Queuing Pricings to Bulk Carriers at the Anchorage

Chen-Hsiu LAIH
Professor
Department of Merchant Marine
National Taiwan Ocean University
2, Pei-Ning Rd, Keelung
204-47 TAIWAN
Fax: +81-3-2431-1270
E-mail: chlaih@mail.ntou.edu.tw

Pey-Yuan SUN
Graduate Student
Department of Merchant Marine
National Taiwan Ocean University
2, Pei-Ning Rd, Keelung
204-47 TAIWAN
Fax: +81-3-5841-8506
E-mail: M97710003@ntou.edu.tw

Abstract: This paper develops the optimal step toll scheme which is levied to bulk carriers for a queuing port. Bulk carriers’ arrival times at the port will be rationally dispersed after pricing this toll scheme. Consequently, the queuing time at the anchorage to all bulk carriers will be rationally decreased. This paper also shows bulk carrier owners’ decisions of arrival time changes under the optimal step toll scheme. Based on these results, we find some bulk carriers that paid no toll under the optimal step toll scheme maintain the same arrival times at the anchorage as they did in the original non-toll equilibrium situation. New arrival times at the anchorage for other bulk carriers that paid the optimal step toll are postponed when compared with their original arrival times in the non-toll equilibrium situation.

Key Words: Optimal step toll scheme, bulk carriers, queuing

1. INTRODUCTION

When a bulk carrier arrives at a queuing port, the port officers will guide her to wait at the anchorage. When a bulk carrier berth becomes vacant and available, a pilot will shepherd her to the berth where it unloads and loads. Generally speaking, this situation is similar to auto-vehicles queuing at a road bottleneck especially in the morning rush hour. In order to make the efficient use of the bulk carrier berth, this paper develops a series of pricing schemes to bulk carriers at a queuing port.

A model of pricing a queuing bottleneck was initially developed by Vickrey (1969) and extended by Small (1982), De Palma and Arnott (1986), Cohen (1987), Braid (1989), Arnott et al (1990, 1993), Tabuchi (1993), Laih (1994, 1997, 2004), Yang and Meng (1998). Among these researches, Laih (1994) first developed a flexible step pricing mechanism to relieve commuting queuing at a road bottleneck. Laih (2004) also provided a methodological framework forecasting the behavior changes of commuters from a non-toll case to the optimal single-and multi-step toll cases. Applying these considerations, the optimal step toll scheme to bulk carriers queuing at the anchorage is derived in this paper. With the toll scheme, arrival times of bulk carriers will be rationally dispersed. Consequently, the queuing time at the anchorage can be decreased. This paper also derives the consequent changes of bulk carriers’ arrival schedules after collecting the optimal step toll. Decisions of changing the arrival time from the non-toll to the tolled cases can be predicted before a queuing port establishes the toll scheme. Port queuing pricing leads to the efficient use of berths especially during heavily congested periods. All of these are important issues for bulk carrier owners and port bureaus if the queuing pricing policy is considered by authorities.
Some literatures related to price a queuing bottleneck with the optimal non-queuing toll and the optimal step toll schemes are reviewed as follows. Laih (1994) looked into the model of charging for queuing road bottleneck and used the static equilibrium analytical method to develop a series of optimal and sub-optimal step toll schemes. This development provided decision makers a set of collecting toll framework with flexibility when they tried to minimize negative effects of congestion at a bottleneck. Furthermore, Laih (1997) proved that the entire queuing time of all auto-commuters could be effectively shortened to 1/2 and 2/3 of the original by collecting the optimal single- and double-step tolls, respectively. This study only displayed an equilibrium outcome of dispersing auto-commuters’ departure rates after the optimal single- and double-step toll schemes applied. However, it was not known clearly about the changes of departure behaviors of auto-commuters from the non-toll to the tolled cases, which led to reduce queuing at a bottleneck. Laih (2004) expanded the analysis of the optimal single- and double- steps to \( n \)th number \((n = 1, 2, 3, \ldots)\) of charging steps. It was realized that when the charging steps increased one by one after detailed derivation, the framework of the optimal step toll, the related equilibrium costs, the equilibrium departure rates and moving tracks of departure time of auto-commuters had all shown regular variation.

These complete and regular information not only facilitate policy makers to apply the optimal step toll scheme, but it can also be used to predict the entire auto-commuters’ behavior in the system of toll collection.

This paper is structured as follows. The basic model to derive the equilibrium cost and the optimal non-queuing toll scheme to bulk carriers at a queuing port are developed in Chapter 2. Based on the basic model, Chapter 3 derives the optimal step toll scheme, inscribed within the optimal non-queuing toll scheme, to bulk carriers for practical purposes. Besides, a framework used to predict bulk carrier owners’ decisions of arrival time changes from the non-toll to the tolled cases are established in Chapter 3. Chapter 4 provides a numerical example to explain the frameworks mentioned in Chapter 3. Finally, the main results obtained in this paper are addressed in Chapter 5.

