INCORPORATING LAND USE, TRANSPORT AND ENVIRONMENTAL CONSIDERATIONS INTO TIME-DEPENDENT TOLLING STRATEGIES

Xiaoqing LI  
Research Student  
Centre for Transport Research and Innovation for People (TRIP)  
Department of Civil, Structural and Environmental Engineering  
Trinity College Dublin  
Dublin 2, Ireland  
Fax: +353-1-677-3072  
E-mail: lixq@tcd.ie

Wai Yuen SZETO  
Assistant Professor  
Department of Civil Engineering  
National University of Singapore  
1 Engineering Drive 2, E1A 07-03  
Singapore 117576  
Fax: +65-67791635  
E-mail: cveswy@nus.edu.sg

Margaret O'MAHONY  
Professor  
Centre for Transport Research and Innovation for People (TRIP)  
Department of Civil, Structural and Environmental Engineering  
Trinity College Dublin  
Dublin 2, Ireland  
Fax: +353-1-677-3072  
E-mail: margaret.omahony@tcd.ie

Abstract: This paper develops a single-level optimization model to determine time-dependent optimal tolls while considering the dynamic relationships between land use, transport, and environment. To illustrate the importance of incorporating land use, transport, and environment considerations in determining time-dependent tolls, and the effect of tightening vehicular emission standards on link tolls, numerical studies are set up. The results show that the tighter the vehicular emission standards, the higher the toll charges are required, and that the vehicular emission standards have direct impacts on the overall vehicular emissions, the operational strategies and profit of public transit, the mode and route choices of travelers, the residential and employment distributions, the profits of land owners, and rents. The government should consider these impacts when determining the vehicular emission standard of each road.

Keywords: Road pricing, vehicular emissions, land use, transport and environment, time-dependent toll design problem

1. INTRODUCTION

The concept of road pricing needs no introduction. It is supported by a long history of economic and transportation analyses (for example, see Hau, 1992), pointing to its potential benefits of travel time minimization, economic benefit maximization, market efficiency related to externality pricing, cost recovery, effective management of transportation demand, etc. Some of these perspectives are summarized in Lo and Hickman (1997).
Previous road pricing studies can be broadly classified into three types: Empirical, simulation, and theoretical studies. Empirical studies (e.g., Lee, 2002) focus on the lessons learnt from toll policy implementations and its impacts to society, economy and environment. Simulation studies (Miyamoto et al. 1996) can simulate the probable results of a certain road pricing strategy while covering a lot of aspects like land use aspects, emissions, and a very detailed land-use transport interaction, but do not prescribe what the strategy ought to be. Theoretical studies focus on the economic theory of the first best pricing problem (e.g. Vickrey, 1969), the cordon-based network congestion pricing (e.g. Zhang and Yang, 2004; Ho et al. 2005), the road pricing under the dynamic traffic assignment framework (e.g. Yang and Huang; 1997; Lo and Szeto, 2005), developing efficient solution algorithms for toll design problems (Meng et al. 2001; Sumalee, 2005), and the impact of road pricing on the transport network reliability (e.g., Chan and Lam, 2005). To our best knowledge, little attention was focused on incorporating environmental and land use considerations into analytical road pricing models. In fact, toll charging can reduce the traffic level and hence emissions. In the long run, tolls will alter the travelers’ residential locations. Until recently, May et al. (2005) and Vold (2005) have carried out some studies on defining optimal land use transport strategies while considering its impacts on vehicular emissions and their environmental costs. However, they did not examine the impact of controlling maximum allowable link emissions on toll charges. Even if the overall vehicular emissions are under control, emissions on some links may exceed their acceptable levels, which are harmful to human health. In addition, the population is changing over time. It is important to consider the time dimension in analytical toll design models in additional to land use and the environment.

This paper develops a single-level minimization model to determine time-dependent optimal tolls while considering the dynamic relationships between land use, transport, and environment as well as the maximum allowable link emissions. This model is in fact bilevel in nature. The upper level formulates the decision maker’s problem whose aims at selecting time-dependent tolls to minimize the total travel cost while ensuring emissions on each link is lower than its maximum allowable emissions. The lower level formulates time-dependent land-use transport problem. This lower level is expressed as two sets of constraints, one for the transport system and the other for the land use system. These constraints are encapsulated into the upper level, resulting in a single level model while considering the land-use transport interaction over time. The resulting model can then be solved by many existing efficient optimization methods. In the following, section 2 describes the formulation of the proposed model. Section 3 is the numerical studies. Finally, section 4 gives some concluding remarks.

2. FORMULATION

We consider a strongly connected multi-modal transportation network with multiple Origin-Destination (OD) flows over the planning horizon \([0, T]\). The planning horizon is divided into \(N\) equal design periods. The network is further divided into \(M\) subnetworks, one for each mode, to account for the unique travelling speed of each mode. The mode here can be an individual mode or a combined mode. The flows of trucks and articulated lorries are treated as background flows and are given. They represent the movements of goods and are converted into Passenger Car Units (PCU). We also make the following assumptions to simplify the analysis: 1.) Basic employment, basic employment growth rate, and the zonal attractiveness are known; 2.) Each zone has only one characteristic, and the residential zone and the employment zone are not mixed; 3.) Traffic assignment follows the user-equilibrium principle,

and; 4.) The link cost and travel demand functions are separable. These assumptions are not restrictive from a modelling perspective and can be relaxed easily in further studies. With these considerations, we can formulate the time-dependent toll design problem as a bilevel problem. The lower level problem is the time-dependent land-use transport problem and the upper level is the decision maker’s problem.

