Abstract: This study proposes a methodology for estimating places where people may perform additional activities with utilizing the concept of space-time prism. Decision about travel behaviour can be affected by time constraints of their life and spatial structure where they live. Considering such constraints is important to estimate which place is likely to be chosen for the additional activities. This study develops a method to estimating the place which people may choose as an additional destination. This method proposes the concept of path constrained potential path area (P-PPA) by incorporating the concept of space-time prism with information of revealed trip trajectories which can be taken by position detection tools such as GPS and so on. An efficient calculation algorithm with considering actual network structure is proposed. An empirical test has been carried out in the real network to check the proposed methodology.

Key Words: Space-time prism, GIS, Probe person surveys

1. INTRODUCTION

Decision about travel behaviour can be affected by time constraints of their life and spatial structure where they live. One of the most important constraints is time for sleep, that is, people must stay at their own home from evening to morning. Many people also commute to their office where they must arrive at certain time in the morning and stays till evening. Such temporal constraints restrict the amount of time which people can spend in moving between different places. Normally people reserve sufficient time to commute between their home and office. However, if they intend to visit another place to achieve additional activities, sometimes they will face the situation where they cannot visit this place due to lack of time for travelling there, meaning that they cannot choose to achieve activities at this place. This restriction on choosing additional activities depends on each person's schedule constraint and spatial structure of transport networks and allocations of the destinations. Such temporal and spatial structure is important factors to consider traveller's behaviour.

To describe such temporal and spatial constraints, Hägerstrand (1970) proposed the concept
of the "space-time prism", which draws these restrictions in a three-dimensional space. The space-time prism depicts the spatial and time area where travellers can visit under their schedule constrains. Assuming that travel time between two places is proportional to the Euclid distance, space-time prism can be described as a combination of two cones whose vertexes correspond to the time and place where the person must stay (e.g. the place of office and work start time). Projecting this time-space prism onto the spatial plane, one can obtain all the possible places and areas where people can achieve the additional activities (Burns 1979). The space-time prism described by cones does not consider the effect of detailed network structure, however, this simple concept is good for understanding the temporal and spatial restrictions which people have. On the other hand, the recent achievement of the Geographical Information Systems (GIS) allows us to calculate the space-time prism with considering detailed structure of transport networks (For example, Weber and Kwan 2003, Kwan and Hong 1998). Such technique will provide us more realistic information on the places where people can go.

Though the concept of space-time prism enumerates all the places where people can go, it can be estimated that people may recognize only the limited places among them due to limited ability of cognition. Kwan and Hong (1998) proposed the cognitive feasible opportunity set (COFS) based on both space-time constraints and cognitive area of each individual. COFS represents the area which individuals can distinguish and also can go to. COFS is more constrained than space-time constraints and therefore it can be expected that COFS represents more proper area as a destination choice set for each individual.

The cognitive area of each individual may be able to be estimated from his/her revealed behaviour. The additional places which people can recognize without performing any extra survey may be mainly located along the routes where they use daily, such as commuting routes to office, school, and so on. Also, people may need less effort to choose the additional places near to the daily routes because only the small change of travel behaviour is needed to visit there. Therefore, it can be assumed that people tend to choose the places near to the daily routes as the places where they perform the additional activities. Recent improvements of position detection instruments such as GPS allow us to observe detailed trajectories of individuals (see Asakura et al 2000, Asakura and Hato 2004 for example), which can be utilized to determine people's daily routes.

This study proposes a concept and methodology to derive a space-time prism with considering revealed daily routes which are obtained by position detection tools. The concept is for determining choice sets of additional destinations which each traveller adds to the original trip. First, the concept of path-constrained potential path area (P-PPA), which describes the space-time prism with considering the effect of daily routes, is proposed in chapter 2. In chapter 3, the concept is mathematically determined with considering network structure. Then, an algorithm for calculating P-PPA on GIS is shown in chapter 4. Empirical examples and conclusion of the method are shown in chapter 5 and 6.

