The Optimal Toll Schemes to Container Ships Queuing at the Anchorage

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Abstract: When arriving at a busy port, a container ship usually need to queue in anchorage waiting for a berth. If the port establishes a toll scheme during the queuing period, container ships’ arrival times at the port will be dispersed, and the queuing situation can be decreased. In this paper an optimal step toll scheme to container ships is designed, and all values of equilibrium queuing costs, equilibrium derivative costs due to queuing, and in patterns of arrival time change under the optimal step toll scheme are obtained. According to these equilibrium results, this paper shows container ships that pay the toll to berth a queuing port are those altered their original arrival times at the port before the toll was established. In addition, container ships’ arrival time change decisions from the non-toll to the optimal step toll cases can be predicted before tolling a queuing port.

Key Words: queuing, container ships, optimal step toll scheme

1. INTRODUCTION AND LITERATURE REVIEW

When a container ship sails to the next appointed port, the shipmaster must notify the shipping company the time of arrival by telegraph or electronic mail. This procedure is convenient for the company to carry out dispatch steps in the container terminal. If arriving at a queuing port, a ship having to wait at anchorage until the berth available is frequent. This situation is very similar to commuting cars queuing in front of a road bottleneck in the morning rush hour. Each car has to queue for passing through the bottleneck one by one. Many transportation economic literatures have verified that this kind of queuing time can be rationally reduced by collecting optimal tolls. Hence, if the same theory applies to arrival container ships waiting at anchorage of the destination port, the similar effect of optimal tolls on reducing the queuing can be expected.

A model of pricing a queuing bottleneck was initially developed by Vickrey (1969) and extended by Small (1982), De Palma and Arnott (1986), Cohen (1987), Braid (1989), Arnott et al (1990, 1993), Tabuchi (1993), Laih (1994, 1997, 2004), Yang and Meng (1998). Among these researches, Laih (1994) first developed a flexible step pricing mechanism to relieve commuting queuing at a road bottleneck. Laih (2004) also provided a methodological framework forecasting the behavior changes of commuters from a non-toll case to the optimal single-and multi-step toll cases. Applying these considerations, the optimal step toll scheme for a queuing port is derived in this paper. With the toll scheme, arrival times of container ships will be rationally dispersed. Consequently, the queuing time at the anchorage can be decreased. This paper also derives the consequent changes of container ships’ arrival schedules after collecting the optimal step toll. Decisions of changing the arrival time from
the non-toll to the tolled cases can be predicted before a queuing port establishes the toll scheme. Port queuing pricing leads to the efficient use of berths especially during heavily congested periods. All of these are important issues for container ship owners and port bureaus if the queuing pricing policy is considered by authorities.

Some literatures related to price a queuing bottleneck with the optimal non-queuing toll and the optimal step toll schemes are reviewed as follows. Laih (1994) looked into the model of charging for queuing road bottleneck and used the static equilibrium analytical method to develop a series of optimal and sub-optimal step toll schemes. This development provided decision makers a set of collecting toll framework with flexibility when they tried to minimize negative effects of congestion at a bottleneck. Furthermore, Laih (1997) proved that the entire queuing time of all auto-commuters could be effectively shortened to 1/2 and 2/3 of the original by collecting the optimal single- and double-step tolls, respectively. This study only displayed an equilibrium outcome of dispersing auto-commuters’ departure rates after the optimal single- and double-step toll schemes applied. However, it was not known clearly about the changes of departure behaviors of auto-commuters from the non-toll to the tolled cases, which led to reduce queuing at a bottleneck. Laih (2004) expanded the analysis of the optimal single- and double- steps to $n$th number ($n = 1, 2, 3, \ldots$) of charging steps. It was realized that when the charging steps increased one by one after detailed derivation, the framework of the optimal step toll, the related equilibrium costs, the equilibrium departure rates and moving tracks of departure time of auto-commuters had all shown regular variation. These complete and regular information not only facilitate policy makers to apply the optimal step toll scheme, but it can also be used to predict the entire auto-commuters’ behavior in the system of toll collection.

The framework of collecting the optimal step toll to container ships in this paper is applicable to all general commercial ports. The non-toll equilibrium cost and the optimal non-queuing toll to container ships queuing at the anchorage are derived in Chapter 2. The optimal step toll scheme and methodological frameworks used to forecast the changes of container ships’ arrival times from the non-toll to the optimal step toll cases are developed in Chapter 3. A numerical example explaining the frameworks mentioned in Chapter 3 is provided in Chapter 4. Finally, practical implications of the main results provided in this paper are addressed in Chapter 5.

2. THE MODEL

Basic assumptions for a queuing port model are as follows. First, the assumed background in this paper is that all berths are no vacancy and all liner container ships have to queue at anchorage until a vacant berth becomes available. Secondly, a large number of container ships will anchor at the port causing queuing during a certain period of time. This may be a result of increase in demand and supply of some goods attracting more ships calling the port, for example, export and import of Christmas merchandise at some ports from September to December. Thirdly, except this queuing port, there are no other alternative ports existed. Fourthly, the cost function to container ships includes the queuing costs at the anchorage and the derivative costs of early or late arrival at the berth due to queuing. Fifthly, the sequence of entering the port follows the principle of first in first serviced.