2.1 Lower Level Time-dependent Land-use Transport Problem
2.1.1 Lower Level Problem Structure
Due to the complexity of the lower level problem, the structure is shown by two figures: figures 1 and 2. Figure 1 shows in details how land-use, transport and the environment interact at a time instant. Decision variables are highlighted in figure 1. Figure 2 illustrates the time dimension in the lower level problem together with the land-use-transport-environment interaction. Based on this structure, the lower level problem can then be described by two sets of mathematical constraints: time-dependent Lowry-based land-use constraints and time-dependent traffic assignment constraints.

Figure 1 Interaction between land use, transport, and environment at a time instant

T_1  T_2  T_3  ....  T_n
Land use  Land use  Land use  Land use
Transport  Transport  Transport  Transport
Environment  Environment  Environment  Environment

Figure 2 The land use transport and environment relationship over time
2.1.2 Time-dependent Lowry-based Land-use Constraints

Time-dependent Lowry-based land-use constraints are developed based on Lowry’s (1964) model, which classifies the land-use into three categories: basic sector, household sector and non-basic sector. The basic sector includes industries, businesses and administrative establishments whose goods and services are exported outside the urban area. It generates a centripetal flow of capital into the city generating growth and surpluses. It is generally assumed that this sector is less constrained by urban location problems since the local market is not the main concern. This consideration is an exogenous element of the Lowry model and must be given. The non-basic sector includes businesses, administrative establishments and other retailing services that deal with providing goods and services for local residential population. Since this sector strictly serves the local / regional demand, the location choice is oriented to the household sector. Employment levels are also assumed to be linked with the local population. The household sector consists of residential population. The number of residents is related to the number of basic and non-basic jobs available. Their residential locations are also closely linked to the place of work. Since the residential and non-basic sector location choices depend on each other, the household and non-basic sectors finally distribute themselves to achieve equilibrium.

The time-dependent Lowry-based land-use constraints extend the Lowry-type equilibrium to a dynamic framework. In each design period, a Lowry-type equilibrium is assumed to be held. The equilibrium is depicted by a number of constraints. The first one describes how to allocate residents who work in employment zone \( i \) to residential zone \( j \) using the gravity-type model:

\[
R_{ij,\tau} = E_{i,\tau}B_{i,\tau}W_{j,\tau}^a \exp(-\beta^r c_{ij,\tau}), \forall i, j, \tau ,
\]

where \( B_{i,\tau} = 1/\sum_j W_{j,\tau}^a \exp(-\beta^r c_{ij,\tau}), \forall i, \tau \).

\( R_{ij,\tau} \) is the number of residents travelling between OD pair \( ij \) in period \( \tau \) or the number of work-to-home trips (or the number of total employment trips) between OD pair \( ij \) in period \( \tau \). This is the number of residents in zone \( j \) that work in zone \( i \); \( E_{i,\tau} \) is the total employment in zone \( i \) in period \( \tau \); \( W_{j,\tau}^a \) is the attractiveness of zone \( j \) in period \( \tau \), which can be represented by the availability of floor space for residential use. The attractiveness of each zone is assumed to follow the following function:

\[
W_{i,\tau+1}^a = W_{i,\tau}^a \left( 1 + \tilde{h}_{w,i} \right), \forall i, \tau ,
\]

where \( \tilde{h}_{w,i} \) is the growth rate of attractiveness of zone \( i \) over time. \( \beta^r \) is the parameter to regulate the effect of transport cost on distribution of residents. A high value of \( \beta^r \) will result in the residents being allocated close to their place of work; if \( \beta^r \) tends to infinity, all residents will live and work in the same zone. On the other hand, if \( \beta^r \) tends to zero, the residents whose work in zone \( i \) will locate to all residential zones equally. \( c_{ij,\tau} \) is the composite travel cost between OD pair \( ij \) in period \( \tau \), representing the inter-zonal impedance and will be defined later. The term \( B_{i,\tau} \) is to ensure a correct allocation of residents to zone \( j \) in period \( \tau \) so that \( \sum_j B_{i,\tau}W_{j,\tau}^a \exp(-\beta^r c_{ij,\tau}) = 1 \) and \( \sum_j R_{ij,\tau} = E_{i,\tau} \). The latter means that the total number of work-to-home trips from employment zone \( i \) in period \( \tau \) must be equal to the
number of jobs available in that zone in period $\tau$, $E_{i,\tau}$ (i.e. the number of people working in employment zone $i$ in period $\tau$ must be equal to the number of jobs available in that zone in the same period).

