Coordination, supply chain optimization and warehouse location selection problem

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Abstract

In this research, an enterprise comprise vendor and buyer with price sensitive linear demand function is considered to find the optimal solution as well as best location for the warehouses. If there is no coordination exists between the members in a supply chain, vendor and buyer will act independently to maximize their own profit and reduce cost which does not guarantee the optimal state of the whole system. A single vendor-buyer with multiple products and consumers problem is formulated, and the coordination between vendor and buyer is emphasized to find not only the optimal network but also the best location for the buyer’s warehouse. Further, instead of constant demand function, a price-sensitive linear demand function is introduced to formulate the constraints. Finally, the formulated the Mixed Integer Problem (MIP) models are solved and hence the sensitivity on buyer’s selling price is analyzed to conclude this research.

Key Words: Warehouse location problem, coordination between vendor and buyer, mixed integer program.

1. INTRODUCTION

The coordination among the members of supply chain is one of the vital issues to overcome the new challenges of the comprehensive enterprise. Without coordination a supply chain system could not be optimal as a whole since each party always try to enhance his own profits only. That is why to ensure the optimal system and to satisfy customer demands in today’s competitive markets, significant information needs to be shared along the supply chain. And a high level of coordination between vendor’s and buyer’s decision making is also required. The concept of Joint Economic Lot Sizing (JELS) has been introduced to filter traditional methods for independent inventory control and to find a more profitable joint production and inventory policy.

The idea of optimizing the joint total cost in a single-vendor and a single-buyer model was first introduced by Goyal (1976). Then, Banerjee (1986) developed the model by incorporating a finite production rate and following a lot-for-lot policy for the vendor. By
relaxing Banerjee’s lot-for-lot assumption, Goyal (1988) proposed a more general joint economic lot-sizing model.

In addition, Viswanathan and Wang (2003) described the effectiveness of quantity discounts and volume discounts as a coordination mechanism in distribution channels with price sensitive demand. They concluded that the effectiveness of volume discounts as a coordination mechanism is higher when the sensitivity of demand to price changes higher and the effectiveness of quantity discounts is higher with lower price demand. In addition, Qin et al. (2007) have considered volume discounts and franchise fees as coordination mechanism in a system of supply chain with single supplier and single buyer with price sensitive demand. Subsequently, they showed that when demand is price sensitive, channel profits achieved by employing volume discounts and franchise fees is larger than achieved by quantity discounts and franchise fees.

Further, Pourakbar et al. (2007) described an integrated four-stage supply chain system, incorporating one supplier, multiple producers, multiple distributors and multiple retailers. Then they determined the optimal order quantity of each stage and shortage level of each stage to minimize the cost of the supply chain. Recently, Wu and Yen (2009) have provided some patch works to enhance the reliability of the integrated single-vendor single-buyer inventory model. Zavanella and Zanoni (2009) have investigated the Consignment Stock (CS) policy on Vendor-Managed Inventory (VMI) model and showed that CS policy is works better than the uncoordinated optimization and is implemented for a industrial case of a single-vendor and multiple-buyer production situation.

At the same time, Jokar and Sajadieh (2009) have described a vendor–buyer integrated production inventory model considering Joint Economic Lot Sizing (JELS) policy with price sensitive demand of the customer. In the meantime, Sajadieh and Jokar (2009) described a JELS model where the shipment; ordering and pricing policy are all optimized. Finally, they investigated the effectiveness of customer price sensitive demand.

Another stream of literate that deals with locational decision-making has led to a strong interest in location analysis and modeling within the operations research and management science community. Undoubtedly, humans have been analyzing the effectiveness of locational decisions since they inhabited their first cave. The term “facility” here is used in its broadest sense. That is, it is meant to include entities such as air and maritime ports, factories, warehouses, retail outlets, schools, hospitals, day-care centers, bus stops, subway stations, electronic switching centers, computer concentrators and terminals, rain gages, emergency warning sirens, and satellites, to name but a few that have been analyzed in the research literature. Holmberg (1999) proposed a solution method for uncapacitated facility location problems where the transportation costs are nonlinear and convex. He developed a branch and bound method based on dual ascent and adjustment procedure and compared to application of a modified Benders decomposition method.