This paper defines three possible schedules of arriving pattern at the destination port: on-time ($t + T_q(i) = \bar{t}$), early arrival ($t + T_q(i) < \bar{t}$) and late arrival ($t + T_q(i) > \bar{t}$) compared with the
liner scheduled berthing time getting ready for cargo work in the port. Among these three situations, \( t \) is the time point when the container ship arrived at the anchorage of the port. \( T_q(t) \) is the length of queuing time at the anchorage and varies in accordance with \( t \). \( \bar{t} \) is the scheduled berthing time at the port, also called Estimated Time of Arrival (ETA) in this paper.

According to the on-time schedule case in Figure 1, \( \bar{t} \) is defined as the arrival time at the anchorage, which allows the ship berthing time is just on ETA after queuing. \( t^* \) is defined as the scheduled departure time at the port, called Expected Time of Departure (ETD) in this paper. The operating time length in unloading/loading cargoes required to meet ETD in this case is \( t^* - \bar{t} \). Meanwhile, the wharfage to the ship is also counted from \( \bar{t} \) to \( t^* \). For the early schedule case in Figure 1, \( T_E(t) \) is defined as the time length of early arrival, i.e., \( \bar{t} - (t + T_q(t)) \), which implies that the ship berthing time is earlier than ETA. Because the early arrived container ship will not unload/load until \( \bar{t} \) in order to avoid additional operating costs, the operating time needed to unload/load cargoes in this case is the same as the on-time schedule case. Meanwhile, the wharfage to the ship is counted from \( (\bar{t} - T_E(t)) \) to \( t^* \). For the late scheduling case, on the other hand, \( T_L(t) \) is defined as the time length of late arrival, i.e., \( (t + T_q(t)) - \bar{t} \), which implies that the ship berthing time is later than ETA.

The length of both the operating time period and the wharfage period in this case is the same and equal to \( t^* - (\bar{t} + T_L(t)) \).

On-Time Schedule: \( \bar{t} + T_Q(\bar{t}) = \bar{t} \)

Time-Early Schedule: \( t + T_Q(t) < \bar{t} \)

Time-Late Schedule: \( t + T_Q(t) > \bar{t} \)

Figure 1 Three possible schedules of arriving patterns at the destination port

In order to develop the cost functions based on Figure 1, we have the following assumptions. Firstly, let \( c_q \) represent the queuing cost per hour to \( T_q(t) \), then \( c_q \cdot T_q(t) \) means the
queuing time cost, which consists of personnel expense, depreciation cost of the ship, expense for repairing, insurance fee, interests, petrol fee for maintenance and desalination fee. These expenses are indispensable while ships anchor during the queuing period \((t_q, t_{q'})\). \(t_q\) and \(t_{q'}\) represent the start and the end times of queuing at the anchorage, respectively. \(t_q\) will vary in accordance with the ship arrival times at the anchorage. With regard to \(t_{q'}\), it will vary in accordance with the time length required to disperse all queuing ships, and depend upon the number of ships queuing, the number of ships being able to be serviced and the entire operation time of a ship at the port in average. Secondly, let \(c_w\) represent the wharfage cost per hour to a container ship, generally speaking, \(c_q\) is larger than \(c_w\) in actual practice. Thirdly, let \(c_{ext}\) represents the extra operating time cost per hour for the late arrival. When a container ship arrives at the port late (later than ETA), the ship owner must bear the extra cost for unloading/loading cargoes, which includes increased costs to port hardware apparatus and longshoremen, in order to meet the scheduled ETD. Therefore, the total extra operating time cost \(c_{ext} \cdot T_L(t)\) is necessary to allow the ships that belong to the Time-Late Scheduling Case leave the port on ETD.

Accordingly, we obtain the total cost \((TC(t))\) that resulted from queuing at the anchorage to all container ships:

(i). On-time Schedule Case (ETA on time)

\[
TC(t) = c_q \cdot T_Q(t) \quad \text{for } t + T_Q(t) = \tilde{t} 
\]

(ii). Time-Early Schedule Case (Earlier than ETA):

\[
TC(t) = c_q \cdot T_Q(t) + c_w \cdot T_L(t) \\
= c_q \cdot T_Q(t) + c_w \cdot \left[\tilde{t} - \left(t + T_Q(t)\right)\right] \quad \text{for } t_q \leq t + T_Q(t) < \tilde{t} 
\]

(iii). Time-Late Schedule Case (Later than ETA):

\[
TC(t) = c_q \cdot T_Q(t) + c_{ext} \cdot T_L(t) \\
= c_q \cdot T_Q(t) + c_{ext} \cdot \left[\left(t + T_Q(t)\right) - \tilde{t}\right] \quad \text{for } \tilde{t} < t + T_Q(t) \leq t_q' 
\]

Equilibrium obtains when no individual ship has an incentive to change the arrival time \((t)\). This implies that the total cost \(TC(t)\) to each ship must be the same at all times during the queuing period \((t_q, t_{q'})\). In other words, the equilibrium condition to the model is \(dC/dt = 0\). For this purpose, we differentiate (1), (2) and (3) with \(t\).

(i). On-time Schedule Case \((\tilde{t} + T_Q(\tilde{t}) = \tilde{t})\):

Unable to differentiate (1), because a time spot \((\tilde{t})\) is not differentiable.

