Comparison of Road Pricing of Optimum and Descriptive Approaches Considering Local Emissions of Road Transportation Network

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Abstract: Road pricing from an economic point of view has largely been motivated by the time costs that each road user imposes on other road users, since the classical discussion of external costs by Pigou (1920). For many years road pricing has been attracting considerable attention of analysts and policymakers as a means of relieving urban traffic congestion and recently for local traffic air-pollution problems. In this paper, a model is proposed considering local traffic emissions and illustrated with hypothetical network. Davidson time-flow function and vehicle speed-emission factor relationship is considered. Optimum road pricings of both optimum and descriptive approaches are compared. The optimum approach can be applied to most of the situations when speeds are below 70 kmph on selected locations. At speeds above 70 kmph, there may be considerable difference between both the approaches and may be used appropriately. The accuracy of the emission factor curves has influence on actual tolls.

Key words: Optimum tolls, Road pricing, External costs, Environmental externalities

1. INTRODUCTION

For many years road pricing has been attracting considerable attention of analysts and policymakers as a means of relieving urban traffic congestion, since the classical discussion of external costs by Pigou (1920). A large number of studies focused on road pricing (McDonald, 1995; Olszewski, 2005; Yang Hai and Huang 2005; Johansson, 1997; Dial, 1999; May, 2000; Verhoef, 2002) as an efficient approach to internalize externalities such as congestion, air pollution. Congestion pricing schemes do not necessarily lead to less traffic emissions (May, 2000; Yin, 2006). In recent years, externalities with particular attention to local air-pollution problems associated with urban transportation (Button and Verhoef, 1998) are seriously considered by policy makers and government agencies. Transportation is responsible for approximately 50% of the emissions of nitrogen oxide and 90% of the carbon monoxide. Any policy system aimed towards the reduction of emissions due to motor vehicles must be taken into consideration of network topology, cost structure, as well as the travel demand structure (Nagurney, 2000). In this paper, a model is proposed considering local traffic emission standards for selected routes and illustrated with hypothetical network. Davidson time-flow equation is considered in the model which estimates accurate speeds over BPR or other functions for flows below capacity (Singh, 1999). The speed-emission factor relationship is introduced in the model for emission estimates to determine further the link environmental capacities that can be allowed in the network. Here, the focus is on road networks with fixed O-D demand and single vehicle type with homogenous users is considered. Further, only the static deterministic equilibrium solution is considered.
2. BACKGROUND

In view of modeling road pricing considering local emissions, it is necessary to overview the existing congestion pricing models and emission related pricing studies. Motorists typically select routes that minimize their travel time or generalized cost. This may entail traveling on longer but faster routes. This raises questions concerning whether traveling along a longer but faster route results in energy and/or air quality improvements (Kyoungho et.al.,2008). The theoretical background of road-use pricing has relied upon the fundamental economic principle of marginal-cost pricing, which states that road users using congested roads should pay a toll equal to the difference between the marginal-social cost and the marginal-private cost in order to maximize the social surplus. (Verhoef, 1996). Marginal-cost pricing theory or the first-best pricing in the literature by pigou and followers was developed based on the demand-supply (or performance) curves for the standard case of homogeneous traffic stream moving along a given uniform stretch of road, such as an expressway, connecting given entry and exit points. In the case of homogeneous users, the first-best congestion pricing theory is established in general traffic networks. In line with this theory, a toll that is equal to the difference between the marginal social cost and marginal private cost is charged on each link so as to internalize the user externalities and thus achieve a system optimum flow pattern in the network (Beckmann, 1965 et.al.). Investigations have been conducted on how this classical economic principle would work in a general congested road network with multiple vehicle types and link flow interactions, with queing, and in a congested network in a stochastic equilibrium. Moreover, Bellei et al. (2002) developed a variational inequality model for network pricing optimization in a multi-user and multi-modal context. Yang hai (2005) formulated system optimum problem for a general congested network keeping a capacity constraint and indicated that environmental problem can be dealt similarly by keeping the constraints, but did not actually modeled considering vehicle emission factors.