Consequently, Teo and Shu (2004) studied a distribution network design problem integrating transportation and infinite horizon multi-echelon inventory cost function. They formulated a set-partitioning integer-programming model and solved by the column generation algorithm which arises pricing sub-problem. Finally, they showed that price sub-problem is NP-complete. Ko (2005) described an integrated decision model to determine the location of distribution faculties. He proposed the hierarchy process and analyzed the survey data based on the location selection criteria and demonstrated the practical applicability of the research findings. Eroglu and Keskintürk investigated a model for locating an economic facility to
determine how many warehouse to set up, where to locate those warehouses. They proposed a genetic algorithm for the warehouse location problem and cities of Turkey are considered where the warehouses construct for minimizing the distribution costs. Sheng et al. (2006) have investigated a transportation model with discontinuous piecewise linear cost function and proposed a genetic algorithm based on the matrix encoding which more efficient than previous algorithm.

For the sake of this study, combining the price sensitive demand and the coordination between the members of supply chain, buyer’s MIP, vendor’s Integer program(IP) and coordinated MIP based models are formulated. A linear price sensitive demand function is considered to formulate the constraints and the sensitivity on the buyer’s selling prices are discussed. The goal of this work is to determine the individual and coordinated profit with the best location for buyer’s new warehouse. Numerical examples presented in this research, which include the sensitivity of the key parameters to illustrate the models. Eventually, the formulated models are solved to obtain not only the optimal enterprise but also the best location for the buyer’s warehouse.

The reminder of this paper is organized as follows. In Section 2, mathematical models are formulated as MIP. In Section 3, a numerical example with solution procedure is provided. In Section 4, the results of these models are discussed. Finally, Section 5 contains some conclusions and scopes of future research.

2. MODEL FORMULATION

It is considered an enterprise consists of single vendor and single buyer with a set of locations candidate for feasible warehouse. The vendor manufactures products and delivers to buyer’s warehouses and then buyer delivers the products to the customer located at different geographical areas as follows:

![Framework of the model](image.png)

Parameters:
Let
I: set of products
J: set of customers
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L: set of location for the warehouses
s_i: buyer’s purchasing price of i-th product ($/unit)
w_l: the maximum capacity for i-th product at warehouse l (unit)
f_l: vendor’s set up cost ($)
h_i: vendor’s holding cost of i-th product ($/unit)
t_{il}: unit transportation cost of i-th product from vendor to warehouse l ($/unit)
p: production rate
fw_l: buyer’s set up cost of l-th warehouse ($)
ht_l: transportation unit cost from warehouse l to customer j ($/unit)
hh_l: holding cost at buyers warehouse l for product i ($/unit)
M: any large scalar
b= (b_i): a constant vector, i in products set
A = (a_{ij}): a matrix of appropriate dimensions, (i,j) are in products set

Decision Variables:
c_i: buyer’s selling price of i-th product ($/unit)

\[ x_l = \begin{cases} 
1, & \text{if warehouse } l \text{ is open,} \\
0, & \text{otherwise} 
\end{cases} \]

n_i: number of shipment of i-th product (unit)
Q_{li}: ordered quantity of i-th product for warehouse l (unit)
z_{lij}: amount of i-th product shipped from warehouse l to customer j (unit)
d_{lj}: the demand of i-th product to j-th customer (unit)
w_{li}': the optimal capacity for i-th product at location l

2.1 Prerequisites

Linear demand function:
Let us consider the linear demand system of the form \( d(p) = b - Ap \), where d and p are the demand and price vectors respectively, b is a constant vector and A is a matrix of appropriate dimensions. By the law of demand, the demand \( d_i \) decreases in its own price \( p_i \). Suppose that a vendor produces m product for n customers, then linear demand could be defined as:

\[ d : R^m \rightarrow R^n, \text{ where, } R^m = \{ p \in R^m : p \geq 0 \}. \]

Since d is linear, it can be restated as follows:

\[ d(p) = b - Ap, \text{ where, } b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix} \text{ and } A = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mm} \end{pmatrix} \]

The diagonal entries \( a_{ii} \) of A are the decrement in demand for product i when the price \( p_i \) increases by one unit, and the (i,j) entries \( a_{ij} \), \( i \neq j \) of A represent the change in the demand for product i as the prices \( p_j \) change.

Assumption: Since demands d are non negation, hence, \( b \geq Ap \).