(ii). Time-Early Schedule Case \((t_q \leq t + T_Q(t) < \tilde{t})\):

\[
\frac{dTC(t)}{dt} = c_q \cdot \frac{dT_Q(t)}{dt} + c_w \cdot \left[1 - \frac{dT_Q(t)}{dt}\right] = 0 \\
\frac{dT_Q(t)}{dt} = \frac{c_w}{c_q - c_w} 
\]
(iii). Time-Late Schedule Case \((\bar{t} < t + \bar{T}_Q(t) \leq t_q')\):

\[
\frac{dT_C(t)}{dt} = c_q \cdot \frac{dT_Q(t)}{dt} + c_{ext} \left( 1 + \frac{dT_Q(t)}{dt} \right) = 0
\]

\[
\frac{dT_Q(t)}{dt} = -\frac{c_{ext}}{c_q + c_{ext}}
\]  

(4) and (5) represent the slopes of the linear relationship between \(T_Q(t)\) and \(t\). Because \(c_q > c_w\), as shown in Figure 2, the positive relationship occurs in early arrival situation and negative when ships arrive late. From this relationship, the equilibrium queuing time length can be easily calculated. Take \(\bar{t}\) for example, \(T_Q(\bar{t})\) can be obtained as \((\bar{t} - t_q) \cdot -\frac{c_{ext}}{c_q + c_{ext}}\).

![Figure 2: The relationship between equilibrium queuing time length and arrival times](image)

Suppose \(T_s\) is defined as the average time of stay at the berth for \(N\) arrival container ships during the queuing period \([t_q', t_q]\), therefore we obtain

\[
t_q' - t_q = T_s \cdot (N - 1)
\]  

(6)

According to \(\bar{t} + T_Q(\bar{t}) = \bar{t}\), the following Equations can be developed based on Figure 2:

\[
\bar{t} + \frac{c_w}{c_q - c_w} \cdot (\bar{t} - t_q) = \bar{t}
\]  

(7)

\[
\bar{t} + \frac{-c_{ext}}{c_q + c_{ext}} \cdot (\bar{t} - t_q) = \bar{t}
\]  

(8)

The decision we are now facing is how to locate \(\bar{t}\), \(t_q\) and \(t_q'\) in equilibrium. By using (6) ~ (8), we obtained
\[ \tilde{t} = \tilde{t} - \frac{c_w \cdot c_{ext}}{c_w + c_{ext}} \cdot T_s \cdot (N - 1) \] (9)

\[ t_q = \tilde{t} - \frac{c_{ext}}{c_w + c_{ext}} \cdot T_s \cdot (N - 1) \] (10)

\[ t_q = \tilde{t} + \frac{c_w}{c_w + c_{ext}} \cdot T_s \cdot (N - 1) \] (11)

Since all container ships queuing during \([t_q, t_q']\] have the same cost in equilibrium, by substituting \(T_Q(\tilde{t}) = \tilde{t} - \tilde{t}\) and (9) into (1), or substituting \(T_Q(t_q) = 0\) and (10) into (2), or substituting \(T_Q(t_q') = 0\) and (11) into (3), the equilibrium total cost per container ship that resulted from queuing at the anchorage can be expressed as

\[ TC^e = \frac{c_w \cdot c_{ext}}{c_w + c_{ext}} \cdot T_s \cdot (N - 1) \] (12)

Next, let’s consider the toll scheme to a queuing port. The optimal non-queuing toll is defined as a series of tolls that will completely eliminate the loss of queuing times without making ship owners worse off than they would be in the non-toll equilibrium. In order to attain such an objective, it is necessary to impose a series of tolls, \(\Omega(t)\), that results in \(T_Q(t) = 0\) and \(TC(t) = TC^e\) for all \(t\) in (1), (2) and (3). Then we obtain a series of the optimal time varying toll, \(\Omega(t)\), listed in details below:

\[ \Omega(\tilde{t}) = TC^e = \frac{c_w \cdot c_{ext}}{c_w + c_{ext}} \cdot T_s \cdot (N - 1) \] (13)

\[ \Omega(t) = TC^e - c_w \cdot (\tilde{t} - t) \]
\[ = \frac{c_w \cdot c_{ext}}{c_w + c_{ext}} \cdot T_s \cdot (N - 1) - c_w \cdot (\tilde{t} - t) \] for \(t_q \leq t < \tilde{t}\) (14)

\[ \Omega(t) = TC^e - c_{ext} \cdot (t - \tilde{t}) \]
\[ = \frac{c_w \cdot c_{ext}}{c_w + c_{ext}} \cdot T_s \cdot (N - 1) - c_{ext} \cdot (\tilde{t} - t) \] for \(\tilde{t} < t \leq t_q'\) (15)

As shown in Figure 3, the shape of the optimal non-queuing toll scheme, \(\Omega(t)\), is triangular \(t_q bt_{q'}\) because of continuously changeable charges throughout the queuing period \([t_q, t_q']\). The maximum optimal non-queuing toll is located at \(\tilde{t} (=\ ETA)\). This is reasonable because ship owners are willing to pay the highest optimal non-queuing toll to arrive on time without incurring any early or late arrival costs. The other triangular \(t_q at_{q'}\) in Figure 3 represents the equilibrium queuing cost \(c_q \cdot T_Q(t)\). Because areas of two triangles are the same, the total optimal time varying toll completely replace all container ships’ equilibrium queuing costs.
3. THE OPTIMAL STEP TOLL SCHEME

The optimal non-queuing toll is capable of eliminating queuing time completely, but has practical difficulties because it requires continuously changeable charges. Therefore a step toll scheme has been considered as an alternative to reduce queuing time. The step toll inscribed in the optimal non-queuing toll triangle was first developed by Laih (1994) for reducing the queuing time to a desired level. A step toll structure with the maximum toll revenue inscribed in the optimal non-queuing toll triangle is defined as the optimal step toll scheme to remove the largest proportion of the total queuing time, and make ship owners no worse off than they would be in the non-toll equilibrium. As shown in Figure 3, the optimal step toll \( \delta = \frac{\Omega(\bar{t})}{2} \) is inscribed in the optimal non-queuing toll triangle \( t_q b t_q' \), and the revenue of this toll scheme is shaped as \( t^+ f' h' t^- \). Laih (1994) has shown that the optimal step toll divides the maximum optimal non-queuing toll into two equal amounts. He also has proved that the effect of the optimal step toll on queuing reduction is \( 1/2 \) of the total queuing time that existed in the non-toll equilibrium.