The total employment in zone $i$ in period $\tau$, $E_{i,\tau}$, in (1), is the sum of the basic employment $E_{i,\tau}^B$ and the service employment or non-basic employment, $E_{i,\tau}^S$, in zone $i$ in period $\tau$:

$$E_{i,\tau} = E_{i,\tau}^B + E_{i,\tau}^S, \forall i, \tau. \quad (4)$$

The basic employment in the employment zone is supposed to grow linearly over time:

$$E_{i,\tau+1}^B = E_{i,\tau}^B \left(1 + \hat{h}_{E,i}\right), \forall i, \tau, \quad (5)$$

where $\hat{h}_{E,i}$ is the growth rate of basic employment. The service employment in zone $i$ in period $\tau$, $E_{i,\tau}^S$, in (4) is equal to the number of service employment trips starting from zone $i$ in period $\tau$ or simply the number of service employees working there in that period:

$$E_{i,\tau}^S = \sum_j E_{y_{ij},\tau}, \forall i, j, \tau. \quad (6)$$

where $E_{y_{ij},\tau}$ is the number of service employees who work in zone $i$ living in zone $j$ (i.e. the number of service employment trips between OD pair $ij$ in period $\tau$). The number of service employment trips $E_{y_{ij},\tau}$ is obtained by:

$$E_{y_{ij},\tau} = sR_{j,\tau}A_{j,\tau}W_{i,\tau}^{\alpha} \exp(-\beta^s c_{y_{ij},\tau}), \forall i, j, \tau, \quad (7)$$

where

$$A_{j,\tau} = 1/\sum_i W_{i,\tau}^{\alpha} \exp(-\beta^s c_{y_{ij},\tau}), \forall j, \tau; \quad (8)$$

$s$ is a service employment-to-population ratio; $W_{i,\tau}^{\alpha}$ is the attractiveness in zone $i$ in period $\tau$, which can be the availability of floor space for commercial use; $\beta^s$ is the parameter to regulate the effect of transport cost on distribution of service employees, and its function is similar to $\beta^r$ in (1). A high value of $\beta^s$ will result in service employment being allocated close to the residential location, and a small value will result in service employment being allocated to all residential locations equally. The term $A_{j,\tau}$ is to ensure a correct allocation of service employment to zone $i$, which has a similar function to the term $B_{i,\tau}$ in (2). The term $sR_{j,\tau}$ in (7) is the total number of employees in zone $j$ in period $\tau$. The total number of residents in zone $j$ in period $\tau$, $R_{j,\tau}$, in (7) is defined as:

$$R_{j,\tau} = \mu \sum_i R_{ij,\tau}, \forall j, \tau, \quad (9)$$

where $R_{ij,\tau}$ is the number of employees who work in zone $i$ and live in zone $j$ in period $\tau$, and $\mu$ is a population-to-employment ratio. According to (9), in each period, the total number of employees in zone $j$, $\sum_i R_{ij,\tau}$ multiplied by the population-to-employment $\mu$ gives the total number of residents in that zone $R_{j,\tau}$.

2.1.3 Time-dependent Traffic Assignment Constraints

The time-dependent traffic assignment constraints represent the transport model in this framework and describe the route and mode choices over time. These constraints are made up
of Wardrop’s conditions, travel cost constraints, as well as the modal split, flow conservation, and non-negativity conditions.

2.1.3.1 Wardrop’s Conditions
These conditions are supposed to be held in each design period for each mode. They require that for each mode \( k \) and for each period \( \tau \), route \( p \) between OD pair \( ij \) will not be used if its travel cost is higher than the lowest travel cost between OD pair \( ij \). Conversely, any used route \( p \) must have its travel cost equal to the lowest travel cost between OD pair \( rs \).

Mathematically, these conditions can be stated as:

\[
\begin{align*}
  f_{p,ij}^k \left[ c_{p,ij}^k - \pi_{y,\tau}^k \right] &= 0, \forall p, i, j, k, \tau, \quad (10) \\
  c_{p,ij}^k - \pi_{y,\tau}^k &\geq 0, \forall p, i, j, k, \tau, \quad (11)
\end{align*}
\]

where \( f_{p,ij}^k \) is the representative hourly flow of mode \( k \) on route \( p \) between OD pair \( ij \) in period \( \tau \); \( c_{p,ij}^k \) is the travel cost on route \( p \) between OD pair \( ij \) by mode \( k \) in period \( \tau \); \( \pi_{y,\tau}^k \) is the lowest travel cost between OD pair \( ij \) by mode \( k \) in period \( \tau \):

\[
\pi_{y,\tau}^k = \min \left[ c_{p,ij}^k, \forall p \right]. \quad (12)
\]

Conditions (10) and (11) constitute the nonlinear complementarity conditions for the route choice assignment principle for each mode \( k \) in each period \( \tau \). According to (10), if route \( p \) carries a positive mode \( k \)'s flow in period \( \tau \), (i.e., \( f_{p,ij}^k > 0 \)), then its associated route cost \( c_{p,ij}^k \) must be equal to the lowest cost \( \pi_{y,\tau}^k \) through the condition \( c_{p,ij}^k - \pi_{y,\tau}^k = 0 \). If no mode \( k \)'s flow is on path \( p \) in period \( \tau \), the term \( c_{p,ij}^k - \pi_{y,\tau}^k \) in (10) is unrestricted and the travel cost \( c_{p,ij}^k \) can be greater than or equal to \( \pi_{y,\tau}^k \) according to (11). Actually, condition (11) ensures \( \pi_{y,\tau}^k \) to be the lowest cost among all the possible routes between OD pair \( ij \) by mode \( k \) in period \( \tau \).