Buyer’s total revenue:
The total revenue of buyer’s is obtained by multiplying the difference of selling price and purchasing cost of each product by the amount of demand of that product. It is assumed that the selling price for a product is not varying from customer to customer depending on the location of the customer.
That is, Buyer’s total revenue = \(\sum_{i}^{m} (b_{i} - \sum_{j=1}^{m} a_{ij}c_{j})(c_{i} - s_{i})\)

Buyer’s total cost:
Buyer’s total cost is the combination of fixed opening cost, holding cost and transportation cost that are briefly discussed as follows:

Opening cost:
Buyer’s fixed opening concern with the land acquisition costs, order processing costs, facility construction costs. For this model, it is assumed that warehouse facilities are to be located on an existing set of potential locations. These potential locations may or may not have a fully functional facility. Moreover, the existing facility may not have sufficient capacities to meet the demand requirements. In this model, the costs related to these constructions or expansion activities are considered to the fixed opening cost which is equal to

\[\sum_{l=1}^{L} \sum_{i=1}^{m} (b_{i} - \sum_{j=1}^{m} a_{ij}c_{j})f_{wl}x_{l} / Q_{li},\]

where \(f_{wl}\) is the fixed set up costs of the potential location of warehouse \(l\) and \(x_{l}\) is the binary variables and \(Q_{li}\) is the ordered quantity.

Buyer’s holding cost:
It is the cost associated with storing the product inventory until it is shipped to the customer for a given unit of time. It may also include the cost of insurance and other factors that are proportional to the amount stored in inventory. The inventory holding cost for buyers is equal to

\[\sum_{i=1}^{L} \sum_{i=1}^{m} Q_{li} h_{li} / 2,\]

where \(Q_{li}\) is the ordered quantity and \(h_{li}\) is the holding cost of per unit product.

Finally, the transportation cost is the cost of shipment per unit product from warehouse to customer. It is assumed that travel time is linearly proportional to the distance between the warehouse and customer which may vary to each warehouse-customer pair depending on the route condition, climate condition, geographical condition etc. Total transportation cost is obtained as follows:

\[\sum_{l=1}^{L} \sum_{i=1}^{m} \sum_{j=1}^{n} t_{lj} \times d_{ji},\]

where \(t_{lj}\) is the unit cost of transportation and \(d_{ji}\) is the demand of the \(i^{th}\) product to \(j^{th}\) customer.

Vendor’s revenue:
After knowing the buyer’s order quantity, vendor’s revenue is obtained by the multiplication of the selling price and demand quantity. It is assumed that vendor’s selling price, \(s_{i}\) is fixed for each product \(i\). Therefore,

\[\text{Vendor’s total revenue} = \sum_{i=1}^{m} (b_{i} - \sum_{j=1}^{m} a_{ij}c_{j})s_{i}\]

Vendor’s cost:
The total cost of vendor is required to satisfy order quantity of buyer as well as customer’s demand for all products. In this model, opening cost, holding cost and transportation cost are considered as vendor’s costs which are briefly discussed as follows:

Opening cost:
Vendor’s fixed opening cost is the cost included the costs such as, land acquisition costs, facility construction costs input cost and manufacturing cost. In this model, it is assumed that
vendor opening cost is fixed is equal to \( \sum_{i=1}^{L} \sum_{i=1}^{m} (b_i - \sum_{j=1}^{m} a_{ij} c_j) * f_v / Q_{hi} * n_i \), where \( f_v \) is vendor’s fixed setup cost and \( n_i \) is the number of shipments of \( i^{th} \) product.

**Holding cost:**
Vendor’s holding cost is the cost associated with storing the product inventory until it is shipped to the buyer’s warehouse. According to Sajadieh and Jokar (2009), the vendor total inventory holding cost could be obtained as follows:

\[
\sum_{i=1}^{L} \sum_{i=1}^{m} (h_i * Q_{hi}) / 2(n_i(1 - \sum_{j=1}^{m} (b_i - \sum_{j=1}^{m} a_{ij} c_j)) / p - 1 + 2 \sum_{i=1}^{m} (b_i - \sum_{j=1}^{m} a_{ij} c_j) / p),
\]

where \( p \) is the production rate and \( n_i \) is the number of shipment of \( i^{th} \) product.