2. THE BASIC MODEL

Basic assumptions for a queuing port model are as follows. Firstly, the assumed background in this paper is that arrived bulk carriers have to queue at the anchorage until a vacant berth for cargo loading/unloading becomes available. This may be a result of increase in demand or supply of bulk cargos attracting more ships calling the port. Secondly, the sequence of entering the berth at a queuing port follows the principle of first come first serviced. Thirdly, except this queuing port, there are no other alternative ports existed. Fourthly, the cost function to bulk carriers in the model includes the queuing costs at the anchorage and the derivative costs of early or late arrival at the berth due to queuing.

In Figure 1, There are three kinds of departure patterns to all bulk carriers: on-time departure \((\bar{t} + T_q(\bar{t}) + T_w = t^*)\), early departure \((t + T_q(t) + T_w < t^*)\) and late departure \((t + T_q(t) + T_w > t^*)\). Among these three situations, \(\bar{t}\) and \(t^*\) are Estimate Time of Arrival (ETA) and Estimate Time of Departure (ETD), respectively, to all bulk carriers. \(\bar{t}\) is defined as a bulk carrier’s arrival time at the anchorage which allows the berthing time just the same as ETA after queuing. \(t\) is the time point when a bulk carrier arrives at the anchorage of the port. \(T_q(t)\) is the length of queuing time period at the anchorage and varies in accordance with \(t\). For simplicity, \(T_q(t)\) is assumed to be a linear function. \(T_w = t^* - \bar{t}\) is the average (fixed) time
length that each bulk carrier staying at a berth for loading/unloading operation for all cases in Figure 1. $T_E(t) = t^* - (t + T_Q(t) + T_w)$ and $T_L(t) = (t + T_Q(t) + T_w) - t^*$ in Figure 1 are defined as the time lengths of early departure period and late departure period, respectively.

On-time Departure: $T_Q(t)$

Early Departure: $t + T_Q(t) + T_w < t^*$

Late Departure: $t + T_Q(t) + T_w > t^*$

According to Figure 1, we obtain the total cost ($TC(t)$) that resulted from queuing at the anchorage to all bulk carriers:

**On-time Departure (ETD on time):**

$$TC(\bar{t}) = c_q \cdot T_Q(\bar{t}), \quad \bar{t} + T_Q(\bar{t}) + T_w = t^* \quad \text{or} \quad \bar{t} + T_Q(\bar{t}) = \bar{t}$$

**Early Departure (Earlier than ETD):**

$$TC(t) = c_q \cdot T_Q(t) + c_{dis} \cdot T_E(t)$$

$$= c_q \cdot T_Q(t) + c_{dis} \cdot \left[ \bar{t} - (t + T_Q(t) - T_w) \right], \quad t_q \leq t + T_Q(t) + T_w < t^* \quad \text{or} \quad t_q \leq t + T_Q(t) < \bar{t}$$

**Late Departure (Later than ETD):**

$$TC(t) = c_q \cdot T_Q(t) + c_{pen} \cdot T_L(t)$$

$$= c_q \cdot T_Q(t) + c_{pen} \cdot \left[ t + T_Q(t) + T_w - t^* \right], \quad t^* < t + T_Q(t) + T_w \leq t^*_q \quad \text{or} \quad \bar{t} < t + T_Q(t) \leq t^*_q$$

$$= c_q \cdot T_Q(t) + c_{pen} \cdot \left[ (t + T_Q(t)) - \bar{t} \right]$$

Figure 1 Bulk carriers’ departure patterns at a queuing port
In (1)~(3), \( c_q \) represents the time cost per hour to \( T_Q(t) \), then \( c_q \cdot T_Q(t) \) means the queuing time cost, which consists of personnel expense, depreciation cost of the ship, expense for repairing, insurance fee, interests, petrol fee for maintenance and desalination fee. These expenses are indispensable while ships queue at the anchorage during the queuing period \([t_q, t_{q'}]\). Where \( t_q \) and \( t_{q'} \) represent the start and the end times of queuing at the anchorage, respectively. \( c_{dis} \) and \( c_{pen} \) in (2) and (3) represent the time cost per hour to \( T_E \) and \( T_L \), respectively. In tramp shipping, the ship owner should pay the dispatch fee to the consignor if the ship’s departure time becomes earlier than ETD due to the consignor’s prompt arrangements for cargos being loaded/unloaded. For the early departure case in Figure 1, the derivative cost of early arrival at the berth due to queuing is the dispatch fee \( c_{dis} \cdot \left[ \tau - (t + T_Q(t)) \right] \). On the other hand, the ship owner has to suffer a penalty cost or revenue loss on delaying the next voyage if the ship’s departure time becomes later than ETD. For the late departure case in Figure 1, the derivative cost of late arrival at the berth due to queuing is the penalty cost or revenue loss \( c_{pen} \cdot \left[ (t + T_Q(t)) - \tau \right] \).

