \dot{z}_a(t) \\ \quad - k_{bz} z_a(t) - c_{bx} \Omega x_a(t) - F_0 \end{cases} \quad (5)$$

$$\begin{cases} u_{11}(t) = c_{bx} \Omega_1 \sin \Omega_1 t - k_{bx} \cos \Omega_1 t, & u_{12}(t) = c_{bz} \Omega \cos \Omega_1 t \\ u_{21}(t) = -c_{bx} \Omega_1 \cos \Omega_1 t - k_{bx} \sin \Omega_1 t, & u_{22}(t) = c_{bz} \Omega \sin \Omega_1 t \\ \vdots & \vdots \\ u_{71}(t) = c_{bx} \Omega_4 \sin \Omega_4 t - k_{bx} \cos \Omega_4 t, & u_{72}(t) = c_{bz} \Omega \cos \Omega_4 t \\ u_{81}(t) = -c_{bx} \Omega_4 \cos \Omega_4 t - k_{bx} \sin \Omega_4 t, & u_{82}(t) = c_{bz} \Omega \sin \Omega_4 t \\ \\ v_1(t) = c_{by} \Omega_1 \sin \Omega_1 t - k_{by} \cos \Omega_1 t \\ v_2(t) = -c_{by} \Omega_1 \cos \Omega_1 t - k_{by} \sin \Omega_1 t \\ \vdots \\ v_7(t) = c_{by} \Omega_4 \sin \Omega_4 t - k_{by} \cos \Omega_4 t \\ v_8(t) = -c_{by} \Omega_4 \cos \Omega_4 t - k_{by} \sin \Omega_4 t \\ \\ w_{11}(t) = -c_{bx} \Omega \cos \Omega_1 t, & w_{12}(t) = c_{bz} \Omega_1 \sin \Omega_1 t - k_{bz} \cos \Omega_1 t \\ w_{21}(t) = -c_{bx} \Omega \sin \Omega_1 t, & w_{22}(t) = -c_{bz} \Omega_1 \cos \Omega_1 t - k_{bz} \sin \Omega_1 t \\ \vdots & \vdots \\ w_{71}(t) = -c_{bx} \Omega \cos \Omega_4 t, & w_{72}(t) = c_{bz} \Omega_4 \sin \Omega_4 t - k_{bz} \cos \Omega_4 t \\ w_{81}(t) = -c_{bx} \Omega \sin \Omega_4 t, & w_{82}(t) = -c_{bz} \Omega_4 \cos \Omega_4 t - k_{bz} \sin \Omega_4 t \end{cases} \quad (6)$$

The left terms of Eq. (4) are expressed as Eq. (5) and can be obtained from the measured data of the rolling test. The right terms in Eq. (4) are given by Eq. (6) and parameters $X_1, \dots, X_8, Y_1, \dots, Y_8, Z_1, \dots, Z_8, F_d$. Thus, the right terms can be calculated if the parameters are given because all terms in Eq. (6) are known. Thus, the squared residuals as the difference between the left term and the right term are defined

as shown in Eq. (7), and the parameters are identified using least squares method.

$$\begin{aligned}
\sum Q(t_i) &= \sum \|F - [X_1 \mathbf{u}_1 + \dots + X_8 \mathbf{u}_8 + Y_1 \mathbf{v}_1 \\
&\quad + \dots + Y_8 \mathbf{v}_8 + Z_1 \mathbf{w}_1 + \dots \\
&\quad + Z_8 \mathbf{w}_8 + F_d \mathbf{i}]\|^2
\end{aligned} \quad (7)$$

where

$$\begin{cases} \mathbf{F} = \begin{Bmatrix} F_1(t_i) \\ F_2(t_i) \\ F_3(t_i) \end{Bmatrix}, \\ \mathbf{u}_1 = \begin{Bmatrix} u_{11}(t_i) \\ 0 \\ u_{12}(t_i) \end{Bmatrix}, \dots, \mathbf{u}_8 = \begin{Bmatrix} u_{81}(t_i) \\ 0 \\ u_{82}(t_i) \end{Bmatrix}, \\ \mathbf{v}_1 = \begin{Bmatrix} 0 \\ v_1(t_i) \\ 0 \end{Bmatrix}, \dots, \mathbf{v}_8 = \begin{Bmatrix} 0 \\ v_8(t_i) \\ 0 \end{Bmatrix}, \\ \mathbf{w}_1 = \begin{Bmatrix} w_{11}(t_i) \\ 0 \\ w_{12}(t_i) \end{Bmatrix}, \dots, \mathbf{w}_8 = \begin{Bmatrix} w_{81}(t_i) \\ 0 \\ w_{82}(t_i) \end{Bmatrix}, \mathbf{i} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \end{cases} \quad (8)$$