2.1.3.2 Travel Cost Constraints
Travel costs depend on flows, network characteristics such as free flow travel times and capacities of links, and out-of-pocket costs such as tolls and fares. Route costs depend on route flows, in which the latter depends on link flows through:

\[
v_{a,\tau} = \sum_p \sum_y f_{p,ij}^k \delta_{a,p}^k, \forall a, k, \tau, \quad (13)
\]

where \( v_{a,\tau} \) is the hourly flow of mode \( k \) on link \( a \) in period \( \tau \), and \( \delta_{a,p}^k \) is a link-path incidence indicator for mode \( k \), which equals one if link \( a \) is on route \( p \), and zero otherwise. Equation (13) states that for each mode \( k \), the link flow in each period is obtained by adding all route flows on that link in that period together. The link time \( t_{a,\tau} \) (such as travel time, waiting time, or walking time) relates link flows through the link performance function:

\[
t_{a,\tau} = t_{a,\tau}^k \left( v_{a,\tau} \right), \quad (14)
\]

where \( v_{a,\tau} \) is the link flow vector in period \( \tau \). This link performance function is non-separable as the link time on link \( a \) depends on the flows on other links. The link performance function adopted in road traffic assignment in this paper is:
where the superscript \( m \) stands for the mode that travels in the road network only (which is different from the superscript \( k \) that is used for representing any mode considered in this paper); \( t_{a,\tau}^{0,m} \) and \( \overline{c}_a \) are the free flow travel time for mode \( m \) and the capacity of link \( a \); \( \alpha_0 = 1 \), \( \alpha_1 = 0.15 \), and \( \alpha_2 = 4 \) are parameters of the link performance function of link \( a \) for mode \( m \). Equation (15) is the typical Bureau of Public Roads (BPR) function, which describes the monotonic relationship between the link travel time \( t_{a,\tau}^m \) and the link flow \( \nu_{a,\tau}^m \).

The route cost \( c_{p,i,j,\tau} \) is the sum of the link-wise additive costs \( g_{p,i,j,\tau}^k \) and the route specific costs \( \theta_{p,i,j,\tau}^k \):

\[
c_{p,i,j,\tau} = g_{p,i,j,\tau}^k + \theta_{p,i,j,\tau}^k, \forall p, i, j, k, \tau.
\]

The link-wise additive costs \( g_{p,i,j,\tau}^k \) are defined by summing up link attributes, which include link tolls \( \rho_{a,\tau}^k \) and congestion-dependent attributes; for instance, travel time (and other costs such as fuel consumption) spending on road networks, or walking, on-board and boarding/alighting time spending on transit networks. The link-wise additive cost \( g_{p,i,j,\tau}^k \) can be written as:

\[
g_{p,i,j,\tau}^k = \sum_a (\psi t_{a,\tau}^k + \rho_{a,\tau}^k) \cdot \delta_{p,i,j,k,\tau}^a, \forall p, i, j, k, \tau,
\]

where \( \psi \) is the cost of unit (travel) time, and therefore \( \psi t_{a,\tau}^k \) is the (travel) time cost on link \( a \) by mode \( k \) in period \( \tau \); \( \rho_{a,\tau}^k \) is the toll for mode \( k \) using link \( a \) in period \( \tau \). The route specific costs \( \theta_{p,i,j,\tau}^k \) are non-linear and/or nonadditive over links; for instance, some types of tolls on road networks (e.g. non-linearly proportional to distance), or waiting time and some fare structures for transit networks (e.g. zone-wise prices).

The composite travel cost between OD pair \( ij \) in period \( \tau \), \( c_{i,j,\tau} \) is defined as:

\[
c_{i,j,\tau} = -\ln \left[ \sum_k \left( \exp(-\beta (\pi_{i,j,\tau}^k + \theta^k)) \right) \right] \left/ \beta \right., \forall i, j, \tau,
\]

where \( \beta \) is the parameter in the logit model to regulate the effect of the mode travel cost \( \pi_{i,j,\tau}^k + \theta^k \); \( \pi_{i,j,\tau}^k \) is the lowest travel cost between OD pair \( ij \) by mode \( k \) in period \( \tau \) defined in (12); \( \theta^k \) is the mode-specific cost. The composite cost is obtained by aggregating the mode travel cost \( \pi_{i,j,\tau}^k + \theta^k \) over all modes. The derivation of this composite cost can be found in Ben Akiva and Lerman (1987).

\[ q_{i,j,\tau}^k = R_{i,j,\tau} \left[ \frac{\exp(-\beta (\pi_{i,j,\tau}^k + \theta^k))}{\sum_g \exp(-\beta (\pi_{i,j,\tau}^g + \theta_g^k))} \right], \forall i, j, k, \tau,
\]
where \( q_{ij}^k \) is the demand for mode \( k \) between OD pair \( ij \) in period \( \tau \), and \( R_{ij,\tau} \) is the number of residents who work in zone \( i \) and live in zone \( j \) defined in (1). The demand for mode \( k \) between OD pair \( ij \) in period \( \tau \), \( q_{ij}^k \), in (19) is equal to the sum of the route flows of that mode between the OD pair in the same period so that route flows are conserved in each mode between each OD pair in each period:

\[
q_{ij}^k = \sum_p f_{p,ij,\tau}^k, \forall i, j, k, \tau .
\]  

(20)

Moreover, route flows in (20) must be non-negative:

\[
f_{p,ij,\tau}^k \geq 0, \forall p, i, j, k, \tau .
\]  

(21)

2.1.4 Lower Level Optimization Model

2.1.4.1 Gap Function

The lower level problem can indeed form an optimization problem by employing a gap function. Many gap functions can serve this purpose. In this paper, we adopt

\[
G = \sum_p \sum_q \sum_\tau f_{p,ij,\tau}^k \left[ c_{p,ij,\tau}^k - \pi_{ij,\tau}^k \right].
\]  

(22)

This gap function must have non-negative values if \( f_{p,ij,\tau}^k \geq 0 \) and \( c_{p,ij,\tau}^k - \pi_{ij,\tau}^k \geq 0 \) because the sum of non-negative numbers must be non-negative. One property of this gap function is that if \( f_{p,ij,\tau}^k \geq 0 \), \( c_{p,ij,\tau}^k - \pi_{ij,\tau}^k \geq 0 \) and the gap function attains its minimum value of zero, the time-dependent Wardrop’s condition (10) is satisfied. This property will be used in developing our lower level optimization model.