Equilibrium obtains when no individual bulk carrier has an incentive to change the arrival time, \( t \). This implies that the total cost \( TC(t) \) to each bulk carrier must be the same at all times during the queuing period \([t_q, t_{q'}]\). In other words, the equilibrium condition is \( dTC/dt = 0 \). For this purpose, differentiating (2) and (3) with \( t \), then we obtain (4) and (5), respectively, as follows:

\[
\frac{dTC(t)}{dt} = c_q \cdot \frac{dT_Q(t)}{dt} + c_{dis} \cdot \left[ -1 - \frac{dT_Q(t)}{dt} \right] = 0
\]

\[
\frac{dT_Q(t)}{dt} = \frac{c_{dis}}{c_q - c_{dis}}, \quad t_q \leq t + T_Q(t) < \bar{\tau} \tag{4}
\]

\[
\frac{dTC(t)}{dt} = c_q \cdot \frac{dT_Q(t)}{dt} + c_{pen} \cdot \left[ 1 + \frac{dT_Q(t)}{dt} \right] = 0
\]

\[
\frac{dT_Q(t)}{dt} = \frac{-c_{pen}}{c_q + c_{pen}}, \quad \bar{\tau} < t + T_Q(t) \leq t_{q'} \tag{5}
\]

Because \((c_q - c_{dis})\) in (4) is positive in practice, (4) and (5) represent the positive and negative slopes of the linear relationship between \( T_Q(t) \) and \( t \) in early and late departure situations, respectively. Using (4) and (5), the equilibrium queuing time length can be easily calculated. Take \( \bar{\tau} \) in Figure 2 for example, \( T_Q(\bar{\tau}) \) can be obtained as \( (\bar{\tau} - t_q) \cdot \frac{-c_{pen}}{c_q + c_{pen}} \).

Following the definition of \( \bar{\tau} \), we have \( \bar{\tau} + T_Q(\bar{\tau}) = \bar{\tau} \), then

\[
\bar{\tau} + \frac{c_{dis}}{c_q - c_{dis}} \cdot (\bar{\tau} - t_q) = \bar{\tau} \tag{6}
\]

\[
\bar{\tau} + \frac{-c_{pen}}{c_q + c_{pen}} \cdot (\bar{\tau} - t_q) = \bar{\tau} \tag{7}
\]
Next, since $T_w$ is defined as the average (fixed) operation time length of the total $N$ arrived bulk carriers during the queuing period, the queuing time length for the queuing period $[t_q', t_q]$ can be obtained as

$$t_q' - t_q = T_w \cdot (N - 1) \tag{8}$$

Solving (6)–(8), we then obtain three arrival time values $\tilde{t}$, $t_q$ and $t_q'$ as follows:

$$\tilde{t} = \bar{t} - \frac{c_{dis} \cdot c_{pen}}{c_q (c_{dis} + c_{pen})} \cdot T_w \cdot (N - 1) \tag{9}$$

$$t_q = \bar{t} - \frac{c_{pen}}{c_{dis} + c_{pen}} \cdot T_w \cdot (N - 1) \tag{10}$$

$$t_q' = \bar{t} + \frac{c_{dis}}{c_{dis} + c_{pen}} \cdot T_w \cdot (N - 1) \tag{11}$$

Because all bulk carriers have the same cost in equilibrium, by substituting $T_Q(\bar{t}) = \bar{t} - \tilde{t}$ and (9) into (1), or substituting $T_Q(t_q) = 0$ and (10) into (2), or substituting $T_Q(t_q') = 0$ and (10) into (3), the equilibrium total cost per bulk carrier that resulted from queuing at the anchorage can be expressed as

$$TC^e = \frac{c_{dis} \cdot c_{pen}}{c_{dis} + c_{pen}} \cdot T_w \cdot (N - 1) \tag{12}$$

$\triangle t_q \cdot at_q'$ in Figure 3 represents the queuing cost, $\alpha \cdot T_Q(t)$, in the non-toll equilibrium. Based on (4) and (5), slopes of $\frac{t_q}{a}$ and $\frac{at_q}{a}$ can be easily obtained as $\frac{c_q \cdot c_{dis}}{c_q - c_{dis}}$ and $\frac{-c_q \cdot c_{pen}}{c_q + c_{pen}}$, respectively.
Next we consider the optimal non-queuing toll scheme to bulk carriers for a queuing port. The optimal non-queuing toll is defined as a series of tolls that will completely eliminate the loss of queuing times without making ship owners worse off than they would be in the non-toll equilibrium. In order to attain such an objective, it is necessary to impose a series of tolls, \( \psi(t) \), that results in \( T_C(t) = T_C^e \) for all \( t \) in (1), (2) and (3). Then we obtain a series of the optimal time varying toll as

\[
\psi(t) = T_C^e - c_{\text{dis}} \cdot \left( t - \bar{t} \right) = \frac{c_{\text{dis}} \cdot c_{\text{pen}}}{c_{\text{dis}} + c_{\text{pen}}} \cdot T_w \cdot (N - 1) - c_{\text{dis}} \cdot \left( \bar{t} - t \right), \quad t_q \leq t \leq \bar{t}
\]

\[
\psi(t) = T_C^e - c_{\text{pen}} \cdot \left( t - \bar{t} \right) = \frac{c_{\text{dis}} \cdot c_{\text{pen}}}{c_{\text{dis}} + c_{\text{pen}}} \cdot T_w \cdot (N - 1) - c_{\text{pen}} \cdot \left( t - \bar{t} \right), \quad \bar{t} < t \leq t_q
\]