The environmental benefits are about 10% of the congestion benefits for London congestion pricing (Prudhomme, 2004). Road user should pay a charge to the increased emissions and fuel consumption of other road users in addition to their own emissions (Johansson, 1997). Johansson (2006) further shown a model that an optimal first-best road charge should not only be differentiated with respect to factors that affect the direct external environmental and time costs from the road-user himself. But also include indirect effects that other’s cars will be more polluting when congestion increases. Link interaction was not considered in that model where the link speed is not independent. Nagurney et.al. (2001) developed a dynamic model of a link-based pollution permit system for urban congested transportation networks but not intended for the day-to-day implementation of the pollution permit system and took constant emission factor without actual speed-emission factor relationship. Yin (2006) has shown a model for emission charging that minimizes the total emissions with an example of CO emission factor. However the charges that minimize the total emissions may not keep the emissions below the permissible levels on individual selected locations based on their sensitiveness of area particularly when selected local air pollutant that may not require minimizing in non-sensitive areas. Kyoungho et.al.(2008) indicated that different emission- and/or energy-optimized assignments should be recommended for each pollution type and fuel consumption and CO2 emission- and energy-optimized assignment is identical to the SO assignment.

The existing models mainly focused on congestion pricing and have some indirect benefits of emission reductions. The existing UE or CP models can be extended for environmental road pricing (ERP) and combined environmental road pricing and congestion pricing (CP+ERP) by
implicitly considering the environmental costs in terms of emission constraints. Very few studies considered local pollution as main objective. Existing models with environmental externalities as main objective are not considered either the actual speed-emission factor curves or the network or the link permissible emission standards parameters. Due to growing importance of local pollution problems and very few studies on it, in this paper local air pollutant (NOx) is considered for the model. Two different approaches i.e. optimum approach with approximation curve and descriptive approach with original curve are considered and the pricings of both are compared to see whether the optimum pricings of both approaches are close enough in order to use approximation curve in general networks for optimization. The optimum model is identical to Yang hai congestion pricing model but with individual link emission constraints using NOx speed-emission factor relationship. The descriptive approach is similar to general UE model where the assignment will be done for fixed pricing. In this paper, model is formulated considering local pollutant as main objective by implicitly considering their external costs with a typical speed-emission factor relationship and link permissible emission standards in a network.

3. MODEL FORMULATION

3.1 Assumptions
Consider fixed demand with route choice only in the model. Consider a single vehicle category with homogeneous users and single pollutant. Both travel time costs and fuel costs are considered in the model. Consider linear relationship between vehicle average speed and vehicle mileage.

3.2 Parameter Description
Consider a network with O-D pairs W, A set of links, Q set of link flows and Rw set of routes in each OD pair w; Consider fixed demand dw (veh/hr) between OD pair w; Qa is link flows; frw is flows on route r between OD pair w; era is cost of route r between OD pair w; Ca is Link Capacities; La is Link Length; ta0 is link free flow travel time(hr/km); tao is link travel time (hr/km); Ma0 is mileage at free flow speed (km/lt); va0 is link free flow speed(kmph); F is fuel cost (price/lt); VOT is value of time (price/hr); vehicle emission factor e(Qa) (gm/km) which depends on speed and speed depends on link flows through speed-flow relationship; vehicle emission factor using approximation curve is ea(Qa) (gm/km) and vehicle emission factor using original curve is eo(Qa) (gm/km); Ea is Link emission standard levels (permissible emissions) (gm/km-hr); Ea(Qa) is total Link Emissions using approximation curve(gm/km-hr), Ea(Qa) is total Link Emissions using original curve(gm/km-hr). Davidson time-flow equation

\[ t_a = \frac{aD}{1 + \frac{J_D X_a}{1 - X_a}} \]  

In equation (1), the first term is link travel time cost and the second term is link fuel consumption cost. The cost function is flow-dependent and explicitly takes into account the fact that traffic congestion will lead to a higher generalized cost through the opportunity cost of waiting in queues or the payment of a congestion toll. The costs are monotonically
increasing with flows and hence it is convex.