3.1 Equilibrium Queuing Costs

The optimal step toll, \( \delta = \frac{\Omega(\bar{t})}{2} \), inscribed within the optimal non-queuing toll \( t_q b t_q' \) in Figure 3, is applied at \( t^+ \) and lifted at \( t^- \). On the other hand, the triangle \( t_q a t_q' \) represents the non-toll equilibrium queuing time cost. Slopes of \( t_q a, a t_q', t_q b \) and \( bt_q' \) can be easily...
obtained as
\[ \frac{c_q \cdot c_w}{c_q - c_w}, \quad \frac{-c_q \cdot c_{ext}}{c_q + c_{ext}}, \quad c_w \quad \text{and} \quad -c_{ext}, \]
respectively from Figures 2 and 3 that have been proposed in Chapter 2. \( \hat{t} \) in Figure 3 means the arrival time which allows the ship to berth on schedule (ETA) under the optimal step toll scheme.

In Figure 3, \( t' \) and \( t^\# \) are two important time spots. \( t' \) is defined as the start time when no ship arrives at the anchorage until \( t^\# \) under the optimal step toll scheme. \( t^\# \) is defined as the time when some ships arrive at the anchorage but decide not to notify the port officer for berthing until \( t^- \) in order to avoid paying the toll. Let’s first interpret \( t' \). Because the queuing cost to the first ship that will pay the toll \( \delta \) at \( t^+ \) is zero, also because the arrival time at the berth to both the last untolled ship and the first tolled ship is almost the same, the last ship that will not pay the toll before \( t^+ \) must incur a queuing cost that is equal to the amount of the toll \( \delta \) for the equilibrium purpose. Consequently the last untolled ship must arrive at the general anchorage \( \frac{\delta}{c_q} \) earlier the first tolled ship. Hence there are no arrivals at the anchorage during the period \([t', t^+]\), and the length between \( t' \) and \( t^+ \) is equal to \( \frac{\delta}{c_q} \).

For the similar reason, because the queuing cost to the last ship that will pay the toll \( \delta \) just before \( t^- \) is zero, also because the arrival time at the berth to both the last tolled ship and the first no tolled ship is almost the same, the first ship that will not pay the toll after \( t^- \) must incur a queuing cost that is \( \delta \) higher than the last tolled ship for the equilibrium purpose. This is impossible unless the first untolled ship has queued for a period of \( \frac{\delta}{c_q} \) before \( t^- \). Therefore, there are some arrival ships possibly waiting at the anchorage from \( t^\# \) until \( t^- \) to avoid being tolled, and prepare to enter the berth free once the toll is lifted on \( t^- \). Consequently, the length of the time period \([t^\#, t^-] \) is also equal to \( \frac{\delta}{c_q} \).

Note that \( t_q \) is now assumed to locate on the origin (i.e., \( t_q = 0 \)) in Figure 3 for the purpose of simplifying computation to other arrival time values without losing the generality. Detailed computations to values of the toll and arrival times appeared in Figure 3 are shown as follows:

\[ \delta = \frac{TC^e}{2} = \frac{c_w \cdot c_{ext}}{2(c_w + c_{ext})} \cdot T_s \cdot (N - 1), \quad t_q = 0, \quad t_q = T_s \cdot (N - 1) \]

\[ \hat{t} = t_q + t^\# = t_q + \frac{TC^e}{c_q \cdot c_{ext}/(c_q - c_w)} = \frac{c_{ext}(c_q - c_w)}{c_q(c_w + c_{ext})} \cdot T_s \cdot (N - 1) \]

\[ \tilde{t} = \hat{t} + T_q(\hat{t}) = \tilde{t} + \frac{TC^e}{c_q \cdot c_{ext}} = \frac{c_{ext}}{c_q \cdot c_{ext}} \cdot T_s \cdot (N - 1) \]

\[ \tilde{t} - T_q(\tilde{t}) = t^* - \frac{\delta}{c_q} = \frac{c_{ext}(2c_q - c_w)}{2c_q(c_w + c_{ext})} \cdot T_s \cdot (N - 1) \]

\[ t^* = t_q + t^\# = t_q + \frac{\delta}{c_w} = \frac{c_{ext}}{2(c_w + c_{ext})} \cdot T_s \cdot (N - 1), \]

\[ t^t = t^t - \frac{\delta}{c_q} = \frac{c_{ext}(c_q - c_w)}{2c_q(c_w + c_{ext})} \cdot T_s \cdot (N - 1), \quad t^- = t_q - t^{-t} = t_q - \frac{\delta}{c_{ext}} = \frac{c_w + 2c_{ext}}{2(c_w + c_{ext})} \cdot T_s \cdot (N - 1), \]
\[
T^* = T - \frac{\delta}{c_q} = \frac{2c_q c_{ext} + c_q c_w - c_w c_{ext}}{2c_q(c_w + c_{ext})} \cdot T_e \cdot (N - 1).
\]

Table 1 illustrates equilibrium results for all arrival intervals under the optimal step toll scheme. Note that a blanket arrival time interval \([t', t')\) is existed because no ship arrives during this time period that mentioned above. Container ships of groups B, C and D arrive at the port during the tolled period \([t', t')\). Except group D escaping from being tolled, groups B and C pay the toll to enter the berth. Groups A and E do not need to pay the toll because they arrive during the no toll periods. In addition, only groups A and B will be alongside the berth earlier than ETA because they arrive at the anchorage before \(i\). These results are arranged as columns (I) ~ (III) in Table 1.