The shape of the optimal non-queuing toll scheme, \( \psi(t) \), is shown as \( \triangle t_q b t_q' \) in Figure 3. Based on (14) and (15), slopes of \( b t_q \) and \( b t_q' \) are \( c_{\text{dis}} \) and \( -c_{\text{pen}} \), respectively. The optimal non-queuing toll scheme has continuously changeable charges throughout the queuing period [\( t_q, t_q' \)]. The maximum optimal non-queuing toll is located at \( \bar{t} \) (= ETA) as shown in (13). This is reasonable because ship owners are willing to pay the highest optimal non-queuing toll to match the on-time schedule (ETA and ETD) without incurring any early or late costs. Since the two triangles \( \triangle t_q a t_q' \) and \( \triangle t_q b t_q' \) in Figure 3 have the same area, the total optimal non-queuing toll completely replace all bulk carriers’ queuing costs at the anchorage in the non-toll equilibrium.

![Figure 3 Equilibrium costs and optimal tolls to bulk carriers queuing at the anchorage](image)
3. EQUILIBRIUM RESULTS UNDER THE OPTIMAL STEP TOLL SCHEME

The optimal non-queuing toll is capable of eliminating queuing time completely, but has practical difficulties because it requires continuously changeable charges. Therefore a step toll scheme, which is first developed by Laih (1994), has been considered as an alternative to reduce queuing time. The step toll scheme, inscribed in the optimal non-queuing toll triangle, is designed to make the toll payers no worse off than they would be in the non-toll equilibrium. As shown in Figure 3, the optimal step toll, \( \rho = \psi(\bar{t}) / 2 \), inscribed within the optimal non-queuing toll, \( \psi' h t', \) is applied at \( t^+ \) and lifted at \( t^- \), and the toll revenue is shaped as the rectangle \( t^+ h' t^- \). Because the optimal step toll, \( \rho \), is just half of the highest optimal non-queuing toll, \( \psi(\bar{t}) \), \( t^+ h' t^- \) is the maximum rectangle inscribed within the optimal non-queuing toll to collect the maximum toll revenue. Consequently the optimal step toll remove the largest proportion of the total queuing time in all step toll cases and make ship owners no worse off than they would be in the non-toll equilibrium. Under this optimal step toll scheme, \( \hat{t} \) in Figure 3 is defined as a bulk carrier’s arrival time at the anchorage which allows the berthing time just the same as ETA after queuing. It is reasonable that \( \hat{t} \) under the optimal step toll scheme is later than \( \tilde{t} \) under the non-toll equilibrium.

In Figure 3, \( t' \) is defined as the start time when no bulk carrier arrives at the anchorage until \( t^- \) under the optimal single step toll scheme. According to this pricing scheme, the queuing cost to the first bulk carrier that will pay the toll, \( \rho \), at \( t^+ \) is zero. It is then clear that no bulk carrier arrives at the anchorage from \( t' \) until \( t^+ \), and the length of the time period \( [t', t^+] \) is equal to \( \rho / c_q \). Consequently the last untolled bulk carrier before \( t^+ \) must arrive at the anchorage \( \rho / c_q \) earlier than the first tolled bulk carrier at \( t^+ \).

On the other hand, \( t'' \) in Figure 3 is defined as the start time when the first bulk carrier arrives at the anchorage but decide not to notify the port officer for berthing until \( t^- \) in order to avoid paying the toll. Also according to this pricing scheme, the queuing cost to the last bulk carrier that will pay the toll \( \rho \) before \( t^- \) is zero. This is impossible unless there are a mass of arrived bulk carriers waited willingly at the anchorage from \( t'' \) until \( t^- \) to avoid being tolled. These speculators are ready to enter the berth free once the toll is lifted on \( t^- \). Consequently, the length of the time period \( [t'', t^-] \) must be equal to \( \rho / c_q \).

Note that \( t_q \) is now assumed to locate on the origin (i.e., \( t_q = 0 \)) in Figure 3 for the purpose of simplifying computation to all arrival time values without losing the generality. Detailed computations to the toll level and arrival times values appeared in Figure 3 are shown as follows:

\[
\rho = \frac{TC^e}{2} = \frac{1}{2} \psi(\bar{t}) = \frac{c_{dis} c_{pen}}{2(c_{dis} + c_{pen})} \cdot T_w \cdot (N - 1), \quad t_q = 0,
\]

\[
t_q = T_w \cdot (N - 1) - t_q = T_w \cdot (N - 1),
\]

\[
\tilde{t} = t_q + t_q t = t_q + \frac{TC^e}{c_q c_{dis} (c_q - c_{dis})} = \frac{c_{pen}(c_q - c_{dis})}{c_q (c_{dis} + c_{pen})} \cdot T_w \cdot (N - 1),
\]

\[
\bar{t} = \tilde{t} + T_q(\bar{t}) = \tilde{t} + \frac{TC^e}{c_q} = \frac{c_{pen}}{c_{dis} + c_{pen}} \cdot T_w \cdot (N - 1),
\]
\[ \hat{t} = \bar{t} - T_Q(\hat{t}) = t^* - \frac{c_q}{c_q} \left( \frac{2c_q - c_{dis}}{2c_q \left( c_{dis} + c_{pen} \right)} \right) \cdot T_w \cdot (N - 1), \]