The minimum of the squared residuals is found by setting the gradient to 0:

$$\begin{aligned}
\frac{\partial \sum Q}{\partial X_1} = 0, \dots, \frac{\partial \sum Q}{\partial X_8} = 0, \frac{\partial \sum Q}{\partial Y_1} = 0, \dots, \frac{\partial \sum Q}{\partial Y_8} = 0, \\
\frac{\partial \sum Q}{\partial Z_1} = 0, \dots, \frac{\partial \sum Q}{\partial Z_8} = 0, \frac{\partial \sum Q}{\partial F_d} = 0
\end{aligned} \quad (9)$$

and the gradient equations become

$$\begin{aligned}
\begin{bmatrix} S_1 \\ \vdots \\ S_8 \\ S_9 \\ \vdots \\ S_{16} \\ S_{17} \\ \vdots \\ S_{24} \\ S_{25} \end{bmatrix} &= \begin{bmatrix} S_{1,1} & \dots & S_{1,8} & 0 & \dots & 0 & S_{1,17} & \dots & S_{1,24} & S_{1,25} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ S_{8,1} & \dots & S_{8,8} & 0 & \dots & 0 & S_{8,17} & \dots & S_{8,24} & S_{8,25} \\ 0 & \dots & 0 & S_{9,9} & \dots & S_{9,16} & 0 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & S_{16,9} & \dots & S_{16,16} & 0 & \dots & 0 & 0 \\ S_{17,1} & \dots & S_{17,8} & 0 & \dots & 0 & S_{17,17} & \dots & S_{17,24} & S_{17,25} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ S_{24,1} & \dots & S_{24,8} & 0 & \dots & 0 & S_{24,17} & \dots & S_{24,24} & S_{24,25} \\ S_{25,1} & \dots & S_{25,8} & 0 & \dots & 0 & S_{25,17} & \dots & S_{25,24} & S_{25,25} \end{bmatrix} \begin{Bmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_8 \\ \mathbf{Y}_1 \\ \vdots \\ \mathbf{Y}_8 \\ \mathbf{Z}_1 \\ \vdots \\ \mathbf{Z}_8 \\ \mathbf{F}_d \end{Bmatrix}
\end{aligned} \quad (10)$$

where

$$\begin{cases} S_1 = \sum(F, \mathbf{u}_1), \dots, S_8 = \sum(F, \mathbf{u}_8), \\ S_9 = \sum(F, \mathbf{v}_1), \dots, S_{16} = \sum(F, \mathbf{v}_8), \\ S_{17} = \sum(F, \mathbf{w}_1), \dots, S_{24} = \sum(F, \mathbf{w}_8), S_{25} = \sum(F, \mathbf{i}) \end{cases} \quad (11)$$

$$\left\{ \begin{array}{l} S_{1,1} = \sum \|u_1\|^2, \dots, S_{8,8} = \sum \|u_8\|^2, \\ S_{9,9} = \sum \|v_1\|^2, \dots, S_{16,16} = \sum \|v_8\|^2, \\ S_{17,17} = \sum \|w_1\|^2, \dots, S_{24,24} = \sum \|w_8\|^2, \\ S_{25,25} = \sum \|i\|^2 \end{array} \right. \quad (12)$$

$$\left\{ \begin{array}{l} S_{1,2} = S_{2,1} = \sum (u_1, u_2), \dots, \\ S_{1,17} = S_{17,1} = \sum (u_1, w_1), \dots, \\ S_{1,25} = S_{25,1} = \sum (u_1, i) \\ S_{2,3} = S_{3,2} = \sum (u_2, u_3), \dots, \\ S_{2,17} = S_{17,2} = \sum (u_2, w_1), \dots, \\ S_{2,25} = S_{25,2} = \sum (u_2, i) \\ \vdots \\ S_{8,17} = S_{17,8} = \sum (u_8, w_1), \dots, \\ S_{8,25} = S_{25,8} = \sum (u_8, i) \\ \vdots \\ S_{8,10} = S_{10,8} = \sum (v_1, v_2), \dots, \\ S_{17,18} = S_{18,17} = \sum (w_1, w_2), \dots, \\ S_{17,25} = S_{25,17} = \sum (w_1, i) \\ \vdots \\ S_{24,25} = S_{25,24} = \sum (w_8, i) \end{array} \right. \quad (13)$$

All terms data in Eq. (11)–(13) can be obtained from the rolling test. Therefore, the parameters can be identified from Eq. (10).