Lastly, the transportation cost is the cost of shipment per unit product from manufacturer to the warehouse. It is assumed that travel time is linearly proportional to the distance between the manufacturer and warehouse which may also vary to each manufacturer-warehouse pair depending on the route condition, climate condition, geographical condition etc. Transportation cost is obtained as follows:

\[
\sum_{i=1}^{L} \sum_{i=1}^{m} t_{li} * Q_{hi}, \text{ where } t_{li} \text{ is the unit cost of transportation and } Q_{hi} \text{ is the ordered quantity of the } i^{th} \text{ product for } l^{th} \text{ location of warehouse.}
\]

**Optimal shipment:**
Vendor’s optimality condition for shipment of each product can be stated as follows (see Sajadieh and Jokar, 2009):

\[
n_i(n_i - 1) = \sum_{j=1}^{m} 2(b_i - a_{ij} c_j) p f_v / \sum_{i=1}^{L} h_i Q_{hi}^2 (p - b_i + a_{ij} c_j) <= n_i (n_i + 1), \forall i \in \text{ products}
\]

**Joint Revenue:**
Suppose that both party agreed to cooperate with each other by the jointly optimal integrated policy. Then total revenue = \( \sum_{i=1}^{m} (b_i - \sum_{j=1}^{m} a_{ij} c_j) c_i \)

**Optimal shipment after coordination:**
The optimum number of shipment of each product can be obtained by the following equation (see Sajadieh and Jokar, 2009):

\[
n_i = \sqrt{\sum_{i=1}^{L} f_v (p h_i Q_{hi} - h_i) / \sum_{i=1}^{L} h_i Q_{hi}^2 (p - b_i + \sum_{j=1}^{m} a_{ij} c_j)}, \forall i \in \text{ products}
\]

The remaining costs concerning terms are the combination of the costs of both vendor and buyer which are described earlier.

**2.2 Buyer’s model**

**MIP1:**
Maximize,

\[
P_d(c_i, Q_{hi}, x_i, z_{ji}) = \sum_{i=1}^{m} (b_i - \sum_{j=1}^{m} a_{ij} c_j)(c_i - s_i) - \sum_{i=1}^{m} (b_i - \sum_{j=1}^{m} a_{ij} c_j) \sum_{i=1}^{L} (f_v x_i / Q_{hi}) - \sum_{i=1}^{m} \sum_{j=1}^{m} \sum_{i=1}^{m} (Q_{hi} h_i Q_{hi} / 2 + \eta g_d Q_{hi} d_{ji})
\]
Subject to
\[ b_i \geq \sum_{j=1}^{m} a_{ij} c_j, \forall i \in \text{product set} \] (1)
\[ \sum_{j=1}^{m} d_{ji} = (b_i - \sum_{i=1}^{m} a_{ij} c_j), \forall i \] (2)
\[ \sum_{i=1}^{l} \sum_{j=1}^{m} z_{iji} = \sum_{i=1}^{l} d_{ji}, \forall j \] (3)
\[ \sum_{i=1}^{l} z_{iji} = d_{ji}, \forall i, j \] (4)
\[ \sum_{i=1}^{l} \sum_{j=1}^{m} z_{ijl} = \sum_{i=1}^{l} d_{ji}, \forall i \] (5)
\[ \sum_{j=1}^{l} \sum_{i=1}^{m} z_{ijl} \leq M x_{jl}, \forall l \] (6)
\[ \sum_{l=1}^{L} Q_{il} = (b_i - \sum_{i=1}^{m} a_{ij} c_j), \forall i \] (7)
\[ \sum_{l=1}^{L} Q_{il} = \sum_{l=1}^{L} (b_i - \sum_{i=1}^{m} a_{ij} c_j) \] (8)
\[ \sum_{l=1}^{L} w'_{il} \geq (b_i - \sum_{i=1}^{m} a_{ij} c_j), \forall i \] (9)
\[ \sum_{l=1}^{L} w'_{il} * x_i \leq \sum_{l=1}^{L} w_{il}, \forall i \] (10)
\[ s_i, w_i, w'_i, f w_i, d_{ji}, \mu_{ij}, \phi_{ij}, \psi_i, z_{ijl}, Q_{il} \geq 0, \ x_i \text{ is binary} \] (11)