2. CONCEPT OF PATH-CONSTRAINED POTENTIAL PATH AREA

2.1 Past Studies of Prism Constraint
Travel behaviour can be described as a path in the three dimensional space which has three axes, that is, two special axes and one temporal axis. Figure 1 shows an example of a three dimensional path observed by GPS. Travel speed of this trajectory is represented as a
gradient of the trajectory shown in this space. Due to the limitation of moving speed of transport modes, the gradient of the trajectory is upper bounded, meaning that the places where people can go in future is restricted by the place where people stay now.

![Figure 1](image)

Figure 1 An example of three dimensional path obtained by GPS

The constraints on travel trajectories can be expressed as the space-time prism, which is also referred as to the "potential path space (PPS)". Figure 2 represents the space-time prism which has two time constraints at the same location. The gradient of this prism indicates maximum speed of travel. The time constraints can be considered as the individual's schedules. Figure 2 (b) describes the area where the individual can go within the schedule, which is referred as to the "potential pass area (PPA)". This area is derived from projecting the prism onto the spatial plane and can be described by a circle with $v(t_j-t_i)/2$ in radius.

![Figure 2](image)

Figure 2 A space-time prism with two time constraints at the same place.
People can have their schedule constraints at different places, such as both home and office, for example. Figure 3 describes the space-time prism with time constraints at the different place. Figure 3 (b) shows the PPA of this prism. The shape of the PPA is ellipse whose foci indicate the two places where the traveller has time constraints.

No information on network structure is considered in the space-time prisms and PPAs in figures 2 and 3. Of course, the shape of PPA differs from the circle or ellipse and should be complicated if the detailed network structure is considered (Burns 1979, Kwan and Hong 1998). Also, the shape of PPA is affected by travel speed on the link within the network. The PPA will be expanded if travel speed increases due to improvements of performance of transport systems.

2.2 Path Constraint Space-time Prism
As discussed in the section 1, it can be estimated that people tend to choose the places which is near to the daily routes as the places where additional activities are made. Assume a person who intends to have a dinner at a restaurant on the way to home, for example. Theoretically, he/she can choose any restaurants within the PPA which can be determined with considering time constraints at the office, home and duration of the stay at the restaurant. However, people may choose the restaurants where they can go without any big effort such as extra investigation of all restaurants situated in the PPA or finding new unfamiliar transport modes or routes. This implies that people tend to choose the restaurant near to the route which people uses in everyday life and not to choose one far from the daily route.

Such tendency of people's behaviour will reduce the possible places where people visit from the original PPA. Iryo et al. (2005) proposed a method to extract the area where additional activities would be made with considering the restriction caused by daily routes. They defined a concept of the "living area" which was derived from the routes observed by GPS in a probe person survey. The living area is defined as the area where people can make a side
trip from the trajectory within certain travel time. Note that the travellers conserve their original trajectory. They go to the additional destination from a point on the trajectory and come back to the same point along the same way. This means that travellers will go to the additional destination with the smallest change of the trajectory. The living area can be drawn as a set of circles whose centres are situated at every trajectory points. The diameter of each circle shows the maximum travel time which people can spend in stopping over the place.

Though the concept of the living area may useful to limit the possibilities of additional destinations, it cuts off too many possible destinations where people intend to go. It is because this method assumes that travellers conserve whole trajectory perfectly that such strong restriction arises. To ease this restriction, this study proposes a concept of the path-constrained possible path area (P-PPA), which is defined as a subset of PPA. The P-PPA is defined as the area where a traveller can go along the route, which is referred as to a possible path, satisfying the conditions such as

1. The possible path must start from the origin so as to satisfy the schedule constraint at the origin.
2. The possible path must arrive at the destination so as to satisfy the schedule constraint at the destination.
3. The possible path must have a certain overlap ratio to the original path, where the original path is the route which the traveller originally chooses before adding an activity.

The first and second conditions are the same as the conditions for determining PPA and therefore P-PPA will be a subset of PPA. The third condition means that a traveller can go to

![Figure 4](image-url)  
Figure 4  An example of space-time prism and P-PPA with 100% overlap ratio
all the places in P-PPA with keeping the overlap ratio greater than a certain threshold, which is referred as to the "least overlap ratio". The concept of “overlap rate” is introduced to describe how the path is changed from the original one. It is defined as the ratio of the number of the road links contained in both the original path and the possible path to the number of the road links contained in the entire original path. Figure 4 shows an example of P-PPA where the overlap ratio is 100 %. In this case, all the possible paths must include the original path and the P-PPA is expressed by the belt with the certain width (figure 4 (b)). Note that the P-PPA is identical to the living area in this case.