3. Excitation test equipment and procedure

Fig. 3 shows the experimental setup of the excitation test. Similar to the rolling test, the tire is pressed against the drum with a preload of 200 N, and an impact hammer (5850B, Dytran Instruments Inc., USA) was used to excite the impact force.

The excitation point was fixed, and the impulsive force was applied to the center of lug. The acceleration responses of the lug center in radial, circumferential, and lateral directions were measured at 18 points corresponding to the lug positions excluding the 2 contact points. The excitation was repeated 8 times at each point, and the resulting data was averaged to improve accuracy. The impact force and acceleration response

Impact hammer is used to excite the impact force and the induced acceleration responses are measured at 18 points corresponding to lug positions excluding 2 contact points.

Fig. 3 Schematic of excitation test

signals were amplified and applied to a signal analyzer (SA-02A4, RION Co. Ltd., Japan). The frequency response function (i.e., accelerance) was obtained using software (ME'scope VES, SYSTEM PLUS Inc., Japan). The natural frequency and natural mode shape were identified from the accelerance.

4. Transfer characteristic analysis procedure

For the transfer characteristic, the amplitude magnification and phase difference between the shaft and lug excitation forces were investigated. For the analysis, the transfer function was first obtained from the time history between the shaft and lug excitation forces when resonance occurred. Then, the amplitude magnification and phase difference at the resonance frequency were identified from the obtained transfer function.

III Results and Discussion

1. Experimental results of shaft displacement and shaft force

Fig. 4 shows the time history of shaft displacement and shaft force at tire revolution of 150 rpm. At this tire revolution, the displacement in longitudinal direction is the largest. The rotation speed of the tire was constant, and no fluctuations were observed. To investigate the shaft force characteristics, the mean value and fluctuation amplitude were identified from the time history of the measured shaft force. Fig. 5 shows the shaft force characteristics by the mean value and fluctuation amplitude corresponding to tire revolution. The mean value is indicated by the dots and the fluctuation amplitude by the error bars.

The mean value of the force in longitudinal and lateral directions is almost 0. In the vertical direction, however, the dynamic force due to the rolling motion is superimposed to the preload 200 N. The dynamic force is considered to be due to the impact load when the tire collides with the drum in the vertical direction during rotation. It can also be seen that the mean value of the vertical force increases linearly as the tire

(a) Shaft displacements: x_a : longitudinal, y_a : lateral, z_a : vertical (b) Shaft forces: F_x : longitudinal, F_y : lateral, F_z : vertical

Fig. 4 Time history of shaft displacements and shaft forces at tire revolution 150 rpm

speed increases. Regarding the fluctuation amplitude, resonance phenomena are observed with some specific rotation speeds in each direction, and it was confirmed that the rotation speed when resonance occur differs between the cases.

2. Identification results of lug excitation force

Fig. 6 shows the time history of the shaft force at tire revolution of 150 rpm. $F_{x \text{ exp}}$, $F_{y \text{ exp}}$, $F_{z \text{ exp}}$ (solid curve) are measured shaft forces while $F_{x \text{ cal}}$, $F_{y \text{ cal}}$, $F_{z \text{ cal}}$ (dashed curve) are calculated ones from Eq. (1). Fig. 6 shows that the measured shaft forces are consistent with the calculated ones.

A little deviation was observed for the vertical direction. The fluctuation of the shaft displacement in vertical direction is not smooth, and the fluctuation of the shaft force is small in the vertical direction at 150 rpm as seen in Fig. 4. These fluctuations might have caused the deviation in the vertical direction. However, Fig. 5 shows that the fluctuation in the vertical shaft force at 150 rpm is relatively small; thus, this deviation is considered acceptable. Therefore, this result confirms that the parameters can be correctly identified, and it proves the effectiveness of this identification method.

F_x : longitudinal, F_y : lateral, F_z : vertical. Mean value is indicated by dot and fluctuation amplitude by error bar. Resonance phenomena are observed with some specific rotation speeds for all forces.

Fig. 5 Shaft force characteristic

Further, the lug excitation forces F_{lugx} , F_{lugy} and F_{lugz} were identified. Similar to shaft force, the lug excitation force can be calculated from Eq. (2).

The lug excitation force characteristics, the mean value, and the fluctuation amplitude were identified for each condition from the time history of the calculated lug excitation force. Fig. 7 shows the lug excitation force characteristics. The resonance phenomena are also confirmed for lug excitation force characteristics. Compared to the shaft force characteristics in Fig. 4, the tendency of the fluctuation amplitude is different, but the number of revolutions at which the resonance phenomenon occurs is relatively well matched.