\[ t^* = t_q + \frac{T_w}{c_q} = t_q + \frac{c_{pen}}{2(c_{dis} + c_{pen})} \cdot T_w \cdot (N - 1), \]

\[ t' = t^* - \frac{c_{pen}}{2(c_{dis} + c_{pen})} \cdot T_w \cdot (N - 1), \]

\[ t^* = t_q' + t^* = t_q' - \frac{c_{pen}}{2(c_{dis} + c_{pen})} \cdot T_w \cdot (N - 1), \]

\[ t^n = t^* - \frac{2c_q c_{pen}}{2c_q (c_{dis} + c_{pen})} \cdot T_w \cdot (N - 1). \]

3.1 Equilibrium Queuing Costs

Table 1 illustrates equilibrium results for all arrival intervals under the optimal step toll scheme to bulk carriers. Note that a blanket arrival time interval \([t', t^*]\) is existed in Table 1 because no bulk carrier arrives at the anchorage during this time period that mentioned before. Bulk carriers of groups B, C and D arrive at the anchorage during the tolled period \([t^*, t^*]\). Except group D, speculators, escaping from being tolled, groups B and C pay the toll to enter the berth. Groups A and E do not need to pay the toll because they arrive at the anchorage during the no toll periods. In addition, only groups A and B will be alongside the berth earlier than ETA because they arrive at the anchorage before \(\hat{t}\). Consequently only groups A and B are early departures because they will leave the berth (depart from the port) earlier than ETD. These results are arranged as columns [1] ~ [2] in Table 1.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>A</td>
<td>(a).Toll Free</td>
<td>( E\hat{Q} = \frac{c_q}{c_q} \cdot T_Q(\hat{t}) )</td>
<td>( [t_q, t'] )</td>
</tr>
<tr>
<td></td>
<td>(b).Early Departures</td>
<td>( E\hat{D}=\frac{c_q}{c_q} \cdot T_Q(\hat{t})(N-1) )</td>
<td>( (\bar{t} = \hat{t}) )</td>
</tr>
<tr>
<td>t ∈ [t', t]</td>
<td>(No Toll Period)</td>
<td>( E\hat{C}=0 )</td>
<td>None</td>
</tr>
<tr>
<td>B</td>
<td>(a).Toll Payers</td>
<td>( E\hat{Q} = \frac{c_q}{c_q} \cdot T_Q(\hat{t}) )</td>
<td>( [t', \hat{t}] \rightarrow [t^*, \hat{t}] )</td>
</tr>
<tr>
<td></td>
<td>(b).Early Departures</td>
<td>( E\hat{D}=\frac{c_q}{c_q} \cdot T_Q(\hat{t})(N-1) )</td>
<td>( (\bar{f} = \bar{f}) )</td>
</tr>
<tr>
<td>t ∈ [t', t]</td>
<td>(Tolled Period)</td>
<td>( E\hat{C}=0 )</td>
<td>None</td>
</tr>
<tr>
<td>C</td>
<td>(a).Toll Payers</td>
<td>( E\hat{Q} = \frac{c_q}{c_q} \cdot T_Q(\hat{t}) )</td>
<td>( [\hat{t}, t^*] \rightarrow [\hat{t}, \hat{t}] )</td>
</tr>
<tr>
<td></td>
<td>(b).Late Departures</td>
<td>( E\hat{D}=\frac{c_q}{c_q} \cdot T_Q(\hat{t})(N-1) )</td>
<td>( (\bar{t} = \bar{t}) )</td>
</tr>
</tbody>
</table>
Since equilibrium will be achieved as long as all bulk carriers have the same total cost throughout the queuing period, there are two kinds of equilibrium conditions for the early and late departures. One is \( TC(t) = TC(t') \) for groups A and B of the early departures, and the other is \( TC(t) = TC(t_q') \) for groups C and D (or E) of the late departures. These equilibrium conditions then can be expressed as follows:

\[
\begin{align*}
\text{E} & \quad t \in [t^*, t_q'] \\
& \quad (\text{No Toll Period})
\end{align*}
\]

<table>
<thead>
<tr>
<th>( E )</th>
<th>[ t \in [t^*, t_q'] ] (No Toll Period)</th>
<th>( t \in [t^*, t_q'] ) (Tolled Period)</th>
<th>( E )</th>
<th>[ t \in [t^*, t_q'] ] (No Toll Period)</th>
<th>( t \in [t^*, t_q'] ) (Tolled Period)</th>
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<td>( E )</td>
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<td>( E )</td>
<td>[ t \in [t^*, t_q'] ] (No Toll Period)</td>
<td>( t \in [t^*, t_q'] ) (Tolled Period)</td>
</tr>
</tbody>
</table>
| \( t \in [t^*, t_q'] \) (Tolled Period) | \( \text{(a). Toll Free} \\
\text{(speculators)} \) | \( \text{(b). Late Departures} \) | \( \text{(a). Toll Free} \\
\text{(speculators)} \) | \( \text{(b). Late Departures} \) |
| \( E \) | \( t \in [t^*, t_q'] \) (No Toll Period) | \( t \in [t^*, t_q'] \) (Tolled Period) | \( E \) | \[ t \in [t^*, t_q'] \] (No Toll Period) | \( t \in [t^*, t_q'] \) (Tolled Period) |
| \( t \in [t^*, t_q'] \) (Tolled Period) | \( \text{(a). Toll Free} \\
\text{(speculators)} \) | \( \text{(b). Late Departures} \) | \( \text{(a). Toll Free} \\
\text{(speculators)} \) | \( \text{(b). Late Departures} \) |