If the least overlap ratio is smaller, the size of P-PPA will be greater because the restriction is loosened. Figure 5 represents “P-PPA at overlap rate 100 %”, “P-PPA at overlap rate 50 %”, and “P-PPA at overlap rate 0 %”. The P-PPA is identical to PPA if the overlap rate is 0 % and the shape of P-PPA is an ellipse. If the overlap rate is 100 %, P-PPA is the same as the living area and therefore it can be described as the set of circles along the original path. If the ratio is 50 %, the border of P-PPA is situated between them.

3. FORMULATION OF POTENTIAL PATH AREA WITH DETAILED ROAD NETWORK STRUCTURE

In the last chapter, the concept of PPA and P-PPA is introduced with the assumption that travel time between two points is proportional to the Euclid distance between them. Travel time on the real networks, however, will be depend on the structure of network and therefore both path areas should be largely depend on the structure of the network. This chapter proposes formulations for solving the exact shape of the PPA and P-PPA with considering road network structure explicitly. Note that only the road transport is considered here.

3.1 Formulation of the Potential Path Area with Road Network Structure

The space-time prism and PPA is defined with considering the road network structure. Assume a road network whose node set is denoted as \( V \) and link set is denoted by \( E \). Travel speed of each link is given as priori and will not change over time. The space-time prism is defined as a set of nodes where travellers can visit at time \( t \) and formulated as
\[ V_{STP}(i, j, t_i, t_j, t) = \left\{ k \mid t_i + \sum_{m \in E_k} \frac{d_m}{v_m} \leq t \leq t_j - \sum_{m \in E_{jk}} \frac{d_m}{v_m} \right\} \]  \hspace{1cm} (1)

where

- \( i \): The node where a traveller starts his/her original trip
- \( j \): The node where a traveller ends his/her original trip
- \( t_i \): Departure time from node \( i \), which represents a schedule constraint at the origin
- \( t_j \): Arriving time at node \( j \), which represents a schedule constraint at the destination
- \( E_{ik} \): Set of the link contained in the shortest path from the node \( i \) to node \( k \)
- \( E_{jk} \): Set of the link contained in the shortest path to location \( j \) from location \( k \)
- \( d_m \): Distance of link \( m \)
- \( v_m \): Travel speed of link \( m \)

The PPA is defined as a projection of \( V_{STP}(t) \) onto the spatial plane and can be described as

\[ V_{PPA}(i, j, t_i, t_j) = \{ k \mid k \in V_{STP}(i, j, t_i, t_j) \text{ for any } t \} . \]  \hspace{1cm} (2)

### 3.2 Formulation of the Path Constrained Potential Path Area with Road Network Structure

This section defines the P-PPA with considering the network structure. In the formulation of P-PPA, the evaluation of overlap ratio is necessary and therefore route enumeration is inevitable. Denote a set of all non-cyclic routes connecting the node \( i \) to node \( j \) as \( R(i, j) \). If all the routes included in \( R(i, j) \) is enumerated, the P-PPA can be calculated as the set \( V_{p-PPA}(i, j, t_i, t_j, \alpha) \), which is defined as

\[ V_{p-PPA}(i, j, t_i, t_j, r_0, \alpha) = \left\{ k \mid R_0(i, j, k, r_0, \alpha) \cap R_t(i, j, t_j - t_i) \neq \emptyset \right\} \]  \hspace{1cm} (3)

where

\[ R_0(i, j, k, r_0, \alpha) = \left\{ r \in R(i, j) \mid k \in V(r), \frac{|E(r_0) \cap E(r)|}{|E(r_0)|} \geq \alpha \right\} \]  \hspace{1cm} (4)

and

- \( V(r) \): Set of the nodes included by the route \( r \)
- \( \alpha \): Least overlap rate
- \( r_0 \): The original path
- \( E(r) \): Links included by the route \( r \)
- \( R_t(i, j, \Delta t) \): Set of routes connecting node \( i \) and \( j \) whose travel time is not greater than \( \Delta t \)

Note that it is necessary to determine the least overlap rate if P-PPA is used in some practical applications. If choice set which is made from P-PPA is used for a destination choice model...

in a travel behaviour analysis, the least overlap rate should be determined by reproducibility of the model.