3.4 Emission Factor and Link Emissions

There are many speed-emission factors curves available in the literature such as ARAI for India, JCAP for Japan, Finland etc. They are different from each other. The amount of local emissions depends on the speed-emission factor curve. Nataraju (2009) compared the variation of optimum pricing using three different emission factor curves. The original shape of average speed-emission factor is non-monotonic in most of the cases such as Finland and ARAI curves. In case of monotonic curves such as JCAP, the optimization approach can be straightforward applied and there is no need to check for pricing difference. The descriptive approach is fixing the pricing on selected emission constraint locations and evaluating the emissions on those selected locations whether or not within permissible limits and changes the pricing and try again. This approach requires many trials to get the unique optimum solution. On the other hand, if the original shape of speed-emission factor curve is approximated by making monotonic curve for optimization approach, there may be pricing difference when the situations occur where changes has been made. This approximation may be different for different curves depend on actual speed-emission factor relationship. So, in this paper, a typical non-monotonic speed-emission factor curve (Finland curve) is considered and approximated to observe those changes in optimum pricings using optimization approach with approximation curve and descriptive approach with original curve. External costs of local pollution may vary with the variation in local emissions.

The original relationship of speed-NOx emission factor curve for diesel car is given by (Finland curves, FHWA (2005)) \( e = -10^{-7} V^3 + 0.0002 V^2 - 0.0245 V + 1.3698 \), where the emission factor will increase at later portion of the curve at higher speeds also. If the original relation is considered, then the network link pricing should be fixed first for which the equilibrium flows will be simulated using general UE model. Emissions are calculated for the resulted flows and constraints will be checked whether or not within permissible limits and pricing will be changed and tried again if necessary. This approach is called as descriptive approach modeling. But for optimization problem, due to difficult in computations and non-convexity of the problem, the vehicle average speed and emission factor relationship is approximated as \( e = 2.7331 V_a^{-0.3692} \), where emission factor decreases monotonically with average speed and hence it is convex. This assumption may result lower tolls than actual at speeds above 70 kmph due to reduced emissions at higher flows which is not true in practice. Figure 1 shows the speed-emission factor relationship. Figure 2 shows the shape of the link flow-link emissions relationship. Total Link emissions are the product of vehicle emission factor and the link flows.

Approximation emission factor  
\[
e_a = 2.7331 \left[ \frac{V_a^0}{1 + \left[ J_D X_a / (1 - X_a) \right]} \right]^{-0.3692}
\]

Total emissions using approximation curve  
\[
E_a^o (Q_a) = 2.7331 Q_a \left[ \frac{\left\{ C_a - Q_a \right\} V_a^0}{C_a - Q_a + J_D Q_a} \right]^{-0.3692}
\]

Original emission factor  
\[
e_o = -10^{-7} \left[ \frac{\left\{ C_a - Q_a \right\} V_a^0}{C_a - Q_a + J_D Q_a} \right]^3 + 0.0002 \left[ \frac{\left\{ C_a - Q_a \right\} V_a^0}{C_a - Q_a + J_D Q_a} \right]^2 - 0.0245 \left[ \frac{\left\{ C_a - Q_a \right\} V_a^0}{C_a - Q_a + J_D Q_a} \right] + 1.3698
\]
Total emissions using original curve

\[ E^a(Q_a) = -10^{-7} Q_a \left[ \frac{(C_a - Q_a) y_a^0}{C_a - Q_a + J a Q_a} \right]^3 + 0.0002 Q_a \left[ \frac{(C_a - Q_a) y_a^0}{C_a - Q_a + J a Q_a} \right]^2 - 0.0245 Q_a \left[ \frac{(C_a - Q_a) y_a^0}{C_a - Q_a + J a Q_a} \right] + 1.3698 Q_a \]

(5)

For any given link emission standards for the optimum approach, the link environmental capacity \((Q^E_a)\) can be determined by solving eq.(3) by any minimization techniques. For any positive flows and emission standards, \(Q^E_a < C_a\). Here the approximation curve is used for optimum solution for which augmented Lagrange algorithm is used.