3. Experimental results of tire vibration characteristics

Fig. 8 shows the agricultural tire vibration characteristic. The tire vibrational modes have been studied through modal analysis with respect to the SWIFT model by Pacejka (2006). The calculated vibrational modes are indicated as natural mode (theory) in Fig. 8. The calculated vibrational modes have 6 degrees of freedom: 4 in-plane and 2 out-of-plane degrees of freedom. To identify the natural modes obtained from the excitation test, the experimental mode shapes were compared with the theoretical one. Consequently, 2 rotational modes are referred to as in-phase rotation of the rim and tire belt (at 48 Hz) and anti-phase rotation (at 68 Hz) of these bodies. Further, 2 translational modes are referred to vertical

$F_{x\text{ exp}}$, $F_{y\text{ exp}}$ and $F_{z\text{ exp}}$ (solid curve) are measured shaft forces. $F_{x\text{ cal}}$, $F_{y\text{ cal}}$ and $F_{z\text{ cal}}$ (dashed curve) are calculated shaft forces.

Fig. 6 Time history of shaft force at tire revolution 150 rpm

F_{lugx} : longitudinal, F_{lugy} : lateral, F_{lugz} : vertical. Mean value is indicated by dot and fluctuation amplitude by error bar. Resonant phenomena are also found and it is confirmed that the number of revolutions at which resonance occurs corresponds to shaft force characteristics.

Fig. 7 Lug excitation force characteristics

mode (at 74 Hz) and longitudinal mode (at 90 Hz). For 2 out-of-plane modes, we can identify camber mode (at 35 Hz) and lateral mode (at 85 Hz).

4. Comparison of characteristic of shaft force and lug excitation force

Fig. 9 shows the spectrum map of shaft force (left side) and lug excitation force (right side). In the fluctuations of the shaft and lug excitation forces, the lug excitation frequency

becomes dominant. The 1st-order lug excitation frequency f_e (Hz) is calculated by the equation:

$$f_e = \frac{NL}{60} \quad (14)$$

where L is the number of lugs and N (rpm) is the number of tire revolutions. In the spectrum map, the lug excitation frequencies of the 1st, 2nd, and 0.5th are indicated by dotted lines, and the tire revolution when resonance occurs in rolling test is indicated by bold line. The dominant frequency of the bold line is identified from the spectrum map. The dominant frequency was found to coincide with the lug excitation frequency in all cases. We defined the dominant frequency as resonance frequency. The resonance frequency of the shaft and lug excitation forces coincides with each other. For the longitudinal direction, 3 resonance frequencies (40, 53, and 69 Hz) were confirmed. For the lateral direction, 2 frequencies (31, 73 Hz) were confirmed, and as for the vertical direction, 1 frequency (65 Hz) was confirmed.

Table 3 shows the comparison between the resonance frequencies obtained from the spectrum map and the natural frequencies specified by the excitation test. The natural frequency of the tire was found to affect the resonance frequency of the shaft and lug excitation forces. That is, the force direction of the resonance matches the vibration direction of the corresponding natural mode in all cases. Furthermore, it can be seen that the resonance frequency is smaller than the natural frequency in both cases. It has been reported that the natural frequency of the tire is significantly reduced in the rolling state compared to the static state (Saguchi, 1998), which is considered an explanation of the difference.

Natural frequency	35 Hz	48 Hz	68 Hz	74 Hz	85 Hz	90 Hz
Natural mode (experimental)						
Natural mode (theory)						
	camber	rotational (in-phase)	rotational (anti-phase)	vertical	lateral	longitudinal

Natural frequencies and natural mode shapes are identified from the acceleration, and the calculated vibrational modes are indicated as natural mode (theory).

Fig. 8 Agricultural tire vibration characteristic

Lug excitation frequency of 1st, 2nd, and 0.5th orders (lateral direction only) are indicated by dotted lines. Dominant frequency when resonance occurs in rolling test (indicated by bold line) is identified from spectrum map.

Fig. 9 Spectrum map of shaft force (left side) and lug excitation force (right side)

Table 3 Comparison of resonance frequencies and natural frequencies (natural modes)

Natural frequency (Hz)	Natural mode (vibration direction)	Resonance frequency (Hz)	Force direction
35	Camber (lateral)	31	lateral
48	Rotational (longitudinal)	40	longitudinal
68	Rotational (longitudinal)	53	longitudinal
74	Vertical (vertical)	65	vertical
85	Lateral (lateral)	73	lateral
90	Longitudinal (longitudinal)	69	longitudinal

As already mentioned, the resonance frequency of the shaft and lug excitation forces is consistent. On the other hand, for the spectrum strength, it can be seen that the characteristics of the shaft force is different from that of the lug excitation force.