The aim of the MIP1 model is to optimize the profit and to find the feasible locations for the warehouses before coordination. The constraints (1) make sure that the linear demands are non negative. The constraints (2) represent the demand of each product. The constraints (3) ensure that the shipment amount satisfy the demand of each customer. The constraints (4) ensure that the total amount of specific product being stored for a particular customer at all warehouses is equal to the demand of the product from that customer. The constraints (5) provide that the total amount of a particular product being stored at all warehouses for all customers is equal to the total demand of that product. The constraints (6) describe that a warehouse is located if and only if there is a demand. The constraints (7) show that the demand of each product is equal to the order quantity of that product. The constraints (8) provide the relation between total ordered quantities of the products and total demand of the products at all locations. The constraints (9) make sure that the capacity of each product is greater than the demand of that product. The constraints (10) stipulate that required capacities cannot exceed the existing capacities for each product. Finally, the constraints (11) guarantee the non-negativity.

2.3 Vendor’s model

IP2: Maximize,
\[ P_v (n_i) = \sum_{j=1}^{m} \sum_{i=1}^{l} d_{ji} * s_i - \sum_{i=1}^{l} \sum_{j=1}^{m} (b_i - \sum_{i=1}^{m} a_{ij} c_j) * f_v / Q_{il} * n_i - \sum_{j=1}^{l} \sum_{i=1}^{m} (h_i * Q_{il}) / 2(n_i - \sum_{i=1}^{l} (b_i - \sum_{j=1}^{m} a_{ij} c_j)) / p - 1 + 2 \sum_{i=1}^{l} (b_i - \sum_{j=1}^{m} a_{ij} c_j) / p + t_{il} * d_{ji} \]
Subject to

\[ n_i (n_i - 1) \leq \sum_{j=1}^{m} 2(b_i - a_i c_j) p f_v / \sum_{l=1}^{L} (h_l Q_{l_i})^2 (p - b_i + a_j c_j) \]  
(12)

\[ \leq n_i (n_i + 1), \forall i \in \text{products} \]

\[ \sum_{j=1}^{n} d_{ji} = \sum_{j=1}^{m} (b_i - \sum_{j=1}^{m} a_j c_j), \forall i \]

\[ s_i, d_{ji}, f_v, t_{li}, h_i \geq 0 \text{ and } n_i \text{ are integer} \]
(14)

The above IP2 model also optimizes the profit of the vendor before coordination. The constraints (12) stipulate the optimality condition of the number of shipment of each product. The Constraints (13) represent the demand of each product by the customer. The final constraints (14) guarantee the non-negativity.

### 2.4 Coordination Model

If both vendor and buyer agree on coordination between themselves, then the coordinated model could be formulated as following MIP problem

**MIP3: Maximize,**

\[ P_{AC}(c_i, Q_{l_i}, n_i, x_i, z_{iji}) = \sum_{j=1}^{m} (b_i - \sum_{j=1}^{m} a_j c_j) c_i - \sum_{j=1}^{m} (b_i - \sum_{j=1}^{m} a_j c_j) \sum_{l=1}^{L} ((f w_i x_i + n_i f_v) / Q_{l_i} n_i) \]

\[ - \sum_{l=1}^{L} \sum_{i=1}^{m} \sum_{j=1}^{m} (Q_{l_i} h_i / 2 + t_{ji} z_{iji}) - \sum_{l=1}^{L} \sum_{i=1}^{m} (h_i * Q_{l_i} / 2(n_i (1 - \sum_{j=1}^{m} (b_i - \sum_{j=1}^{m} a_j c_j) / p - 1 + 2)) \sum_{j=1}^{m} (b_i - \sum_{j=1}^{m} a_j c_j) / p + t_{ji} * Q_{l_i}) \]  

Subject to

\[ b_i \geq \sum_{j=1}^{m} a_j c_j, \forall i \in \text{product set} \]
(15)

\[ \sum_{j=1}^{n} d_{ji} = (b_i - \sum_{j=1}^{m} a_j c_j), \forall i \]
(16)

\[ \sum_{l=1}^{L} \sum_{i=1}^{m} z_{iji} = \sum_{j=1}^{n} d_{ji}, \forall j \]
(17)

\[ \sum_{l=1}^{L} z_{iji} = d_{ji}, \forall i, j \]
(18)

\[ \sum_{l=1}^{L} \sum_{ij} z_{iji} = \sum_{j=1}^{n} d_{ji}, \forall i \]
(19)