2.1.4.2 Lower Level Optimization Formulation

Given the time-dependent tolls, the lower level model is formulated as follows:

**Lower Level Model**

\[
\min_{f, E, R} G,
\]

subject to

- time-dependent Lowry-based land-use model constraints (1)-(9), and;
- time-dependent traffic assignment constraints (10)-(21).

where \( f, E, R \) represent, respectively, the vectors of path flows, service employment trips, and residential trips. Since \( f_{p,ij,\tau}^k \geq 0 \) and \( c_{p,ij,\tau}^k - \pi_{ij,\tau}^k \geq 0 \) are ensured by the time-dependent traffic assignment constraints (11) and (21), path flows, service employment trips, and residential trips will satisfy (1)-(21) when the gap \( G \) is zero.

2.2 Upper Level Problem

The upper level is the decision maker’s problem. This study assumes that the decision maker (or the government) is primarily concerned with the total travel cost (TTC), and secondly with vehicular emissions and design (e.g. toll) or regulation (e.g. link emission) constraints when making decisions.

2.2.1 Total Travel Cost (TTC)

TTC is mathematically formulated as:

\[
TTC = \sum_\tau \sum_k \sum_p f_{p,ij,\tau}^k c_{p,ij,\tau}^k .
\]  

(23)

Equation (23) states that TTC is the sum of the products of route flows and their corresponding travel costs over all routes, modes, OD pairs and time. This TTC function forms the objective function of the bilevel model.
2.2.2 Vehicular Emissions
There are two types of vehicular emissions: link and network (or overall). The link vehicular emission is defined through the link emission factor approach:

\[ Q_{a,\tau} = \sum_m Q_{a,\tau}^m - \sum_m h_{a,\tau}^m v_{a,\tau}^m, \forall a, \tau, \]  

(24)

where \( Q_{a,\tau} \) is the vehicular emissions for traffic mode \( m \) on link \( a \) in period \( \tau \); \( v_{a,\tau}^m \) represents the hourly traffic flow for mode \( m \) on link \( a \) in period \( \tau \); \( h_{a,\tau}^m \) is the emission factor for mode \( m \) on link \( a \) in period \( \tau \), which is assumed to be given for all links. The factors affecting the value of \( h_{a,\tau}^m \) are discussed in Nagurney (2000). This link emission factor approach has been adopted by Nagurney et al. (1998) and others. According to (24), the vehicular emissions for mode \( m \) on a particular link is the product of the link flows of mode \( m \) and the corresponding emission factor, and the total vehicular emissions on this link is the sum of vehicular emissions for all modes traveling on this link. The overall vehicular emissions are the sum of the vehicular emissions on each link:

\[ Q_{\tau} = \sum_a Q_{a,\tau}, \forall \tau. \]  

(25)

2.2.3 Link Emission Constraint
With the definition of link vehicular emissions, we can define an link emission constraint, which is to ensure that link emissions is smaller than or equal to the maximum allowable emissions on link \( a \) in period \( \tau \):

\[ Q_{a,\tau} \leq \tilde{Q}_{a,\tau}, \forall a, \tau, \]  

(26)

where \( \tilde{Q}_{a,\tau} \) are the maximum allowable vehicular emissions on link \( a \) in period \( \tau \).

2.2.4 Toll Constraints
For political and other reasons it is not always feasible to set the toll charge too high while in practice only non-negative tolls are possible and charged at certain links. Mathematically, these conditions are expressed as:

\[ \rho_{a,\tau}^k \geq 0, \forall a, k, \tau, \]  

(27)

\[ \rho_{a,\tau}^k \leq \hat{\rho}_{a,\tau}^k, \forall a, k, \tau. \]  

(28)

Condition (27) ensures that all tolls are greater than or equal to zero. Condition (28) requires that the toll charged on link \( a \) in period \( \tau \) be less than the maximum allowable toll \( \hat{\rho}_{a,\tau}^k \) for that link. If the link is not allowable to charge any toll, \( \hat{\rho}_{a,\tau}^k \) is set to zero.

2.3 The Proposed Bilevel Model
The proposed bilevel model is formulated as follows:

Bilevel Model

\[ \min_{f, E, R, \rho} TTC, \]  

subject to

- time-dependent Lowry-based land-use constraints (1)-(9);
- time-dependent traffic assignment constraints (10)-(21);
- link emission constraint (26), and;
- toll constraints (27)-(28).

The proposed bilevel model is formulated as a single level minimization program where the lower level problem is expressed as constraints (1)-(21) instead of the optimization model, Lower Level Model, described in Section 2.1.4.2. The advantage of formulating the bilevel problem as a single level problem is that we can use the existing single optimization
algorithms and packages to solve for solutions. In this study, Bilevel Model is solved using the Generalized Reduced Gradient (GRG) method (Abadie and Carpentier, 1969).