<table>
<thead>
<tr>
<th>Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
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<tr>
<td>Arrival Time Intervals at the Anchorage</td>
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<tr>
<td>(II)</td>
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<tr>
<td>Types of Arrivals at the Berth</td>
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<tr>
<td>Equilibrium queuing costs ((EQC)): (c_q \cdot T_Q(t))</td>
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<tr>
<td>(IV)</td>
</tr>
<tr>
<td>Equilibrium derivative costs due to queuing ((EDC)): (TC^* - EQC - \delta)</td>
</tr>
</tbody>
</table>

- **A**
  - \(t_q \leq t < t'\) (No Toll Period)
  - (a). Toll Free
  - (b). Early Arrivals
  - \(EQC = \frac{c_q \cdot c_w \cdot t}{c_q - c_w} \cdot T_e \cdot (N - 1)\)
  - \(EDC = \frac{c_q \cdot c_w \cdot (c_w + c_{ext})}{2(c_q + c_{ext})(c_q - c_w)} \cdot T_e \cdot (N - 1)\)

- **B**
  - \(t' \leq t < \hat{t}\) (Tolled Period)
  - (a). Toll Payers
  - (b). Early Arrivals
  - \(EQC = \frac{c_q \cdot c_w \cdot t}{c_q - c_w} \cdot T_e \cdot (N - 1)\)
  - \(EDC = \frac{c_q \cdot c_w \cdot (c_w + c_{ext})}{2(c_q + c_{ext})(c_q - c_w)} \cdot T_e \cdot (N - 1)\)

- **C**
  - \(\hat{t} < t < t'\) (Tolled Period)
  - (a). Toll Payers
  - (b). Late Arrivals
  - \(EQC = \frac{c_q \cdot c_w \cdot t}{c_q - c_w} \cdot T_e \cdot (N - 1)\)
  - \(EDC = \frac{c_q \cdot c_w \cdot (c_w + c_{ext})}{2(c_q + c_{ext})(c_q - c_w)} \cdot T_e \cdot (N - 1)\)

- **D**
  - \(t^* \leq t < t'\) (Tolled Period)
  - (a). Toll Free
  - (b). Late Arrivals
  - \(EQC = \frac{c_q \cdot c_w \cdot t}{c_q + c_{ext}} \cdot T_e \cdot (N - 1)\)
  - \(EDC = \frac{c_q \cdot c_w \cdot (c_w + c_{ext})}{2(c_q + c_{ext})(c_q - c_w)} \cdot T_e \cdot (N - 1)\)

- **E**
  - \(t' \leq t \leq t_q\) (No Toll Period)
  - (a). Toll Free
  - (b). Late Arrivals
  - \(EQC = \frac{c_q \cdot c_w \cdot t}{c_q + c_{ext}} \cdot T_e \cdot (N - 1)\)
  - \(EDC = \frac{c_q \cdot c_w \cdot (c_w + c_{ext})}{2(c_q + c_{ext})(c_q - c_w)} \cdot T_e \cdot (N - 1)\)

Since equilibrium will be achieved as long as all container ships have the same total cost throughout the queuing period, there are two kinds of equilibrium conditions for the early and late arrivals. One is \(TC(t) = TC(t_q)\) for groups A and B of the early arrival, and the other is \(TC(t) = TC(t_q)\) for groups C, D and E of the late arrival. These equilibrium conditions then
can be expressed as follows:

\[ c_q \cdot T_Q(t) + c_w \cdot \left[ t - (t + T_Q(t)) \right] = c_w \cdot \bar{t}, \quad \text{for } t_q \leq t \leq t' \]  \hspace{1cm} (16)

\[ c_q \cdot T_Q(t) + c_w \cdot \left[ t - (t + T_Q(t)) \right] + \delta = c_w \cdot \bar{t}, \quad \text{for } t^* \leq t < \hat{t} \]  \hspace{1cm} (17)

\[ c_q \cdot T_Q(t) + c_{ext} \cdot \left[ t + T_Q(t) - \bar{t} \right] + \delta = c_{ext} (t_q - \bar{t}), \quad \text{for } \hat{t} < t < t^- \]  \hspace{1cm} (18)

\[ c_q \cdot T_Q(t) + c_{ext} \cdot \left[ t + T_Q(t) - \bar{t} \right] = c_{ext} (t_q - \bar{t}), \quad \text{for } t^* \leq t < t^- \quad \text{or} \quad t^- \leq t \leq t_q \]  \hspace{1cm} (19)

The above equilibrium conditions (16) ~ (19) are established to groups A, B, C and D (or E), respectively. The values of \( TC(t_q) \) and \( TC(t^*_q) \) are \( c_w \cdot \bar{t} \) and \( c_{ext} (t_q - \bar{t}) \), respectively, because \( t_q = 0 \) and \( T_Q(t_q) = T_Q(t_q^*) = 0 \).