Due to the route enumeration, \( V_{P-PPA} \left( i, j, t_i, t_j, r_0, \alpha \right) \) cannot be easily calculated and some approximation calculation algorithm is necessary, which is introduced in the next section.

4. COMPUTATION METHODOLOGY AND PROCEDURES

This section proposes an approximation method for calculating the P-PPA defined in the last session. First, an approximation to the original set \( V_{P-PPA} \left( i, j, t_i, t_j, r_0, \alpha \right) \) with no route enumeration is formulated as

\[
V_{P-PPA} \left( i, j, t_i, t_j, r_0, \alpha \right) = \left\{ k \in V_{PPA} \left( m, n, t_{p_1} \left( t_i, r_0, m \right), t_{p_2} \left( t_j, r_0, n \right) \right) \mid (m, n) \in V \left( i, j, r_0, \alpha \right) \right\}
\]

(5)

where

\[
V \left( i, j, r_0, \alpha \right) = \left\{ (m, n) \in V \left( r_0 \right) \mid \frac{|E(r, \left( i, m, r_0 \right))| + |E(r, \left( n, j, r_0 \right))|}{E(r_0)} \geq \alpha \right\}
\]

(6)

and

- \( t_{p_1} \left( t_i, r_0, m \right) \) : Time when a traveller departing from the origin at time \( t_i \) and travelling along the route \( r_0 \) passes through the node \( m \)
- \( t_{p_2} \left( t_j, r_0, n \right) \) : Time when a traveller travelling along the route \( r_0 \) and arriving at the destination at time \( t_j \) passes through the node \( n \)
- \( r_p \left( k, l, r_0 \right) \) : Route connecting from the node \( k \) to node \( l \) and completely overlapping the route \( r_0 \)

This approximation tries to calculate the nodes within P-PPA by setting virtual start node and end node on the original set (see figure 6). Any possible paths can be detached only from the original path between the node \( m \) and \( n \) defined by the equation (6), meaning that the overlaps between the start node \( i \) and the node \( m \) and between the node \( n \) and the end node \( j \) are guaranteed. This set will be a subset of the original set \( V_{P-PPA} \left( i, j, t_i, t_j, r_0, \alpha \right) \).
The set of \( V_{p-PPA}^* (i, j, t_i, t_j, r_0, \alpha) \) can be calculated without any route enumeration. A calculation step is shown below:

**STEP 1.1**
Calculate travel time from all nodes included by the original path to all nodes in the network with Dijkstra's algorithm.

**STEP 1.2**
Calculate travel time from all nodes in the network to all nodes included by the original path with Dijkstra's algorithm.

**STEP 2.1**
Define \( \alpha = p \), \( p_{OLD} = -1 \) and \( L(k, -1) = 0 \) for any \( k \).

**STEP 2.2**
Set \( (m, n) \in V_n (i, j, r_0, p) \). Calculate an index defined as follow for all nodes \( k \)

\[
L(k, p) = \begin{cases} 1 & \text{if } t_{p1}(t_i, r_0, m) + T(m, k) + T(k, n) + t_{p2}(t_i, r_0, n) \leq t_j - t_i \text{ or } L(k, p_{OLD}) = 1 \\ 0 & \text{otherwise} \end{cases}
\]

Where \( T(i, j) \) is shortest travel time between the node \( i \) and \( j \).

This calculation must be made for any pair of \( (m, n) \in V_n (i, j, r_0, p) \).

**STEP 3**
If \( p \) is equal to 1 then proceed to **STEP 4**
Otherwise, let \( p_{OLD} = p \), \( p = \left( p \left| E \left( r_0 \right) \right| + 1 \right) / \left| E \left( r_0 \right) \right| \) and return to **STEP 2.2**

**STEP 4**
Search the node \( k \) where \( L(k) \) is equal to 1. This result of the searching is expressed as \( V_s = \{ k | L(k) = 1, k \in V_s \} \). This \( V_s \) is the same as \( V_{p-PPA}^* (i, j, t_i, t_j, r_0, \alpha) \).
Note that the calculation results of shortest travel time can be utilized for calculating PPAs with different least overlap ratios.