3. NUMERICAL STUDIES

Two scenarios are set up in this section. The first one is to illustrate the effects of tolls on the overall vehicular emissions, link emissions, total travel cost, transit revenue, residents’ working and living locations, and the importance of considering land use, transport and environment (in terms of link emissions) over time in determining optimal time-dependent tolls. The second one is to study the impact of maximum allowable link emissions on link tolls.

3.1 The Impacts of Time-dependent Tolls on the Overall Vehicular Emissions, Link Emissions, Total Travel Cost, Transit Revenue, and Residents’ Working and Living Locations

The scenario network is shown in figure 3 with five nodes, six (road) links and six origin-destination (OD) pairs. The six OD pairs are E1-R3, E1-R4, E1-R5, E2-R3, E2-R4, and E2-R5 respectively. E1 and E2 represent employment zones 1 and 2 respectively. R3, R4, and R5 correspondingly represent residential zones 3, 4, and 5. There is public transit operated between each OD pair (but the transit is not shown here). Link E1-R3 is the sole toll link. No truck flows are considered here for the sake of simplicity. The parameters adopted in this scenario are shown below.

Transport model parameters: Value of time: $\psi = €10/hour$; free flow travel times: $t_{13}^{0,\text{car}} = t_{24}^{0,\text{car}} = 20 \text{ mins}$, $t_{15}^{0,\text{car}} = t_{23}^{0,\text{car}} = t_{35}^{0,\text{car}} = t_{45}^{0,\text{car}} = 15 \text{ mins}$; Car-specific constant: $\theta_{\text{car}} = 0.8$; free flow travel times: $t_{13}^{0,\text{transit}} = t_{24}^{0,\text{transit}} = 12 \text{ mins}$, $t_{14}^{0,\text{transit}} = t_{23}^{0,\text{transit}} = 20 \text{ mins}$, $t_{15}^{0,\text{transit}} = t_{25}^{0,\text{transit}} = 10 \text{ mins}$; transit fares: $\theta_{13,\text{transit}} = \theta_{24,\text{transit}} = \theta_{14,\text{transit}} = \theta_{23,\text{transit}} = \theta_{15,\text{transit}} = \theta_{25,\text{transit}} = \€2$, $\theta_{15,\text{transit}} = \theta_{25,\text{transit}} = \€1$; Transit-specific constants: $\theta_{13}^{\text{transit}} = \theta_{24}^{\text{transit}} = 1.8$, $\theta_{14}^{\text{transit}} = \theta_{23}^{\text{transit}} = \theta_{15}^{\text{transit}} = \theta_{25}^{\text{transit}} = 1$. Land use model parameters: Basic employment: $E_{1,1}^{B} = 1000 \text{ jobs}$, $E_{2,1}^{B} = 800 \text{ jobs}$; Population to employment ratio: $\mu = 5$; Service employment to population ratio: $s = 0.1$; Accessibility parameters: $\beta^{r} = 0.8$, $\beta^{s} = 0.6$; parameter for the composite cost: $\beta = 1$; Attractiveness: $W_{0,1}^{a} = W_{2,1}^{a} = 1000 \text{ jobs}$; $W_{3,1}^{a} = W_{4,1}^{a} = W_{5,1}^{a} = 1000 \text{ houses}$; growth rates of basic employment and attractiveness: $\tilde{h}_{E,j} = \tilde{h}_{W,j} = 0.04$. Environment parameters: Emission factors: $h_{13,\text{car}}^{\text{emission}} = 0.7 \text{ liter/veh}$, $h_{24,\text{car}}^{\text{emission}} = 0.6 \text{ liter/veh}$, $h_{35,\text{car}}^{\text{emission}} = 0.4 \text{ liter/veh}$, $h_{45,\text{car}}^{\text{emission}} = 0.3 \text{ liter/veh}$, $h_{15,\text{car}}^{\text{emission}} = h_{25,\text{car}}^{\text{emission}} = 0.5 \text{ liter/veh}$; overall maximum allowable vehicular emissions: $Q = 1680 \text{ liters/hour}$. Planning parameter: Planning horizon: 3 years, length of each period: 1 year.
In this scenario, Lower Level Model is employed to study the impact of tolls on transport, land use and environment systems and is solved by the GRG method. For illustrative purposes, the tolls are constant over time. The toll levels on link 1 are varied from €0 to €5. The overall vehicular emissions on the entire network are plotted in figure 4. From this figure, we can observe that the tolls can be implemented as a measure to control the overall vehicular emissions. However, the decision maker has to select the toll level carefully, because not all tolls can achieve the overall vehicular emissions to be less than the maximum allowable vehicular emissions. For example, when the toll is greater than €2.4, the overall vehicular emissions are under the maximum allowable vehicular emissions of 1.68E3 liters/hour as shown by the horizontal line in the figure. In particular, when the toll is greater than €3, the overall vehicular emissions remain at the same level. This raises an issue for the government on how to select tolls to regulate the overall vehicular emissions. An analytical model is needed for this purpose.