Equilibrium queuing costs (\( EQC : c_q \cdot T_Q(t) \)), to groups A~E, listed in column (IV) of Table 1 are obtained based on (16) ~ (19). As shown in Figure 3, the equilibrium queuing costs to groups A~E under the optimal step toll scheme are green lines \( \bar{t}_q, \bar{t}'_q, \bar{t}^-_q, \bar{t}^+_q \) and \( \bar{t}_q^* \), respectively. The slope of \( \bar{t}_q^* \) and \( \bar{t}'_q \) for all early arrivals is \( \frac{c_q \cdot c_w}{c_q - c_w} \), which is the same as the slope of the equilibrium queuing cost \( \bar{t}_q^* \) to all early arrivals in the non-toll case. Note that there is no green lines of equilibrium queuing costs through the arrival period \([t', t^+]\) since no ship arrives during this period. In addition, the length of the queue will be reduced to zero at \( t^+ \) because of \( T(t') = \delta / c_q = t^+ - t' \). On the other hand, the slope of \( \bar{t}'^-_q \), \( \bar{t}_h \) and \( \bar{t}_q^- \) for all late arrivals is \( \frac{-c_q \cdot c_w}{c_q + c_{ext}} \), which is the same as the slope of the equilibrium queuing cost \( \bar{a}_q^- \) to all late arrivals in the non-toll case.

In Figure 3, the total equilibrium queuing cost in the original non-toll case is \( \Delta t_q \bar{a}_q^* \), and the total equilibrium queuing cost under the optimal step toll scheme is composed of \( \Delta t_q \bar{a}_q^* \), \( \Delta t^*_q \bar{g}^- \) and \( \Delta t^*_q \bar{h}_q^- \). Because \( \Delta t^*_q \bar{g}^- \) can be moved to \( \Delta fah \) due to the same area, it is then clear that the effect of the optimal step toll on queuing reduction is just half of the total queuing cost in the non-toll equilibrium.

Incorporating ICT (and/or ITS) in the system at ports is already an acceptable factor in enhancing LOS at ports. If the port authority implement the optimal step toll scheme with ICT (and/or ITS) system, it can be expected that both the ship owner and port bureau will be more adaptable.

3.2 Decisions of Arrival Time Change

Because the optimal step toll derived from our model is simply the money cost to the toll payers who require to save the same amount of queuing costs, the equilibrium derivative costs due to queuing (EDC) in the tolled case must be the same as that in the original non-toll case to maintain the equilibrium cost, \( TC^e = c_q T_Q(\bar{t}) = \Omega(\bar{t}) \). For this purpose, decisions of arrival
time change to all container ships can be investigated by "the invariant equilibrium derivative cost principle".

Since the results of equilibrium queuing costs (EQC) to all arrival intervals under the optimal step toll scheme have been shown in column (IV) of Table 1, the corresponding values of equilibrium derivative costs due to queuing (EDC) required to achieve the equilibrium cost $TC^*$ can be easily obtained by $TC^* = EQC + \delta$, as shown in the same column. EDC for groups A~E under the optimal step toll scheme are drawn as the red lines $\bar{cf}$, $\bar{f}\hat{t}$, $\bar{h}\hat{t}$, $\bar{he}$ and $\bar{ed}$, respectively in Figure 3. The slope of $\bar{cf}$ and $\bar{f}\hat{t}$ to all early arrivals is identical and equal to $-\frac{c_q \cdot c_w}{c_q - c_w}$. This is the same as the slope of $\bar{ct}$ that represents all early arrivals’ EDC in the non-toll case. On the other hand, the slope of $\bar{h}\hat{t}$, $\bar{he}$ and $\bar{ed}$ to all late arrivals is identical and equal to $\frac{c_q \cdot c_{ext}}{c_q + c_{ext}}$. This is also the same as the slope of $\bar{td}$ that represents all late arrivals’ EDC in the non-toll case.

Next, the detailed discussion of all container ships’ decisions to change arrival time will be made as follows. Firstly, group A ships will not change their original arrival times in the non-toll case when the port is priced with the optimal step toll, because the equilibrium derivative cost $\bar{cf}$ in both the non-toll and optimal step toll cases coincide during the arrival period $(t_q, t')$. Secondly, because the equilibrium derivative cost $\bar{f}\hat{t}$ in the optimal step toll case and the equilibrium derivative cost $\bar{f}\tilde{t}$ in the non-toll case are two identical and parallel lines, all group B ships that originally arrive during the period $(t',\tilde{t})$ in the non-toll case will change their arrivals to the period $(\hat{t}, t')$ in the optimal step toll case. Similarly, because the equilibrium derivative cost $\bar{h}\hat{t}$ in the optimal step toll case and the equilibrium late cost $\bar{h}t$ in the non-toll case are two identical and parallel lines, all group C ships that originally arrive during the period $[\tilde{t}, t'']$ in the non-toll case will change their arrivals to the period $[\hat{t}, t']$ in the optimal step toll case. Thirdly, because the equilibrium derivative cost $\bar{he}$ in both the non-toll and optimal step toll cases coincide during the arrival time period $[t^*, t^-]$, group D ships will not change their original arrival times in the non-toll case if the port is priced with the optimal step toll. Since $[t^*, t^-]$ exists within $[\hat{t}, t^-]$, group C and D ships arrive simultaneously during $[t^*, t^-]$. Finally, because the equilibrium derivative cost $\bar{ed}$ in both the non-toll and optimal step toll cases coincide during the arrival time period $[t^*, t_q]$, group E ships will not change their original arrival times in the non-toll case if the port is priced with the optimal step toll.