### 3.5 Model Formulation

System Optimum model to minimize the total system cost that keeps the local emissions below permissible emissions is given by

\[
\text{Min} \sum_{a \in A} Q_a T_a(Q_a)
\]

Subjected to

\[
\sum_{r \in R_w} f_{rw} = d_w, \quad w \in W
\]

(7)

\[
f_{rw} \geq 0, \quad r \in R_w, \quad w \in W
\]

(9)

Eq.(6) minimizes system cost, Eq.(7) restricts the link flows below environmental capacity (which is simulated from eq. (3) for optimum approach), Eq.(8) checks the sum of all route flows should be equal to OD demand and Eq.(9) checks the flows should be non-negative.

\[ \Omega = \{Q | Q = \Delta f, \Delta f = d, f \geq 0\} \].

Link flows, \(Q = (Q_a, a \in A)^T\), are defined by \(Q_a = \sum_{w \in W} \sum_{r \in R_w} f_{rw} \delta_{ar}, a \in A\); \(\Delta = [\delta_{ar}]\) is link/path incidence matrix. \(f\) is path flow vector defined as \(f^* = (f^*_{rw}, r \in R_w, w \in W)^T\). \(\Lambda = [\Lambda_{rw}]\) is OD pair/path incidence matrix, where \(\Lambda_{rw} = 1\) if \(r \in R_w\) and 0, otherwise. \(d = (d_w, w \in W)^T\) is a column vector of all OD demands. View link flow \(Q\) as a function of path flow vector \(f\) as defined by link-path flow relation and construct Lagrangean function

\[
\min L(f, \mu, \lambda) = \sum_{a \in A} Q_a T_a(Q_a) + \sum_{a \in A} \lambda_a \left[ Q_a - Q^E_a \right] + \sum_{w \in W} \mu_w \left[ d_w - \sum_{r \in R_w} f_{rw} \right]
\]

(10)

The first order optimality conditions are

\[
f_{rw} \frac{\partial L(f^*, \mu, \lambda)}{\partial f_{rw}} = f^*_{rw} (\delta_{rw} - \mu_w) = 0, \quad r \in R_w, w \in W
\]

(11)
\[
\frac{\partial f^*}{\partial f^w} c_{rw} - \mu_w \geq 0, r \in R, w \in W \\
= \frac{\partial f^*}{\partial \mu_w} \sum_{r \in R} f^w_r - d_w = 0, w \in W \\
= \frac{\partial f^*}{\partial \lambda_a} Q_a - Q_a^T \leq 0; Q_a \leq Q_a^T, a \in A \\
= \lambda_a (Q_a - Q_a^T) = 0, a \in A \\
\lambda_a \geq 0, a \in A \\
c_{rw} = \sum_{a \in A} (t_a(Q_a) + Q_a dt_a(Q_a)/dQ_a + \lambda_a) \delta_{ar} .
\]

\(\lambda_a, a \in A,\) and \(\mu_w\) are the Lagrange multipliers associated with constraints (7) & (8). \(\mu_w\) is the minimum travel cost between OD pair \(w \in W\) associated with constraint (8) and \(\mu = (\mu_w, w \in W)^T\). From optimality conditions (11)–(16), for any optimum solution, the following relations can be obtained.

\(c_{rw} = \mu_w\) if \(f^w_r > 0; c_{rw} \geq \mu_w\) if \(f^w_r = 0, r \in R, w \in W\) and \(\lambda_a = 0, if Q_a^* < Q_a^T; \lambda_a \geq 0, if Q_a^* = Q_a^T, a \in A\). Route cost \(c_{rw} = \sum_{a \in A} \tilde{t}_a(Q_a) \delta_{ar}, r \in R, w \in W\), where Link cost \(\tilde{t}_a(Q_a) = \tilde{t}_a(Q_a) + \lambda_a\).