Equilibrium queuing costs at the anchorage \( (EQC : c_q \cdot T_Q(t)) \) to groups A–E, listed in column [3] of Table 1 are obtained based on (16)–(19). As shown in Figure 3, the equilibrium queuing costs to groups A–E under the optimal step toll scheme are thick lines \( f_{\tilde{t}} \), \( f_t \), \( g_{\tilde{t}} \) and \( g_t \), respectively. The slope of \( f_{\tilde{t}} \) and \( f_t \) for all early departures is \( \frac{c_q \cdot c_{\text{dis}}}{c_q - c_{\text{dis}}} \), which is the same as the slope of the equilibrium queuing cost \( t_q \Delta \) to all early departures in the non-toll case. Note that there is no thick lines of equilibrium queuing costs through the arrival period \( [t^*, t^+] \) since no bulk carrier arrives at the anchorage during this period. In addition, the length of the queue will be reduced to zero at \( t^+ \) because of \( T_q(t') = \rho / \alpha = t^+ - t' \). On the other hand, the slope of \( g_{\tilde{t}} \), \( \overline{hi} \) and \( \overline{t_q} \) for all late departures is \( \frac{-c_q \cdot c_{\text{pen}}}{c_q + c_{\text{pen}}} \), which is the same as the slope of the equilibrium queuing cost \( \overline{aat_{q'}} \) to all late departures in the non-toll case.

In Figure 2, the total equilibrium queuing cost in the original non-toll case is \( \Delta t_q \Delta \), and the total equilibrium queuing cost under the optimal step toll scheme is composed of \( \Delta t_q f' \),
3.2 Decisions of Arrival Time Change

Because the optimal step toll derived from our model is simply the money cost to the toll payers who require to save the same amount of queuing costs, the equilibrium derivative costs due to queuing \((EDC)\) in the tolled case must be the same as that in the original non-toll case to maintain the equilibrium cost, \(TC^e = c_q T_q(\hat{t}) = y(\hat{t})\). For this purpose, decisions of arrival time change to all container ships can be investigated by “the invariant equilibrium derivative cost principle”.

Since all results of equilibrium queuing costs to bulk carriers at the anchorage \((EQC)\) under the optimal step toll scheme have been shown in column [3] of Table 1, the corresponding values of equilibrium derivative costs due to queuing \((EDC)\) required to achieve the equilibrium cost \(TC^e\) can be easily obtained as shown in the same column. \(EDC\) to groups A~E bulk carriers under the optimal step toll scheme are drawn as the doubled lines \(\bar{cf}, \bar{tf'}, \bar{ih'}, \bar{he}\) and \(\bar{ed}\), respectively in Figure 3. The slope of \(\bar{cf}\) and \(\bar{tf'}\) to all early departures is identical and equal to \(-\frac{c_q \cdot c_{dis}}{c_q - c_{dis}}\). This is the same as the slope of \(\bar{ct}\) that represents all early departures’ \(EDC\) in the non-toll case. On the other hand, the slope of \(\bar{ih'}, \bar{he}\) and \(\bar{ed}\) to all late departures is identical and equal to \(-\frac{c_q \cdot c_{pen}}{c_q + c_{pen}}\). This is also the same as the slope of \(\bar{id}\) that represents all late departures’ \(EDC\) in the non-toll case.

Bulk carrier owners’ decisions of arrival time changes under the optimal step toll scheme are shown as column [4] in Table 1. Firstly, group A ships will not change their original arrival times in the non-toll case when the port is priced with the optimal step toll, because \(cf\) in both the non-toll and optimal step toll cases coincide during the arrival period \([t, t']\).

Secondly, because \(\bar{tf'}\) in the optimal step toll case and \(\bar{tf}\) in the non-toll case are two identical and parallel lines, all group B ships that originally arrive during the period \([t', \hat{t})\) in the non-toll case will change their arrivals to the period \([\hat{t}, t')\) in the optimal step toll case.

Similarly, because \(\bar{ih'}\) in the optimal step toll case and \(\bar{ih}\) in the non-toll case are two identical and parallel lines, all group C ships that originally arrive during the period \([\hat{t}, t')\) in the non-toll case will change their arrivals to the period \([\hat{t}, t')\) in the optimal step toll case.