The implementation of tolls on links not only can lower the overall traffic emissions but also has a significant impact on a public transit system and the land use pattern. Figure 5 depicts the number of passengers on OD pair E1-R3 on the public transit system and the total transit revenue over the entire planning horizon. It is demonstrated from this figure that the toll implementation on link E1-R3 causes travelers between OD pair E1-R3 to change their mode choice and travel by public transit instead of private cars. When the toll level rises, more passengers are attracted to the public transit system in order to decrease their travel costs. Beyond the toll level of €3, the number of passengers on transit between OD pair E1-R3 goes stable. Figure 5 also reveals that the implementation of tolls can generate more transit revenue. When there is no toll on the transport network, the total transit revenue is only €1.35E7. However, after charging a toll of €3 on link E1-R3, the revenue is increased by 21.5% due to the increase in passengers. It is concluded that the implementation of tolls can divert some travelers to take public transit and raise its revenue. This has important implications for transit operators on their operational strategies and profits.

Figure 6 shows the rearrangement of employment due to the changes in tolls. This is because their travel costs are altered after the toll implementation and people are sensitive to travel costs in picking up their jobs. Figure 7 indicates the change in the number of residents in
residential zones 3, 4 and 5 over time. From figure 7, it is revealed that there are dramatic changes in the numbers of residents in zones 3 and 5 over three years when the tolls are ranged from €0-€3. This is again due to the travel costs. People will decide their residential locations based on their travel costs to their works. However, not all tolls can affect the land use pattern in terms of residential and employment distributions. As shown in the above two figures, the land use pattern remains stable after the toll value of €3. To sum up, these two figures clearly demonstrate that tolls can alter the choices of working and living locations of people, which has a strong implication for land owners’ profits, the rents of residents and the living environment. This is because the rents, the profits, and the living environment depend on the population size or the number of residents. More residents will result in higher rents and hence higher land owners’ profits. More residents will also result in few spaces for recreational uses and higher competition for resources and facilities, and hence poor living environment. Tolls must be carefully selected to deal with the concerns of land owners and residents.

Figure 6 Total employments in zones 1 and 2 in each design year

The total travel cost before and after tolling are revealed in figure 8a. It is observed that the total travel cost is increased substantially (more than 13 %) when charging tolls on link E1-R3. This illustrates that the implementation of tolls can have a negative impact on the total travel cost. In particular, when we compare figure 8a with figure 4, we observe that there is a tradeoff between the overall vehicular emissions and the total travel cost. The higher the toll, the fewer the emissions are but the higher the total travel cost is (although the increase in total travel cost is diminishing quickly). The total travel cost should be taken into account in a modeling stage in addition to the overall vehicular emissions. Figure 8b shows that the link emissions on link E1-R5 as a result of tolls on link E1-R3. This figure shows that the higher the tolls, the higher the vehicular emissions on link E1-R5, because more traffic will be diverted to link E1-R5. In the extreme case, the vehicular emissions on link E1-R5 can rise more than 15 %. In addition, their relationship is not linear and two kinks are observed in this figure due to the sudden changes in the overall network flow pattern. One more key observation is that although the overall vehicular emissions are acceptable when the tolls are greater than €2.8 (see figure 5), the link emissions exceed the allowable vehicular emissions if the allowable emissions on link E1-R5 are 6500 liters. Therefore the government should regulate not only the overall vehicular emissions but also link vehicular emissions since link emission standards can be violated even though the overall emission standard is satisfied. To conclude, the total travel cost and link emissions should be considered in determining optimal time-dependent tolls.
3.2 The Impact of Maximum Allowable Link Emissions on Link Tolls

With all the considerations in the first scenario, we set up another scenario to study the effects of the maximum allowable link emissions on its link toll. The proposed Bilevel Model is employed for this purpose and solved by the GRG method. A simple network is shown in figure 9, which has three nodes, two OD pairs, two road network links and one transit link. The two OD pairs are OD pairs E1-R2 and E1-R3. The three nodes are E1, R2 and R3 representing employment zone 1, residential zones 2 and 3 respectively. The two road links are link E1-R2 and link E1-R2. Link E1-R3 has a higher emission factor due to its longer distance compared with link E1-R2. The transit link connects OD pair E1-R3. The government tries to lower the vehicular emissions on link E1-R3 by charging tolls on this link over time, and also regulates the vehicular emissions on link E1-R2 so that the emissions are smaller or equal to its maximum allowable emissions. The following parameters are adopted:

Transport model parameters: Value of time: $\psi = €10/hour$; free flow travel times: $t_{12}^{0,car} = 18$ mins, $t_{13}^{0,car} = 30$ mins; Car-specific constant: $\theta^{car} = 0.8$; free flow travel times: $t_{13}^{0,transit} = 10$ mins; transit fares: $\theta^{transit} = €3$; Transit-specific constants: $\theta^{transit} = 1.8$; maximum allowable toll: $\tilde{\rho}_a^k = €10$. Land use model parameters: Basic employment: $E_{1,1}^B = 1000$ jobs; Population to
employment ratio: $\mu = 5$; Service employment to population ratio: $s = 0.15$; Accessibility parameters: $\beta^e = 0.8$, $\beta^r = 0.6$; parameter for the composite cost: $\beta^c = 1$; Attractiveness: $W_{11}^a = 1000$ jobs; $W_{21}^a = W_{31}^a = 1000$ houses; growth rates of basic employment and attractiveness: $h_e, h_w = 0.04$. Environment parameters: Emission factors: $h_{12}^{car} = 0.5$ liter/veh, $h_{13}^{car} = 1.1$ liters/veh; maximum allowable vehicular emissions: $\bar{Q}_{12} = 4000$ liters/hour. Planning parameter: Planning horizon: 2 years; length of each period: 1 year.