The above outcomes are listed in Table 2. It is clear that container ships choose the same arrival times at the anchorage as they did in the original non-toll case are not the toll payers in the tolled case. These ships are Type I early arrivals as well as Type IV late arrivals in Table 2. The other part of Type II early arrivals and type III late arrivals that change their original arrival times at the anchorage are the toll payers.
Table 2 Arrival time changes under the optimal step toll scheme

<table>
<thead>
<tr>
<th>Types</th>
<th>Change Arrival times or not</th>
<th>Toll Payers or not</th>
<th>Early or Late Arrivals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I Ships</td>
<td>No, Still Arriving at $[t_q, t'_q]$</td>
<td>No</td>
<td>Early</td>
</tr>
<tr>
<td>Type II Ships</td>
<td>Yes, $[t', t_q) \rightarrow [t^*, t)$</td>
<td>Yes</td>
<td>Early</td>
</tr>
<tr>
<td>Type III Ships</td>
<td>Yes, $[t^<em>, t^</em>_q) \rightarrow [t', t)$</td>
<td>Yes</td>
<td>Late</td>
</tr>
<tr>
<td>Type IV Ships</td>
<td>No, Still Arriving at $[t^*, t_q)$</td>
<td>No</td>
<td>Late</td>
</tr>
</tbody>
</table>

4. A NUMERICAL EXAMPLE

The methodology employed in our model does not have a limit to any port. In other words, our methodology is applicable to all busy ports in the world. There is a numerical example for explaining our model. According to the statistical data of Keelung port in Taiwan, container ships in gross tonnage between 10,000 and 19,999 are always the most part of the total container ships visiting Keelung port per year. Therefore, the scenario of this case describes the ships of the above category arriving at Keelung port.

The wharfage ($c_w$) based on ship gross tonnage is charged to about US$66 per ship per hour for the category in accordance with the rate of Keelung port charge. The queuing time cost includes personnel expense of ship crew, fresh water fee, fuel oil fee, ship depreciation cost, insurance fee, etc. The fixed cost of a ship in gross tonnage between 10,000 and 19,999 is about US$3600 every day and listed in Table 3. Consequently the queuing time cost ($c_q$) is US$150 per hour.

Table 3 The distribution of ship fixed cost every day

<table>
<thead>
<tr>
<th>Items</th>
<th>Cost ($USD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crew salaries (including salaries, meals, welfare, etc.)</td>
<td>500</td>
</tr>
<tr>
<td>Maintenance (including annual survey, repairing, maintaining, etc.)</td>
<td>320</td>
</tr>
<tr>
<td>Insurance (including hull insurance and P&amp;I)</td>
<td>210</td>
</tr>
<tr>
<td>Spare parts</td>
<td>100</td>
</tr>
<tr>
<td>Fresh water</td>
<td>30</td>
</tr>
<tr>
<td>Lubricator</td>
<td>120</td>
</tr>
<tr>
<td>Ship depreciation</td>
<td>1,400</td>
</tr>
<tr>
<td>Fuel oil consumption in port</td>
<td>620</td>
</tr>
<tr>
<td>Management</td>
<td>200</td>
</tr>
<tr>
<td>Other fixed cost</td>
<td>100</td>
</tr>
<tr>
<td>Total</td>
<td>3,600</td>
</tr>
</tbody>
</table>

The average operation fee for loading/unloading a 20 feet container is about US$55 in Keelung port. The loading/unloading rate of one working gang is about 27 containers per hour. The standard operating time cost is assumed to US$2,970 per hour under two working gangs. The extra operating time cost ($c_{ex}$) is assumed to 1.5 times of the standard operating time cost, about US$4,455 per hour, in this case due to additional work for longshoremen and equipment.

According to the statistical records of Keelung port from 2003 until 2008, the average berthing time ($T_b$) in the port of all container ships in gross tonnage between 10,000 and
19,999 is approximately 10 hours.

The number of container ships \( N \) waiting in the anchorage is assumed to 5 ships. Then we have the following results to develop Table 4: 

\[
\delta = \frac{TC}{2} = \frac{c_w \cdot c_{ext}}{2(c_{w} + c_{ext})} \cdot T_s \cdot (N - 1) = \text{US$1300.73} \; ;
\]

\[
\tilde{t} = \frac{c_{ext}(c_q - c_w)}{c_q(c_{w} + c_{ext})} \cdot T_s \cdot (N - 1) = 22.07hr \; ;
\]

\[
\tilde{t} = \frac{c_{ext}(2c_q - c_w)}{2c_q(c_{w} + c_{ext})} \cdot T_s \cdot (N - 1) = 30.75hr \; ;
\]

\[
t' = \frac{c_{ext}(c_q - c_w)}{2c_q(c_{w} + c_{ext})} \cdot T_s \cdot (N - 1) = 11.04hr \; ;
\]

\[
t^* = \frac{c_w + 2c_{ext}}{2(c_{w} + c_{ext})} \cdot T_s \cdot (N - 1) = 19.71hr \; ;
\]

\[
t^* = \frac{2c_q c_{ext} + c_q c_w - c_w c_{ext}}{2c_q(c_{w} + c_{ext})} \cdot T_s \cdot (N - 1) = 31.04hr .
\]