\(\tilde{t}_a(Q_a) = t_a(Q_a) + Q_a dt_a(Q_a)/dQ_a + \lambda_a\). (17)

Emissions exceed only \((\lambda_a > 0)\) when flows reached \(Q_a^* = Q_a^T\); below environmental capacity, link costs will be solely given by \(\tilde{t}_a(Q_a)\). \(\lambda_a, a \in A,\) are above the standard levels and should be substituted by an additional toll. Consequently, the optimal link toll to be charged is given by \(u_a = Q_a dt_a(Q_a)/dQ_a\) \(Q_a = Q_a^{EC} + \lambda_a, a \in A\); Where \(Q_a^{EC}, a \in A\) denotes the optimum solution to the System optimum problem with fixed demand and emission constraints; \(\lambda_a = 0\) if \(Q_a^{EC} < Q_a^T\) and \(\lambda_a \geq 0\) if \(Q_a^{EC} = Q_a^T, a \in A\). This formulation can be called as combined congestion and environmental pricing (CP+ERP). Other pricing options such as CP and ERP options can be formulated with slight changes in the above formulation. If the constraint (7) is neglected in the program (6)-(8), then it becomes congestion pricing problem (CP). Link costs \(\tilde{t}_a(Q_a) = t_a(Q_a) + Q_a dt_a(Q_a)/dQ_a\) (18)

In eq.(18), link costs are the sum of average cost and congestion toll (external delay costs). If the external delay costs are not considered, then the Link costs \(\tilde{t}_a(Q_a) = t_a(Q_a) + \lambda_a\) (19)

In eq. (19), link costs are the sum of average cost and environmental road pricing. These are similar to the link costs of UE problem with link emission constraints. It can be called as Environmental Road Pricing (ERP). This option is considered in the further analysis using hypothetical network in this paper.

4. SOLUTION ALGORITHM

The approximation curve is used for optimum approach solution. Many algorithms employ a strategy that converts constraint problem into a un-constraint problems through a penalized/dualization of the constraints, so that various existing efficient approaches for program can be applied for the solution. Augmented Lagrangian algorithm is one of the
efficient and locally convergent methods for the optimization problem with nonlinear constraints (Bertsekas, 1982, Yang Hai et.al, 2005). The discontinuity of the second derivatives of the sub problem of Augmented Lagrangian algorithm is not a serious inconvenient in practical computations and was the best one among the various methods (Birgin et.al, 2005).

Program \( (6-7) \) can be written as \( L_\rho(Q, \lambda) = \sum_{a \in A} Q_a T_a(Q_a) + \sum_{a \in A} \frac{1}{2\rho} \left[ \max \left\{ 0, \lambda_a + \rho(Q_a - Q^\pi_a) \right\} - \lambda^2_a \right] \)

Where \( \lambda \) is the vector of dual variables associated with emission constraints. Augmented Lagrangean dual algorithm can be summarized as below.

**Step0.** (Initialization) Initialize the feasible set of link flows \( Q^{(0)}_a \). Select the initial values for \( \lambda_a \), e.g., let \( \lambda^{(0)}_a = T_a(Q^{(0)}_a) \) if \( Q_a > Q^\pi_a \) and 0 otherwise for all links \( a \in A \). Herein, \( Q^{(0)}_a \) is the solution of the un-constrained emission traffic assignment problem. Here, the initial link flows can be selected in between zero and physical capacity. Choose \( \rho^0 > 0 \) and set the iteration counter \( n = 0 \). Initial \( \rho \) can be taken as 0.01.

**Step1.** Solve traffic assignment problem with link costs \( T_a(Q_a) + \max \left\{ 0, \lambda^{(n)}_a + \rho^{(n)}(Q_a - Q^\pi_a) \right\} \) for all links \( a \in A \), to get link flow pattern \( Q^{(n)}_a \). Frank-wolf Algorithm can be used for traffic assignment and standards error \( \varepsilon = \left( \sum_{a \in A} (Q_a^{(n+1)} - Q_a^{(n)})^2 \right) / \sum_{a \in A} Q_a^{(n)} \) can be taken as 0.000001. Here \( \rho \) is the iterative steps in assignment.