Thirdly, because \(\bar{he}\) in both the non-toll and optimal step toll cases coincide during the arrival time period \([\hat{t}, t')\), group D ships will not change their original arrival times in the non-toll case when the port is priced with the optimal step toll. Since \([t', \hat{t})\) exists within \([\hat{t}, t')\), groups C and D ships arrive simultaneously during \([t', \hat{t})\). Finally, because \(\bar{ed}\) in both the non-toll and optimal step toll cases coincide during the arrival time period \([\hat{t}, t')\),
group E ships will not change their original arrival times in the non-toll case when the port is priced with the optimal step toll.

It is clear from the above outcomes that bulk carrier owners who choose the same arrival times at the anchorage as they did in the original non-toll equilibrium case are not the toll payers in the tolled case. As shown in Figure 3, these bulk carriers are groups A, D and E. On the other hand, other groups of bulk carriers that postpone their original arrival times at the anchorage are the toll payers. As shown in Figure 3, \([t', \tilde{t} \rightarrow [t', \tilde{t}^*])\) for group B ships, and \([\tilde{t}, t^* \rightarrow [\tilde{t}, t])\) for group C ships.

4. A NUMERICAL EXAMPLE

A case study is raised to examine the above theory. According to the statistical data of Keelung port in Taiwan, coal ships in gross tonnage between 20,000 and 39,999 are always the most part of the total bulk carriers visiting Keelung port per year. Therefore, the scenario of this case describes the ships of the above category arriving at Keelung port. The queuing time cost per day at the anchorage to the above size of coal ships includes personnel expense of ship crew:\$779, maintenance:\$283, insurance:\$277, spare parts:\$108, fresh water and lubricator:\$95, ship depreciation:\$1675, fuel oil consumption in port:\$902, management:\$163, and other fixed cost:\$950. Consequently the queuing time cost at the anchorage is US\$218 (\(=5232/24= c_q\)) per hour.

In tramp shipping, the ship owner should pay the dispatch fee to the consignor if the ship’s departure time becomes earlier than ETD due to the consignor’s prompt arrangements for cargos being loaded/unloaded. In general, the dispatch fee is half of the total fixed cost to the coal ship (except for the fuel oil consumption in port), then the dispatch fee for the early departure case, is equal to $90.21 (\(=(5232-902)/48= c_{dis}\)) per hour. On the other hand, the ship owner has to suffer the penalty cost or revenue loss on delaying the next voyage if the ship’s departure time becomes later than ETD. For the late departure case, the rent for chartering a coal ship may be considered as the penalty cost or revenue loss. In the shipping market, it takes $13000 per day to charter a coal ship of 30000~35000 gross tonnage, Then penalty cost or revenue loss is equal to $541.67 (\(=13000/24= c_{pen}\)) per hour.

The speed for a machine loading/unloading coal is about 540.05 tons per hour in Keelung port. Then it takes 31 hours (\(=33000/(540.05 \times 2)= T_u\)) to completely unload a full loading coal ship with 33000 tons of coal by two machines.

The number of coal ships \((N)\) waiting in the anchorage is assumed to be 3 ships. Then we have the following results under the optimal step toll scheme as shown in Figure 3: \(T_w \cdot (N-1) = 31 \times (3-1) = 62hrs\); \(t_q = 0:00\) (the 1st day); \(t_q = 62hrs = 14:00\) (the 3rd day);

\[
\rho = \frac{TC^*}{2} = \frac{1}{2} \psi(t) = \frac{c_{dis} \cdot c_{pen}}{2(c_{dis} + c_{pen})} \cdot T_w \cdot (N-1) = 2397.27
\]

\[
t' = \frac{c_{pen} (c_q - c_{dis})}{2c_q (c_{dis} + c_{pen})} \cdot T_w \cdot (N-1) = 15.58hrs
\]

\[
t^* = \frac{c_{pen}}{2(c_{dis} + c_{pen})} \cdot T_w \cdot (N-1) = 26.57hrs
\]
\[ i = \frac{c_{pen}(c_q - c_{dis})}{c_q(c_{dis} + c_{pen})} \cdot T_w \cdot (N - 1) = 31.16\text{hrs} \]
\[ \hat{i} = \frac{c_{pen}(2c_q - c_{dis})}{2c_q(c_{dis} + c_{pen})} \cdot T_w \cdot (N - 1) = 42.15\text{hrs} \]
\[ t^o = \frac{2c_qc_{pen} + c_qc_{dis} - c_{dis}c_{pen}}{2c_q(c_{dis} + c_{pen})} \cdot T_w \cdot (N - 1) = 46.58\text{hrs} \]
\[ \hat{t} = \frac{c_{pen}}{c_{dis} + c_{pen}} \cdot T_w \cdot (N - 1) = 53.15\text{hrs} \]
\[ t^- = \frac{c_{dis} + 2c_{pen}}{2(c_{dis} + c_{pen})} \cdot T_w \cdot (N - 1) = 57.57\text{hrs} \]