\[ \sum_{l=1}^{L} \sum_{ij} z_{iji} \leq M x_i, \forall l \]
(20)

\[ \sum_{l=1}^{L} Q_{l_i} = (b_i - \sum_{j=1}^{m} a_j c_j), \forall i \]
(21)

\[ \sum_{l=1}^{L} Q_{l_i} = \sum_{i=1}^{m} (b_i - \sum_{j=1}^{m} a_j c_j) \]
(22)

\[ \sum_{l=1}^{L} w'_{ji} \geq (b_i - \sum_{j=1}^{m} a_j c_j), \forall i \]
(23)
\[
\sum_{i=1}^{l} w_{ji}^* x_i \leq \sum_{i=1}^{l} w_{ji}, \forall i
\]
\[
Q_{hi}^* x_i \leq w_{ji}, \forall l, i
\]
\[
\sum_{i=1}^{l} f_i (p(hh_{hi} - h_i) + 2h_i (b_i - \sum_{j=1}^{m} a_{ji} c_j)) - \sum_{i=1}^{l} h_i f_{w_i} (p - b_i + \sum_{j=1}^{m} a_{ji} c_j)
\]
\[
s_i, w_{ji}^*, f_{vi}, t_{hi}, h_i, s_i, w_{hi}, f_{w_i}, d_{ji}, Q_{hi}, n_{ij}, hh_{hi}, t_{hi}, c_i, z_{ji} \geq 0, \ x_i \text{ is binary}
\]

Where, the constraints (15)-(27) are the combination of the constraints of vendor’s and buyer’s models.

3. NUMERICAL EXAMPLE AND SOLUTION PROCEDURE

Suppose that a vendor-X which manufactures the 3 products in batches at a finite yearly rate 3200 with a selling prices 100, 105 and 100, and delivers in equal-sized transfer lots to the buyer-Y having a set containing 5 feasible locations for warehouses with fixed opening cost (4500, 3500, 4000, 3000, 2500) and maximum capacities are (300, 200, 300), (120, 800, 200), (150, 200, 170), (100, 100, 100) and (500, 200, 500). Buyer-Y satisfies his two customers with a price-sensitive demand having constant vector b and appropriate matrix
\[
\begin{pmatrix}
2 & -3 & 2 \\
1 & 4 & -1 \\
-1 & 2 & 1
\end{pmatrix}
\]

In order to obtain the sensitivity of linear demand function, it is considered three set of constant vectors of b such as (1030 1080 980), (1050 1100 1000) and (1070 1120 1020). Further, each warehouse has a yearly holding cost (1.0, 1.2, 1.2), (1.3, 1.2, 1.3), (1.3, 1.4, 1.2), (1.5, 1.4, 1.4), (1.2, 1.3, 1.2) and the transportation cost for two customer are (1.5, 1.6, 1.6, 1.0, 1.5) and (1.7, 1.4, 1.5, 1.2, 1.5). Finally, vendor’s transportation cost for three products are (0.5, 0.6,0.6,0.5,0.5), (0.7,0.4,0.5,0.4,0.5) and (0.6,0.5,0.5, 0.45,0.5).

The aim of this work is to provide a consistent logistics support to the buyer-Y as well as to find the best feasible locations for the warehouses among the given set of locations which optimize the supply chain network.

The formulated buyer’s MIP1 model has been solved by the method of branch and bound algorithm by using AMPL with Bonmin and Couenne. Further, the vendor’s IP2 has been solved by the method of branch and bound algorithm by using Cplex. Finally, the joint model MIP3 has been solved by the method of branch and bound algorithm by using AMPL with Bonmin and Couenne. A program has been written to represent the objective function and the constraints. The program consists of two main parts; the main module containing the actual program and the data file containing data for the various parameters. The program was executed on a Pentium IV machine with a 3.0 GHz processor and 1.0 GB RAM.
4. RESULT ANALYSIS

Significant finding regarding the numerical example of the proposed model is described in table-1. Table-1 also provides the comparative analysis for the model before and after coordination. The percentage of change of profit after coordination is obtained by

\[ PI(\%) = \left( \frac{P_{AC} - P_{BC}}{P_{BC}} \right) \times 100 / P_{BC} \]

where, \( P_{BC} \) and \( P_{AC} \) are the profit before and after coordination between vendor and buyer. The individual benefit of vendor and buyer is calculated from the formula described by (Sajadieh and Jokar, 2009; Goyal, 1976), that is:

\[ P_{V}(\text{After coordination}) = \frac{P_{V}}{P_{BC}} \times P_{AC} \]

and

\[ P_{B}(\text{After coordination}) = \frac{P_{B}}{P_{BC}} \times P_{AC} \]

where, \( P_{V} \) and \( P_{B} \) stand for profit of vendor and buyer.