### Table 4 Numerical results under the optimal step toll scheme to container ships

<table>
<thead>
<tr>
<th>The time of container ships arriving at the anchorage before the toll established.</th>
<th>The time of container ships arriving at the anchorage after the toll established.</th>
<th>Actions of the ships after the toll established.</th>
</tr>
</thead>
<tbody>
<tr>
<td>([t_q, t'])</td>
<td>([t_q, t']) no toll period</td>
<td>(1) Maintain the same arrival time at the anchorage.</td>
</tr>
<tr>
<td>00 : 00 (the first day) ~ 11 : 02 (the first day)</td>
<td>00 : 00 (the first day) ~ 11 : 02 (the first day)</td>
<td>(2) Arrival time at the berth is earlier than ETA.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3) No need to pay the toll.</td>
</tr>
<tr>
<td>([t', \tilde{t}])</td>
<td>([t', \tilde{t}]) toll period</td>
<td>(1) Changing the arrival time at the anchorage.</td>
</tr>
<tr>
<td>11 : 02 (the first day) ~ 22 : 04 (the first day)</td>
<td>19 : 42 (the first day) ~ 06 : 45 (the next day)</td>
<td>(2) Arrival time at the berth is earlier than ETA.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3) Need to pay the toll.</td>
</tr>
<tr>
<td>([\tilde{t}, \tilde{t}^*])</td>
<td>([\tilde{t}, \tilde{t}^*]) toll period</td>
<td>(1) Changing the arrival time at the anchorage.</td>
</tr>
<tr>
<td>22 : 04 (the first day) ~ 07 : 02 (the next day)</td>
<td>06 : 45 (the next day) ~ 15 : 42 (the next day)</td>
<td>(2) Arrival time at the berth is later than ETA.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3) Need to pay the toll.</td>
</tr>
<tr>
<td>([t^*, t^-])</td>
<td>([t^*, t^-]) toll period</td>
<td>(1) Maintain the same arrival time at the anchorage.</td>
</tr>
<tr>
<td>07 : 02 (the next day) ~ 15 : 42 (the next day)</td>
<td>07 : 02 (the next day) ~ 15 : 42 (the next day)</td>
<td>(2) Arrival time at the berth is later than ETA.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3) No need to pay the toll (avoiding).</td>
</tr>
<tr>
<td>([t^-, t'q])</td>
<td>([t^-, t'q]) no toll period</td>
<td>(1) Maintain the same arrival time at the anchorage.</td>
</tr>
<tr>
<td>15 : 42 (the next day) ~ 16 : 00 (the next day)</td>
<td>15 : 42 (the next day) ~ 16 : 00 (the next day)</td>
<td>(2) Arrival time at the berth is later than ETA.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3) No need to pay the toll.</td>
</tr>
</tbody>
</table>
The first ship arriving at the anchorage for waiting berth is the start of queuing, and the time is assumed at 00:00 of the first day. After 40 hours, the last ship enters the port to berth means the end of queuing, and the time is at 16:00 of the next day. The optimal step toll is US$1,300.73 per ship. The time of starting to pay the toll is at 19:42 of the first day, 19.71 hours after the queuing start time. The time of ending the toll is at 15:42 of the second day, 39.71 hours after the queuing start time. The total time interval of paying the toll is 20 hours, just half of the total queuing time period. Based on the conclusion that we have obtained above, the effect of the optimal step toll on queuing reduction is just half of the total queuing time in the non-toll situation. In case the optimal step toll is established, the results of container ships’ decisions of arrival time change are listed in Table 4.

5. PRACTICAL IMPLICATIONS AND CONCLUSIONS

In this paper, an optimal step toll scheme to container ships is designed for a queuing port. All values of equilibrium queuing costs, equilibrium derivative costs due to queuing, and in patterns of arrival time change under the step toll scheme are obtained. With this toll scheme, arrival times of container ships at the queuing port will be rationally dispersed. Consequently, the queuing time at the anchorage can be decreased. This paper also derived the consequent changes of container ships’ arrival schedules at the queuing port after collecting the optimal step toll. Decisions of changing the arrival time from the non-toll to the tolled cases can be predicted before a queuing port establishes the toll scheme. All of these are important issues to the port bureau for implementation of the optimal step toll scheme. Also, they are issues that have not been discussed previously.

Conclusions obtained from the paper and their practical implications are illustrated as follows:

1. A complete optimal step toll structure is established. All values of equilibrium queuing cost and derivative costs due to queuing under the optimal step toll scheme are derived. These results are useful for authorities to make decisions and to handle the scheme easily.
2. Whether ship owners change ship arrival time will depend on the equilibrium derivative cost. Ship owners who choose the same arrival times at the port as they did in the original non-toll case are not the toll payers in the tolled case. These ships are Type I early arrivals and Type IV late arrivals. The other part of container ships, i.e., Type II early arrivals and type III late arrivals that change their arrival times are the toll payers in the tolled case.
3. According to the above results, the general characteristics to container ships whether pay the toll or not become predictable. For example, container ships neither change their original arrival times nor pay the toll at a queuing port can be investigated by surveying Type I and Type IV ships that arrive during the early arrival period \( [t_q, t'] \) and the late arrival period \( [t', t''] \), respectively, before the toll established. On the other hand, ships of the other groups that are both the arrival time changer and the toll payer can be investigated by surveying Types II and Type III ships that arrive during the early arrival period \( [t', \tilde{t}] \) and the late arrival period \( [\tilde{t}, t''] \), respectively.

If the port establishes the optimal toll scheme, all container ships may rationally adjust sailing speed and time to save the cost. Consequently container ships’ arrival times at the port will be dispersed, and the queuing situation can be decreased. This result certainly improves the level of service of intermodal logistics network in the context of sub-regional transport cooperation.
REFERENCES