The first coal ship arriving at the anchorage for waiting berth is the start of queuing, and the time is assumed at 00:00 of the first day. After 62 hours, that the last coal ship enters the port to berth means the end of queuing, and the time is at 14:00 of the third day. Equilibrium queuing costs (EQC) to all coal ships under the optimal step toll scheme as shown in Figure 3 are 153.89 \text{t} for \( t_q \), 153.89 \text{t} - 4089.56 for \( t_g \), -155.44 \text{t} + 8949.42 for \( g' t \) and -155.44 \text{t} + 9637.36 for \( h' \) and \( i't_q \). Equilibrium derivative costs due to queuing (\( EDC = \psi(\hat{i}) - EQC - \rho \)) to all coal ships under the optimal step toll scheme as shown in Figure 3 are -153.89\text{t} + 4794.54 for \( cf \), -153.89\text{t} + 6486.82 for \( tf' \), 155.44\text{t} - 6552.15 for \( h't \), and 155.44\text{t} - 4842.82 for \( he \) and \( ed \). Furthermore, bulk carrier owners’ decisions of arrival time changes under the optimal step toll scheme are listed in Table 2.

<table>
<thead>
<tr>
<th>The time of bulk carriers arriving at the anchorage before the toll established</th>
<th>The time of bulk carriers arriving at the anchorage after the toll established</th>
<th>Actions of the bulk carriers after the toll established</th>
</tr>
</thead>
<tbody>
<tr>
<td>([t_q', t'])</td>
<td>([t_q', t']) no toll period</td>
<td>(1) Maintain the same arrival time at the anchorage.</td>
</tr>
<tr>
<td>00 : 00 (the first day) ∼ 15 : 35 (the first day)</td>
<td>00 : 00 (the first day) ∼ 15 : 35 (the first day)</td>
<td>(2) Departure time is earlier than ETD.</td>
</tr>
<tr>
<td>15 : 35 (the first day) ∼ 07 : 10 (the second day)</td>
<td>02 : 34 (the second day) ∼ 18 : 09 (the second day)</td>
<td>(3) No need to pay the toll.</td>
</tr>
<tr>
<td>([t', \hat{i}]) toll period</td>
<td>([t', \hat{i}]) toll period</td>
<td>(1) Changing the arrival time at the anchorage.</td>
</tr>
<tr>
<td>15 : 35 (the first day) ∼ 07 : 10 (the second day)</td>
<td>02 : 34 (the second day) ∼ 18 : 09 (the second day)</td>
<td>(2) Departure time is earlier than ETD.</td>
</tr>
<tr>
<td>07 : 10 (the second day) ∼ 22 : 35 (the second day)</td>
<td>18 : 09 (the second day) ∼ 09 : 34 (the third day)</td>
<td>(3) Need to pay the toll.</td>
</tr>
<tr>
<td>([\hat{i}, t^-]) toll period</td>
<td>([\hat{i}, t^-]) toll period</td>
<td>(1) Changing the arrival time at the anchorage.</td>
</tr>
<tr>
<td>07 : 10 (the second day) ∼ 22 : 35 (the second day)</td>
<td>18 : 09 (the second day) ∼ 09 : 34 (the third day)</td>
<td>(2) Departure time is later than ETD.</td>
</tr>
<tr>
<td>22 : 35 (the second day) ∼ 09 : 34 (the third day)</td>
<td>22 : 35 (the second day) ∼ 09 : 34 (the third day)</td>
<td>(3) Need to pay the toll.</td>
</tr>
<tr>
<td>([t^o, t^-]) toll period</td>
<td>([t^o, t^-]) toll period</td>
<td>(1) Maintain the same arrival time at the anchorage.</td>
</tr>
<tr>
<td>22 : 35 (the second day) ∼ 09 : 34 (the third day)</td>
<td>22 : 35 (the second day) ∼ 09 : 34 (the third day)</td>
<td>(2) Departure time is later than ETD.</td>
</tr>
<tr>
<td>22 : 35 (the second day) ∼ 09 : 34 (the third day)</td>
<td>22 : 35 (the second day) ∼ 09 : 34 (the third day)</td>
<td>(3) No need to pay the toll (avoiding).</td>
</tr>
</tbody>
</table>
5. CONCLUSIONS

This paper considered a pricing model for bulk carriers anchoring a queuing port. According to bulk carriers’ departure patterns at a queuing port, we derived the non-toll equilibrium total cost per bulk carrier that resulted from queuing at the anchorage. We also developed a series of optimal non-queuing tolls that eliminate the total queuing times without making bulk carrier owners worse off than they would be in the non-toll equilibrium. Because the optimal non-queuing toll scheme has practical difficulties, the optimal step toll inscribed in the optimal non-queuing toll is established in this paper as an alternative pricing scheme to bulk carriers at a queuing port.

This paper has shown all arrival time values, equilibrium conditions, equilibrium queuing costs at the anchorage, and the equilibrium derivative costs due to queuing under the optimal step toll scheme. By following the invariant equilibrium derivative cost principle, this paper provided a framework to predict bulk carrier owners’ decisions of arrival time changes under the optimal step toll scheme. We have found that bulk carrier owners who maintain the same arrival times at the anchorage as they did in the original non-toll equilibrium case are not the toll payers under the optimal step toll scheme. The other part of bulk carriers that postpone their original arrival times in the non-toll equilibrium case are the toll payers under the optimal step toll scheme.

REFERENCES