**Step2.** (Termination) if \( \max_{a \in A} \left| Q_a^{(n)} - Q^\pi_a \right| \lambda^{(n)}_a \right| < 0.0001 \), stop and accept \( (Q^{(n)}, \lambda^{(n)}) \) as the solution; otherwise, continue. Termination criteria check the slackness condition in terms of link flows and multiplier. Tolerance may vary as per desired degree of accuracy.

**Step 3.** (Updating multiplier and penalty) : Lagrange multipliers \( \lambda^{(n+1)} \) be updated based on the assigned flows in the previous iteration. Let \( \lambda^{(n+1)}_a = \max \left\{ 0, \lambda^{(n)}_a + \rho^{(n)}(Q_a^{(n)}(\rho, \lambda^{(n)}) - Q^\pi_a) \right\} \) for all links \( a \in A \). The idea of updating of penalty parameter is that the penalty parameter \( \rho \) will not be changed if the last inner iteration produced a significative improvement both of feasibility and complementarity.

\[
\rho^{(n+1)} = \begin{cases} 
\kappa \rho^{(n)}, & \text{if } \sum_{a \in A} \max \left\{ \frac{\lambda^{(n)}_a}{\rho^{(n)}}, Q^{(n)}_a - Q^\pi_a \right\} > \gamma \sum_{a \in A} \max \left\{ \frac{\lambda^{(n+1)}_a}{\rho^{(n+1)}}, Q^{(n+1)}_a - Q^\pi_a \right\} \\
\rho^{(n)}, & \text{otherwise}
\end{cases}
\]

Where \( 2 \leq \kappa \leq 10 \) and \( \gamma = 0.25 \) can be chosen. Set \( n = n + 1 \) and go to Step 1.

Program: \( \min L_\rho(Q, \lambda) \) subject to \( Q \in \Omega_Q \) is strictly convex, so all existing efficient algorithms for solving standard traffic assignment problems can be used in Step 1. Step 2 carries out a safe-guard termination based on infeasibility of environmental capacity (emission standard levels).

### 5. HYPOTHETICAL DATA ANALYSIS

Hypothetical data analysis aimed to illustrate the model and to compare the optimum pricings of both approximation and original curves and the applicability of approximation curve in general networks. Consider the hypothetical network as shown in Figure 3 with two OD pairs (AC & BC) with equal demands. A common link (5) also chosen to see the effect of link interactions on optimum tolls. Assume delay parameter \( J_D = 0.1 \), fuel cost = 1 price/lt, Value
of Time = 20 Price/hr, Mileage is 35km/lt at free flow speed 120 kmph. Consider link capacities (veh/hr)  
$C_1=4000$, $C_2=3000$, $C_3=3000$, $C_4=4000$ veh/hr, $C_5=3500$ and link lengths  
$L_1=5km$, $L_2=4km$, $L_3=4km$, $L_4=4km$, $L_5=3km$ respectively for link1, link2, link3, link4 and  
link5. The network routes are $R_1=L_1$, $R_2=L_2+L_5$ for the OD pair AC and $R_3=L_3+L_5$, $R_4=L_4$ for  
OD pair BC; Tolls are in price units.