Figure 10 illustrates the effect of the maximum allowable link emissions of link E1-R3 on its link toll. As you can see, higher maximum allowable link emissions result in a lower link toll. It is reasonable as a tighter link emission standard only allows few vehicles on that link, and few vehicles on that link can be achieved by setting a higher toll on it. The implication is that the maximum allowable link emissions can have direct impacts on the transport system (in terms of the number of transit passengers, transit revenue, the total travel cost), the land use system (in terms of population and employment distributions) and the environmental system (in terms of link and overall emissions). It is because the maximum allowable link emissions determine link tolls, and the link tolls can have impacts on the transport, land use and environmental systems over time as shown in the previous scenario. The government should consider the impacts on land use and transport systems in addition to the environmental system when determining the maximum allowable link emissions.

4. CONCLUDING REMARKS

In this paper, we proposed a bilevel model to determine the optimal tolls over time to control vehicular emissions while capturing the land-use transport interaction over time. To illustrate the importance of considering land use, transport and environment over time in the determining optimal time-dependent tolls, a numerical study was set up. The results show that: 1. The implementation of tolls can have a negative impact on the total travel cost although the tolls can lower the overall vehicular emissions. There is a tradeoff between the overall vehicular emissions and the total travel cost. The total travel cost should be taken into account in a modeling stage in addition to the overall vehicular emissions. 2. Link emission standards can be violated even though the overall emission standard is satisfied. Link vehicular emissions should also be considered in determining optimal time-dependent tolls. 3. The implementation of tolls can generate more transit revenue and more transit passengers. This has important implications for transit operators on their operational strategies and profits. 4. Tolls can alter traveler’s choices in terms of their working and living locations, which has strong implications for land owners’ profits, the rents of residents and the general living
environment. Tolls must be carefully selected so as to consider the concerns of land owners and residents in addition to the concern of transit operators. Another study was also performed to study the effect of maximum allowable link emissions (or link emission standard) on link tolls. The results show that higher maximum allowable link emissions lead to a lower toll on that link. This observation together with the previous four findings implies that the maximum allowable link emissions can have direct impacts on the road and transit systems, the land use system and the environmental system. The government should consider the impacts on land use, transport and environmental systems when determining the maximum allowable link emissions.

The proposed model in fact has great potentials in terms of developing equilibrium analysis based on real data (such as for a concrete city). The three sub-models that are integrated into our frame work are respectively the Lowry-based land use model, the transport model and the vehicular emission model. Each of them can be calibrated based on real data through different techniques. In terms of the calibration of the Lowry model, Wong et al. (1998) have developed a new methodology to calibrate the Lowry model using HK as a case study. They proposed a three-stage calibration procedure which can be utilized in our Lowry-based land use forecasting model. With respect to the transport model, the BPR link performance function is the main function requiring calibration. Sheffi (1985) has clearly stated that least squares and Maximum Likelihood (ML) are the most commonly used techniques to calibrate the BPR link performance function. Regarding the vehicular emission model, the key of estimating vehicular emissions is the relationship that volume of emissions is equal to the product of an emission factor and link load (DeCorla-Souza et al., 1995). This emission factor obtained using MOBILE model proposed by the Environmental Protection Agency (EPA) is based on the federal test procedure (FTP), typical driving conditions for an urban vehicle trip (DeCorla-Souza et al., 1995).

This paper raises a lot of future research directions. Firstly, the movements of goods are considered to be given in this study. Relaxing this assumption is one of future research directions. Secondly, from the equity point of view, it is not fair to charge the same toll to travellers who have different values of time, therefore it would be vitally important to consider optimal tolls in multi-class, multi-criteria transportation networks in which travellers consider multi-criteria of travel cost, travel time and incurred vehicular pollution. Finally, extending this framework to consider capacity expansion, road construction and the gradually upgraded transit network is another possible direction, as many parts of Asia are in an active phase of a massive transport development.

ACKNOWLEDGMENT

This research is funded under the Programme for Research in Third-Level Institutions (PRTLI), administered by the Higher Education Authority. The authors thank the constructive comments of the referees.

REFERENCES

MA.
Reliability, *Journal of Eastern Asia Society for Transportation Studies, Vol. 6, No. 1*,
2060-2075.
emissions with environmental protection agency's MOBILE model, *Transportation
Research Record 1444*, 118-125.
Washington D.C., 1-96.
7-9*, 813-834.
*Cross Strait Conference on Intelligent Transportation Systems*. CD-ROM
of Transportation Engineering, Vol. 123*, 316-324.
Optimal land use transport strategies: methodology and application to European cities,
*Transportation Research Record 1924*, 129-138.
Meng, Q., Yang, H. and Bell, M.G.H. (2001) An equivalent continuously differentiable model
and a locally convergent algorithm for the continuous network design problem,
Miyamoto, K., Udomsri, R., Sathyaprasad, S. and Ren, F. (1996) A decision support system
for integrating land use, transport and environmental planning in developing metropolises,
equilibrium model with emission pollution permits: compliance versus noncompliance,
Sumalee, A. (2005) Optimal implementation-path of road pricing schemes with time-
6*, 624-639.
Vold, A. (2005) Optimal land use and transport planning for the Greater Oslo area,
Yang, H. and Huang, H.J. (1997) Analysis of the time-varying pricing of a bottleneck with
elastic demand using optimal control theory, *Transportation Research Part B, Vol. 31,
No. 6*, 425-440.