For example, for longitudinal direction, the 3 levels are different from one another for the lug excitation, whereas those levels are almost same for the shaft. The lug excitation force is transmitted to shaft force through the tire's sidewall.

Numerical values in amplitude are resonance frequencies and the corresponding natural modes are also shown. Plots distinguish the order of lug excitation frequency.

Fig. 10 Transfer characteristics between lug excitation force and shaft force

Therefore, the transmission of the lug excitation force to the shaft force is not uniform. In this sense, there is a possibility that the transfer characteristics with the input of the lug excitation force and the output with the shaft force affect the difference between the lug excitation force and the shaft force characteristics. Additionally, transfer characteristics may be influenced by tire vibration characteristics.

5. Transfer characteristics analysis results

The amplitude magnification and the phase difference were

also investigated from the transfer function for all tire revolutions when resonance was confirmed. The transfer characteristics for each direction are shown in Fig. 10. The numerical values in Fig. 10 of amplitude magnification are resonance frequencies, and corresponding natural modes are also shown. As the amplitude magnification is obtained as the ratio of shaft force to lug excitation force, when the amplitude magnification is greater than 1, the lug excitation force is increased and transmitted to shaft force. However, when it is

less than 1, lug excitation force is decreased and transmitted. The plots distinguish the order of the lug excitation frequency.

In the longitudinal direction, all resonances occur at the 2nd lug excitation frequency. For the amplitude magnification, it is greater than 1 in the rotational (in-phase) and longitudinal modes. As these modes correspond to the mode swinging with respect to the contact point, it is considered that the lug excitation force is increased and transmitted to shaft force. On the other hand, it is less than 1 in the rotational (anti-phase) mode. This mode corresponds to that in which the tire and the rim rotate in reverse phases. Therefore, it is considered that the lug excitation force is decreased. Regarding phase difference, all modes were in-phase.

In the lateral direction, resonances occurred at 0.5th, 1st, and 2nd lug excitation frequencies. The amplitude magnification is greater than 1 at all orders of lug excitation frequency in camber mode. On the other hand, the amplitude magnification was found to be greater than 1 at the 1st order but almost 0 at the 2nd order in lateral mode. As the camber and lateral modes also correspond to the mode of swinging to the contact point, the lug excitation force is considered expanded. For lug excitation frequency, the 1st order was defined by lug number. Hence, the 2nd order corresponds to the input by 2 stitch lugs, and the direction of force generation becomes alternate in the lateral direction. This may lead to force cancellation, and it may explain why the amplitude magnification becomes almost 0 at the 2nd order in lateral mode. Regarding phase difference, all modes were anti-phase.

In the vertical direction, resonances occurred at 1st and 2nd lug excitation frequencies. The mode was only the vertical mode and the magnification of the amplitude was smaller than 1 both at the 1st and 2nd orders. It is considered that the lug excitation force decreased as the motion in the vertical direction is constrained in the vertical mode. Phase difference was anti-phase.

The transfer characteristics of lug excitation and shaft forces were investigated, and it was found that lug excitation force tends to increase in the longitudinal and lateral directions, whereas it decreases in the vertical direction. The lug excitation force increases or decreases in correlation to the shape of the corresponding vibration mode. Therefore, it is considered that the vibration characteristics of the tire influence the transfer characteristics of the lug excitation force and the shaft force. However, these results are not absolute and may vary depending on the tire type and other factors.

IV Conclusion

In this study, we proposed a new technique to identify lug excitation force. By examining the characteristics of the lug

excitation force and the shaft force, it was confirmed that the resonance frequencies of these forces match well, and that the tire vibration characteristics affect their resonance. Further, by investigating the correlation between the transfer characteristics and the vibration mode shape, it was ascertained that the increase or decrease of the lug excitation force correlates with the shape of the corresponding vibration mode. The results of this study show that the reduction of lug excitation force by optimizing the lug shape and tire properties and matching the lug excitation force with the transfer characteristics are valid for reducing tractor vibration. The presented new technique is a powerful tool for understanding the lug excitation force characteristics and is very useful for reducing the vibration of future high-speed tractors.

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Conflict of interest

The authors declare no conflicts of interest.

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