![Figure 3: Hypothetical Network](image)

5.1 Comparison of Road Pricing
The road pricings of optimum approach using approximation curve by augmented Lagrange  
algorithm are compared with the optimum pricings of descriptive approach using original  
curve with standard UE model under the same conditions. The optimum tolls of  
approximation curve are obtained by the augmented lagrangean algorithm. The optimum tolls  
of descriptive approach are obtained by choosing the solution which is lowest system cost that  
satisfies the emissions constraints on all selected links in the combinatorial approach of  
varying link pricings and their corresponding link emissions. A suitable link price range of 0  
to 1 is considered with a stepwise increment of 0.05 in the illustration. So the actual optimum  
may differ in fractions less than 0.05. Table 1 shows the comparison of road pricings of both  
approaches for the above data for the speeds above 70 kmph on selected links where the  
difference between approximate curve and original curve is considerable. By comparing both  
the approaches link pricings in Table 1, it can be observed that the difference is considerable  
and the optimum approach under estimates the pricing when the network link speeds are  
higher above 70kmph. This is due to the differences in both the curves are more at higher  
speeds above 70kmph where the original curve has been modified to use in general networks  
for optimum approach. So, if the approximation curve is used in general networks for  
convexity at higher speeds above 70 kmph, the corresponding resulting emissions of original  
curve may exceed the standards sometimes in practice as there is considerable difference in  
both the approaches.

<table>
<thead>
<tr>
<th>Demand (veh/hr)</th>
<th>$E_1=1000, E_4=2000$</th>
<th>$E_1=1500, E_4=2000$</th>
<th>$E_1=1800, E_4=1800$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimum</td>
<td>Descriptive</td>
<td>Optimum</td>
</tr>
<tr>
<td>5500</td>
<td>$p_1=0.324$</td>
<td>$p_1=0.65$</td>
<td>$p_1=0$</td>
</tr>
<tr>
<td></td>
<td>$p_4=0$</td>
<td>$p_4=0.70$</td>
<td>$p_4=0$</td>
</tr>
<tr>
<td>6500</td>
<td>$p_1=0.367$</td>
<td>No sol*</td>
<td>$p_1=0.136$</td>
</tr>
<tr>
<td></td>
<td>$p_4=0$</td>
<td>$p_4=0.85$</td>
<td>$p_4=0$</td>
</tr>
</tbody>
</table>

*For the permissible emissions, the maximum possible flows are less than the OD demand. So there is no  
feasible solution for these emission standards at this demand using original curve with descriptive approach

Table 2 shows the comparison of road pricings of both approaches for the above data for the  
speeds below 70 kmph on selected links where the difference between approximate curve and
original curve is negligible. By comparing both the approaches link pricings in Table 2, it can be observed that the difference is very less and is due to the stepwise increment of 0.05 adopted in descriptive approach and also due to minor difference of curves below 70kmph. So, if the approximation curve is used in general networks for convexity at speeds below 70 kmph, the difference in both the pricings may be negligible and the corresponding resulting emissions of original curve may also be similar.

<table>
<thead>
<tr>
<th>Demand (veh/hr)</th>
<th>Optimum</th>
<th>Descriptive</th>
<th>Optimum</th>
<th>Descriptive</th>
<th>Optimum</th>
<th>Descriptive</th>
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<tr>
<td>10000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td></td>
<td>0.187</td>
<td>0.20</td>
<td>0.110</td>
<td>0.10</td>
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<td>0</td>
</tr>
<tr>
<td>10500</td>
<td>0.427</td>
<td>0.50</td>
<td>0.161</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.804</td>
<td>0.90</td>
<td>0.554</td>
<td>0.45</td>
<td>0.385</td>
<td>0.30</td>
</tr>
</tbody>
</table>

6. CONCLUSIONS

At higher speeds, beyond 70 kmph, there may be considerable difference in emissions between both the curves and hence in the optimum pricing. For these speeds and at low demands, the approximation curve may not have tolls where as the original curve may have considerable tolls. So, the pricing that satisfies the local emission constraints using approximation curve may not satisfy the constraints using original curves at higher speeds above 70kmph. At speeds below 70 kmph, approximation curve may be applied for general networks due to closeness of link emissions with the actual emissions of original curve. In practical situations, the emission problems may be high and concerned at lower speeds. The improvement of link flow-link emissions relationship may improve the solution at different speeds. Speed control mechanism may be another alternative to solve the emission problems. The optimum model with approximation curve may be used in most of the situations.

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