5. AN EMPIRICAL EXAMPLE

5.1 Methods
An empirical example with the trajectories taken by GPS equipments is shown in this section. This study uses the data set of probe person survey which was collected by Hanshin Express Corporation in 2004. Trajectories of the vehicles whose drivers cooperate to the survey have been collected. Each driver agreed to carry the mobile phone equipped by GPS unit which records the position data at every 5 minutes in average. Trajectories have been fitted onto the road networks with the method proposed by Asakura et al. (2000). This study employs three trajectories for exhibiting the proposed method, which is shown in figures 7 to 9. The trajectories in the figures 7 and 8 have the same origin and destination, whereas the trajectory in the figure 9 has a different OD pair.

A digital network data is used to calculate shortest paths in the network. Travel speed is assumed as 50 km/h for expressways, 30 km/h for arterial roads whose width is not less than 5.5 m, and 15 km/h for other roads. PostGIS is used for managing the network data.

Several P-PPAs are calculated with the assumption that \( t_j - t_i \) is 10 minutes longer than travel time of the original paths. Three least overlap ratio, 100%, 50% and 0% are employed.

5.2 Results
The results are shown in figures 10 to 12. It can be confirmed that the 0% P-PPAs in figure 10 is larger than that in figure 11 though their OD pairs are same. This is because they have different travel time on their original paths. The travel time in the trajectory of figure 7 is longer than that in figure 8. Both the 50% and 100 % P-PPAs in those figures also show the different shapes. This result exhibits that the travellers may choose the different place as the additional destination if they use different routes. The 0% P-PPA in figure 11 also has bigger area than in figure 10, meaning that the shape of P-PPA depends on the detailed network structure around the original path.

5.3 Discussion
The results show that the shapes of P-PPAs are greatly influenced by both route and travel time of original paths, and network structure around the original paths. Especially, P-PPAs may have a tendency to become bigger where the expressways are denser. It means that the shapes of P-PPA are strongly influenced by the speed in the network, and it will be changed significantly if the information of travel speed in the entire real network links can be adopted.

The calculation time for extracting 0%, 50% and 100% P-PPAs is about 4 minutes per trajectory by the computer equipped with 1800 MHz Opteron 265×2. Note that much of calculation time is for calculating the shortest paths, meaning that P-PPAs in different least overlap ratios can be obtained much more quickly once P-PPA in certain least overlap ratio has been calculated.
Figure 7  A trajectory of vehicle 1

Figure 8  A trajectory of vehicle 2

Figure 9  A trajectory of vehicle 3

Figure 10  P-PPAs of vehicle 1
6. CONCLUSIONS AND DISCUSSIONS

This study proposes a concept of the path-constrained potential path area (P-PPA) to extract the places where people will visit for achieving additional activities. This concept is an extension of the potential path area (PPA) which is defined as the projection of the space-time prism proposed by Hägerstrand (1970), whereas a concept of the overlap ratio to the original path is introduced to extract the areas where people likely to visit. This study also provides a methodology to extract the nodes belonging to the P-PPA with considering detailed structure of road networks. The method is implemented on a computer and a few example calculations have been achieved with the real trajectory data obtained by GPS in the real network.

This method can determine the area where travellers likely to visit more efficiently. The PPA without path constrains tend to extract places where possibility of traveller's visiting seems to be smaller. On the other hand, the proposed method can limit the area with considering the current behaviour exhibited by travellers, which can be measured by position detection tools such as GPS and so on. Also, the area extracted by this method may be more realistic than the PPA because this method utilizes information of revealed behaviour of travellers. It would appear that P-PPA can be applied to determine the destination choice sets which are used for the choice models in behaviour analysis.

Network structure is explicitly taken into account in the proposed algorithm. Considering network structure should be necessary to evaluate how the performance of the network will affect the potential path area and therefore can be useful to evaluate the effect of network improvements. This study proposes an efficient algorithm to deal with the real network whose calculation time is feasible.

This study assumes that travel time of roads is constant. Of course, people may choose the additional destinations so as to avoid heavy traffic congestion. Information on road congestion should be taken into account in future to obtain more realistic result. Also, incorporating public transport networks should also be considered in future analysis.
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