Table 1 Comparative analysis of the results before and after coordination

<table>
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<tr>
<th>Cases</th>
<th>Prices</th>
<th>( P_{V} )</th>
<th>( P_{B} )</th>
<th>( P_{BC} )</th>
<th>Prices</th>
<th>( P_{V} )</th>
<th>( P_{B} )</th>
<th>( P_{AC} )</th>
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<td>8904</td>
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<td>135291</td>
<td>20135</td>
<td>155426</td>
<td>22.93</td>
</tr>
</tbody>
</table>

The following figure shows the distribution of different products for different customers before coordination for the three cases:

![Figure 2 Optimum allocation for case-1 (before coordination)](image-url)
The following figure shows the distribution of different products for different customers after coordination for all the cases.
Therefore, from the figure-2, 3 and 4, it is concluded that if both buyer and vendor have no coordination, the binary values (1 0 0 1 1), (0 0 1 1 1) and (1 0 1 0 1) represent the feasible location sets for the warehouses. On the contrary, after coordination, for the three cases, the binary values (1 0 0 1 1) represent the feasible location set for the warehouses. Moreover, the optimal distributions of the three products for two customers are also remained identical which is depicted in figure 5. However, before coordination the optimal number of shipments for the three cases are (67 80 43), (66 77 42) and (66 76 42). Whereas, after coordination the optimal numbers of shipment in all cases become (5, 4, 6). In fact, from the above figure-2, 3, 4 and 5, it is summarized that if both buyer and vendor have no coordination, then buyer’s selling prices can manipulate the optimal solution though after coordination the effectiveness of buyer’s selling prices could be reduced.

The following figure shows the price and reduction of price of product-1 after coordination for different cases.

![Figure 6 Price reduction of product-3](image)

The following figure shows the price and reduction of price of product-2 after coordination for different cases.

![Figure 7 Price reduction of product-3](image)

The following figure shows the price and reduction of price of product-3 after coordination for different cases.
The following graph demonstrates the effect of sensitivity of buyer’s selling price on total profit and profit enhancement before and after coordination:

Indeed, figure-6, 7 and 8 show that all the cases, the prices of different products after coordination is smaller than the prices of before coordination. Consequently, at least 5% of the consumer purchasing price as well as buyer’s selling price could be reduced by coordination. In addition, figure-9 shows that the profit after coordination always remains higher than before coordination. The most of the cases the percentage of profit increment is mounting, in particular, for this example, it lies between 9% and 23%. Consequently, after coordination the profit of buyer and vendor could be increased as well as the purchasing price of consumer could also be reduced which is the significance advantages of the coordinated supply chain.
5. CONCLUSION

In this research, a two echelon-supply chain network of single vendor-buyer is investigated. This study is combined the coordination mechanism among the members of supply chain and warehouse location problem so that the model could achieve the optimum solution as well as select the best feasible locations for the warehouses. Using a multiproduct and multi-customer linear price-sensitive demand function, the nonlinear MIP models are formulated and hence solved the models using appropriate software (AMPL, Bonmin, Couenne, Matlab). Some of the significance findings of this research are as follows:

Firstly, it is observed that after coordination the individual profits could be increased without any extra investment. In the same way, coordination among the member of an enterprise could reduce the consumer purchasing price as well as buyer selling price. Besides this, it is observed that after coordination the consumer allocation of different products from different warehouses is remained identical though it changes before coordination. Further, before coordination the model is achieved different set of locations but after coordination the model achieved the same set of locations for buyer’s warehouse though the price sensitive demand is changing. Above all, it could be concluded that coordination among the members of an enterprise will be more beneficial in the current competitive environments.

Some future research might be of interest. One of the future research scopes is to apply multi vendor-buyer policy. Sensitivity of different inventory policies for coordinated supply chain is also planned. Further, analysis of the lead-time effects and batches of different sizes are also proposed.

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